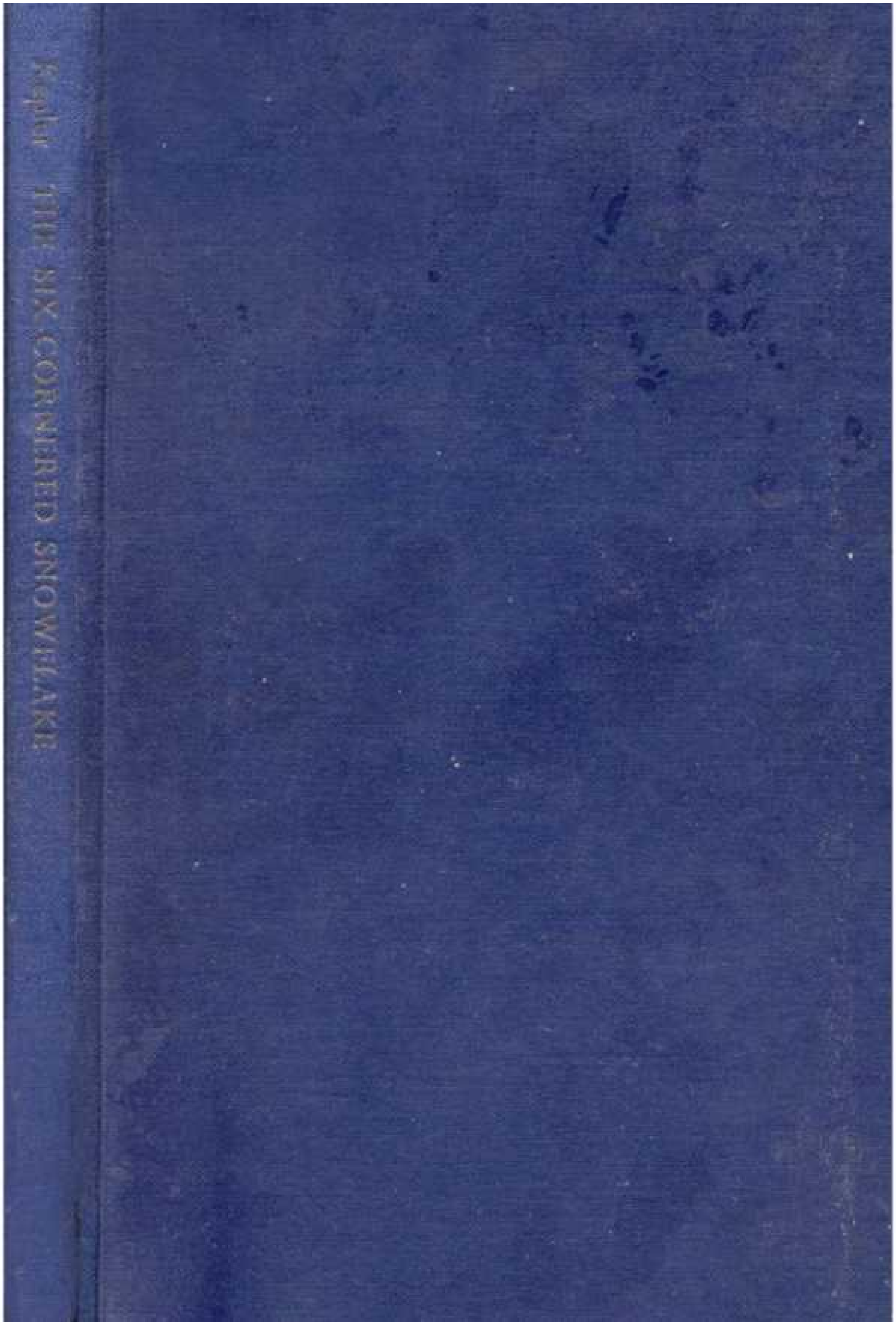

THE SIX-CORNERED SNOWFLAKE

THE SIX-CORNERED SNOWFLAKE

JOHANNES KEPLER

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FOREWORD

KEPLER's Latin booklet of 1611, *A New Year's Gift, or On the six-cornered Snowflake*, is here presented in an English translation for the first time. Though it was known to scholars in the seventeenth century, it does not appear to have been appreciated again until 1907 when the first German translation appeared. Yet in several respects it is a document of outstanding interest.

To the historian of ideas it displays in compact form the antithesis between the medieval outlook and the new mathematical method. The reader of these pages experiences these loyalties competing in Kepler's mind as he pondered on the problem, why snowflakes are hexagonal, two centuries before the first successful steps were taken towards its solution. Yet in this essay the objective scientific aim is dominant. It is as though Kepler, in leaving astronomy, where his temperament and ambitions were deeply involved, to meditate on the snowflake, were able to give freer expression to his desire for objective mathematical solutions of special problems. There is here less of the mystical–emotional elements which colour the *Harmonice*, for example, and though Kepler considers his much-loved regular solids he discards them without a pang as irrelevant to the plane hexagons of his snowflakes.

To the historically oriented physicist and mathematician Kepler's essay provides the first published evidence, in diagrams as well as text, of the ideas of regular arrangements and close-packing which have proved fundamental to crystallography, although it seems that about 1600 Thomas Harriot studied the close-packing of spherical atoms.

This would be enough to justify attention to this gem of disciplined speculation on the part of one of the greatest figures of exact science. But these retrospective features of the *New Year's Gift* are supplemented by another, which points ahead. *This essay is the first recorded step towards a mathematical theory of the genesis of inorganic or organic forms*, a theory which still lies in the future. In 1610 Kepler recognized that the apparently perfect hexagonal form of a few snowflakes that fell on his coat one day in Prague presented a challenge to the new mathematical science that was struggling to birth in his own mind: Why six? What was the physical cause of the six? What principle selected six from the other possible numbers? Moreover he regarded this as a special case of the general problem of the genesis of forms. In effect Kepler challenged those who followed him to discover the mathematics of the emergence of visible forms in crystals, plants, and animals.

Water has long been regarded as the basis of much that happens in this universe and the snowflake is now recognized as an important clue to the shaping agencies of nature, both in the formation of perfect micro-structures and in the formative and destructive power of glaciers and thunderstorms. The interest in snow is old. 'For the Lord spake unto Job:—Hast thou entered into the treasures of the snow? Out of whose womb came the ice?' In the second century B.C., a Chinese scholar, Han Ying, observed that while plants often had five petals, snow starlets possessed six rays.

The purpose of this volume is to display against its historical background the historical, literary, scientific, and philosophical treasures of Kepler's *New Year's Gift*, but above all its scientific intent. For beneath its humour and allusive style, and apparently casual repetitiveness, it displays a scientific judgement of the highest calibre. Kepler recognizes a genuine problem, discusses several alternative solutions, rejects them all, and passes the problem *to the chemists* for them to solve in the future! It is striking to see all this in a mind still attached to much in the medieval tradition, and compelled by his failure to answer his own question to fall back on a universal *facultas formatrix* which lay beyond his mathematical understanding.

For the convenience of the reader the book opens with a Synopsis, with references to the pages of the 1611 Latin text.

There follows the modernized text of the 1611 Latin edition, with an English translation by Mr. Colin Hardie of Magdalen College, Oxford, on the opposite pages.

A single series of notes, printed at the end and indicated by superior numerals in the Latin and English texts, covers (i) textual matters (C. H.); (ii) historical and literary references (mainly C. H.); and (iii) mathematical and physical questions, often by reference to the following two essays (L. L. W.).

Professor B. J. Mason discusses the scientific meaning and validity of Kepler's arguments, and their relation to the history of crystallography and of space-filling. This is supplemented by my examination of Kepler's *facultas formatrix* in relation to the history of philosophical and scientific ideas on the genesis of forms, illustrated by the history of the 'problem of the six' from 1611 to 1962.

The Bibliography gives information on editions, German translations, and commentaries. No attempt is made to refer to the vast literature on snow and ice.

The three authors have worked in close collaboration, and wish to express their indebtedness to the German commentators and to the translation by Strunz and Borm.

I wish also to thank Professor Cyril S. Smith of M.I.T. for calling my attention to Kepler's *New Year's Gift*.

L. L. W.

ACKNOWLEDGEMENTS

WE offer our thanks to Dr. Martha List, Pforzheimerstrasse 3, Weil der Stadt, Germany, for information about the German translations, and the loan of one of them, to Professor Cyril S. Smith of the Massachusetts Institute of Technology for information and advice, to Professor Dorothy Hodgkin, Somerville College, Oxford, for advice and encouragement, and to the Bodleian Library, Oxford, for permission to reproduce the text of its copy of the 1611 edition of Kepler's book.

CONTENTS

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SYNOPSIS

Kepler's pages | *Pages of English Version*

3 KEPLER wishes to offer his benefactor, Counsellor Wackher, 3
who is 'a devotee of Nothing', a gift which shall itself be practically
nothing. One of Epicurus' atoms would not do, for that would be
nothing at all. So he reviews the Elements.

4 Earth, Fire, Air, and Water are each considered, but rejected 5
as unsuitable. So are small animals, such as the mite.

5 But by a happy chance some specks of snow fell on his coat. 7
Here was something smaller than any drop, yet with a pattern.
This was the ideal New Year's gift for a mathematician to give:
it comes from heaven and looks like a star.

Socrates considered how far a flea can jump; Kepler's problem
is why snowflakes have six corners. He imagines that the Psalmist
probably observed these little stars of snow on the fleeces of his
sheep.

Now to business. There must be a cause why snow has the
shape of a six-cornered starlet. It cannot be chance. Why always
six? The cause is not to be looked for in the material, for vapour
6 is formless and flows, but in an agent. 9

What is that agent? An immanent form, or an efficient cause
acting from outside? Did it stamp the shape on the stuff as the
stuff demanded, or out of its own nature as expressing either the
beauty of the hexagon or the purpose it subserves?

Some examples, considered geometrically, may help to a clear
decision.

Honeycombs are built on a six-cornered plan, each cell surrounded
by six others, as a close-packing of rhomboids. Each cell
shares six side-walls with six neighbours, and three end-planes with
three other cells.

7 Can any quasi-regular solid be constituted of such rhombi? 11
This can be done in two ways. These are analysed, and compared
with the cube. The first provides a space-filling solid, rather similar
to the rhomboid cells of the honeycomb.

8 The pomegranate is formed of rhomboid-shaped loculi (cells). 13
What is the cause of their rhomboid shape? It cannot be the
material. In the pomegranate one cause must lie in the soul of
the plant, but this is not sufficient, and is assisted by material

9 necessity, such as close-packing. These planes may be stacked one 15
above the other in two manners, each pellet resting either directly
10 on one pellet below, or on a trio or quartet of pellets. Thus 17
different types of close-packing are obtained.

One of these arrangements explains the form of the pome-
granate, but bees, in making their combs, are oriented in one direc-
tion, and follow an instinct which has nothing to do with the wax,
or with the processes of growth.

11 But what purpose had God in putting these canons of architec- 19
ture into the bees? Three possibilities can be imagined. The
hexagon is the roomiest of the three plane-filling figures (triangle,
square, hexagon); the hexagon best suits the tender bodies of the
bees; also labour is saved in making walls which are shared by
two; labour would be wasted in making circular cells with gaps.
These reasons are enough; Kepler need not speak of the beauty
of the rhomboid or of the nature of the diminutive soul of the bee.

In the case of the pomegranate it is also unnecessary to look
beyond the material necessity which we have recognized, and to
consider the inner life-principle which guides it.

12 Yet where plants have fivefold patterns, a consideration of their 21
souls is in place.

For patterns of five appear in the regular solids, and so involve
the ratio called the golden section, which results from a self-
developing series that is an image of the faculty of propagation in
plants. Thus the flower carries the authentic flag of this faculty,
the pentagon. But all this is only to prepare us for the snowflake.

The first external cause to consider is the cold, which by
producing condensation may seem to be the cause of the star-like
13 shape. Yet cold could not discriminate so as to produce six-sided 23
forms with symmetrical radii. In Turkish baths the meeting of
steam and cold air produces great stripes on broken windows, but
this does not help us with the snowflake. Why plumes at six
points? What is the origin of the number six? Who carved the
14 nucleus, before it fell, into six horns of ice? The cold cannot do 25
this; it must be some internal cause related to the vapour.

But why are the prongs not spherically distributed, rather than
15 in a plane? Indeed are these stars not perhaps three feathered 27
diameters uniformly arranged on a sphere? This idea is tried out.
But why is the condensation on particular radii, rather than on
a perfect sphere? Here we are trying to coax truth from the com-
parison of false trails.

Packing of balls of vapour produces patterns: say square on a
plane and cubic in a solid, and such patterns may guide the con-
densing vapour. But what makes the balls choose one pattern,
rather than another? Cold may play a part, by penetrating the
gaps and producing condensation.

16 The problem is complex: it may be a question of the total 29
regimentation of the material *en masse*, or of the internal organiza-
tion of each ball, or of both. There are arguments pointing to the
shaping of each drop by itself.

17 Thus far we have made no progress. The possibly three- 31
dimensional character of the snowflake is considered and con-
trasted with the undoubtedly basic role of three dimensions in
animals, in relation to habitat, gravity, and an unobstructed view.

18 Sorting his ideas out, Kepler concludes that the cause of the 33
hexagonal shape of the snowflake is the same as that of shapes of
plants and of the numerical constants, nothing being random but
all guided by a Supreme Reason.

There is, in fact, a *facultas formatrix* (shaping art, formative
faculty) in the Earth, carried in vapour, so that all vapour is de-
termined by a formative principle.

One might argue that a purpose is at work in the formation of
natural bodies and plants. Yet no purpose can be observed in the
particular shape of a snowflake; for example, it does not con-
tribute to its stability.

Kepler's answer is that the form-creating reason does not act
only for a purpose, but sometimes simply to adorn. It may play
with the passing moment. Even heat, Kepler believes, maintains
a certain order of its own, distributing its outposts in good array
19 along certain lines of battle. It takes care 'not to fall in an ugly 35
or immodest fashion'.

It may be objected that each single plant must have its own
animating principle. To ascribe an individual soul to every single
starlet of snow is absurd, and the shapes of snowflakes cannot be
attributed to the operation of Soul as can the shapes of plants.

Kepler regards the analogy of plant and snowflake as being
closer than such an objector could imagine. For all forms are the
offspring of one universal principle inherent in the Earth.

Life without Philosophy is Death! The adulteress of Aesop's
fable need not have lost her bastard under her husband's rage; she
might have said that she had conceived from a snowflake.

Kepler passes from the author of the shape to the shape itself.

It may arise from the three-dimensional crossing of three diameters, or it may have been six-cornered from the start. If three-
20 dimensional, the numerical ratios of the regular solids must hold 37
a clue, for they express the mind of God in the language of quantity.

But then why octahedral with six points, rather than cubic with eight?

Jewellers say that perfect octahedra are found in diamonds, which would support our argument.

21 We have speculated too much. Kepler has fought shy of the 39
soul of the mite, only to find himself now exhibiting the very Soul of the Earth in the mote of a snowflake! He must beat a retreat and stop countering trivial arguments with others as trivial.

However, it has begun to snow again and Kepler has been busy examining the little flakes. They are of two kinds: the majority are complex globular clusters, but among them are six-cornered starlets, every one of them flat. So what he has reasoned (about three-dimensional stars) is nothing at all. Thus
22 Kepler returns to the true problem. Why flat? This may be due 41
to cold touching warm vapour on a plane. And why six-cornered?

Is this because this is the first of the plane regular figures which cannot form a solid? Or because hexagons fill a plane? This is excluded, for so do triangles and squares. Or because the hexagon is nearest to the circle? Or do triangles and hexagons build sterile shapes, while pentagons form fruitful ones such as plants? Or, finally, is the shaping faculty, of its very nature, in this case six-cornered?

23 Kepler analyses these five possible causes, and finds difficulties 43
in each.

The fifth possible cause: a six-corneredness of the general shaping faculty will not do, for this faculty is well practised in the whole of geometry and produces other forms as well, not only six-cornered.

24 Kepler concludes that the formative faculty is different in 45
different liquids. The chemists must tell us if there is salt in snowflakes, what kind of salt it is, and what other shapes this salt can assume. And so having knocked at the door of Chemistry, and realizing how much remains to be discovered before the cause can be identified, Kepler prefers to hear what Wackher thinks about it, rather than to tire himself with further discourse.

NOTE ON TEXT

KEPLER's Latin text is here presented in modernized form, and no attempt has been made to produce a facsimile or 'diplomatic' text. Caspar modernized his text to some extent, but we have gone further than he, not only expanding all abbreviations and ampersands, as he did, but also reducing Kepler's lavish use of capitals (which Caspar extended by printing proper names throughout in capitals) except where his capital N draws attention to his joke about Nihil; spelling 'precij' as 'pretii'; ceasing to accentuate adverbs on the last syllable; and consistently distinguishing between vocalic u and consonantal v. This distinction is not the practice of modern Latin texts, such as those of the Oxford Classical Texts, but it is here introduced as convenient for non-specialists who may prefer to distinguish at a glance between, e.g., voluit and volvit.

Caspar's nineteen corrections of misprints in Kepler's text of 1611 have been adopted, and several other textual changes introduced, as indicated at the foot of the page or discussed in the notes.

Kepler's page-divisions and the numbers of his pages have, however, been preserved. Caspar does not preserve the page-division but indicates it in the margin of his text.

IOANNIS KE-PLERI S.C. MAIEST. MATHEMATICI STRENA

Seu

De Niue Sexangula.

Cum Priuilegio S. Cæſ. Maieſt. ad annos xv.

FRANCOFVRTI AD MOENVM,
apud Godefridum Tampach.

Anno M. DC. XI.

JOHANN KEPLER,

MATHEMATICIAN TO HIS IMPERIAL MAJESTY

A NEW YEAR'S GIFT
or
On the Six-Cornered Snowflake.

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for fifteen years.

Published by Godfrey Tampach at
FRANKFORT ON MAIN,
in the year 1611.

AD ILLUSTREM SACRAE CAESAREAE MAIESTATIS CONSILIARIUM IMPERIALEM AULICUM, DOMINUM IOANNEM MATTHAEUM WACKHERIUM[1] *a Wackhenfels, Equitem Auratum, &c. Literatorum & Philosophorum Maecenatem Dominum meum beneficum.*

Cum non sim nescius, quam tu ames Nihil,[1] *non quidem ob pretii vilitatem, sed propter lascivi passeris lusum argutissimum simul et venustissimum: facile mihi est coniicere, tanto tibi gratius et acceptius fore munus, quanto id Nihilo vicinius.*

Quicquid id est quod aliqua Nihili cogitatione tibi allubescat; id et parum et parvum et vilissimum, et minime durabile, hoc est, paene nihil esse oportet. Qualia cum in rerum natura multa sint, est tamen inter ea delectus. Cogitabis fortasse de uno ex atomis Epicuri: verum id Nihil est. Nihil vero a me habes antea.[2] *Eamus itaque per elementa, hoc est per ea, quae sunt in unaquaque re minima.*

Primum de terra, hoc est de Archimedis[3] *mei thesauris ne somnies, qui terram in arenas resolvit, qui pulvisculorum dena millia possidet in uno grano papaveris. Unum enim si subtraxero, numerorum illi myriadumque rationes plane confudero. Adde quod huiusmodi corpusculorum figura nec oculis videtur, nec ab Archimede*

pro-

22 papaveri *K(epler,* 1611)

To the illustrious Counsellor at the Court of His Sacred Imperial Majesty, John Matthew Wacker[1] of Wackenfels, Knight Bachelor, etc., Patron of Men of Letters and of Philosophers, my Master and Benefactor.

I am well aware how fond you are of Nothing,[1] not so much for its low price as for the sport, as delightful as it is witty, that it affords your pert sparrow; and so I can readily guess that the closer a gift comes to Nothing the more welcome and acceptable it will be to you.

Whatever it is that attracts you by some suggestion of Nothing must be both exiguous and diminutive, very inexpensive and ephemeral, in fact almost nothing. Although there are many such things in the world of Nature, the choice between them is open to us. You will perhaps think of one of Epicurus' atoms: but that is simply Nothing. You have had Nothing from me, however, before now.[2] So let us review the Elements, that is, whatever is smallest in each kind.

First Earth: do not dream of that treasury of my admired Archimedes,[3] who resolved the earth into grains of sand and is the owner of ten thousand particles of dust in every poppy-seed. For if I take but one away, I shall have thrown his system of numbers, his tens of thousands, into utter confusion. Moreover the shape of such particles is neither visible to the eye, nor divulged by Archimedes. There is thus nothing in them to prompt *proditur. Nullum igitur in iis ingenium, nulla rei non visae cupido. Est et res durabilis, pulvis, ut quae trabibus vetustate subactis, carie confectis insidens dominatur. Nimium igitur dedero, hoc si dedero.*

Ignis porro scintillae, etsi parvae et evanidae, nunquam tamen sunt minores arenulis pyritarum, quae conflictu deteruntur, aut strigmentis prunarum: quae iam inter pulvisculos reieci. Figurales itaque pyramidas, quas nunquam vidi, Platoni[4] *relinquo, ut ex iis arbitratu suo concinnet ignem. Veniendum ad elementa intermedia.*

Ventum et fumos dare possem, sed hi venduntur; neque hoc tantum in utribus[5] *Islandicis, sed et in chartis, quin et in verbis, idque passim per orbem terrarum. Res itaque preciosa fumus, et quae magno mihi constat. Neque haec apta ingenio, quia rudis et informis.*

Ad aquas igitur devolvimur. Haerentem in urna guttam sacrosancti vates[6] *pro re contemptissima reputant. Et Germani*[7] *nostri nil minus habent illa vini guttula, quae post cyathum exhaustum super unguem excutitur, ibique haerens mole sua stat. Si hanc obtulero guttam, minus sane dedero, quam ille Persa, vola manus Choaspem*[8] *regi affundens; honestius etiam munus, vini gutta ex ungue Germani, quam derosum ramentum de ungue infrendentis, et vel tantillum negantis Itali:*[9] *denique figura guttae globosa iam speculationem pollicetur geometricam: sed vereor, ne et hoc tibi sit nimium, qui tantopere delectaris Nihilo.*

Quid si ad animalia fiat transitio? Vereor ut noctuas Athenas.[10] *Nuper enim apud te vidi volumina rerum singularium et rararum, eius qui ex veteri Parmenidis*[11] *schola motum tollit, quia motus partem unam (scilicet praeteritam) perfectam non habet. Quo in opere cum insint monstra pleraque, haud reor defutura animalcula, prodigiosa exilitate. Quanquam nihil opus uti coniecturis. Habes animadversiones Scaligeri*[12] *in Cardani*[13] *Subtilitates. Invenies ibi animal minimum Exercitatione CXCIV num. 7 Cuniculum subcutaneum. Est vero et hoc nimium. Nam cum incedat id animalculum, anima non caret. Igitur animam tibi cur offeram, cui etiam inanimem guttam dare refugio? Nisi forte ex secto grassatricis bestiae cadavere nova aliqua deprehendi posse speras: de quo viderit Dominus Iessenius*[14] *anatomicus. Talia*

wit, nothing to excite the desire for the unknown. Furthermore, dust is a thing that lasts, settling as it does on beams and getting the upper hand when they are subdued by old age and rotted by decay. If this is my gift to you, I shall give you too much.

As for Fire, sparks, however small and evanescent, are none the less never smaller than the powder that flints lose when struck or the fluff on charcoal embers; and these I have already rejected in rejecting dust particles. So I leave to Plato[4] the pyramidal shapes that I have never seen, from which to concoct fire as he thinks fit. We must pass to the elements that fall between.

Air I could give you in wind and smoke; but they are put up for sale, not only in Icelandic skins[5] but on paper too and even in the spoken word, and that too all over the world. So smoke is a valuable commodity, and costs me a lot. Nor is it apt for wit, raw and shapeless as it is.

And so we are reduced to Water. The last drop to cling to the jar is treated by venerable bards[6] as beneath contempt. Our Germans[7] too count nothing so low as that droplet of wine which is jerked out from the empty cup on to a finger-nail and there stays put by its own weight. If I offer this drop, I shall certainly give less than the famous Persian who poured the Choaspes[8] before his king from the hollow of his hand. Even a drop of wine from the finger of a German is a nobler gift than the sliver gnawed off the finger-nail of the Italian[9] who gnashed his teeth and refused to make even so small an offering. Lastly, the spherical shape of a drop in itself gives promise of a geometrical disquisition. But I am afraid that even this will be too much for you, who so delight in Nothing.

What if we pass on to animals? I am afraid it may be a case of 'owls to Athens'.[10] For I saw recently in your house the volumes on unique and uncommon objects by the author who abolishes motion in accordance with the ancient school of Parmenides,[11] since motion cannot keep the one thing that is complete about it, its past. As this work includes most prodigies, I do not imagine that small animals of portentous fineness will be missing from it. But there is no need to resort to conjecture. You possess Scaliger's[12] *Animadversions on Cardanus's*[13] *Subtleties*. You will find there, in exercise 194, no. 7, the smallest animal, the mite that burrows under the skin. But even this is too big. As the creature can crawl, it is not lifeless. Why then should I offer you a life, when I shrink from giving you even a lifeless drop—unless perhaps you hope that some discoveries could be made by dissecting the corpse of that footpad among insects. Let the anatomist, Dr. Jessen,[14] look into that!

Talia dum meditans anxie, pontem[15] *transeo, confusus super incivilitate mea, qui coram te sine strena comparuissem; nisi quod eadem perpetuo chorda oberrans identidem Nihil affero, nec invenirem, quidnam esset Nihilo proximum, quod ingenii pateretur acumen; commodum accidit, ut vaporibus vi frigoris in nivem coeuntibus, flocculi sparsim in vestem meam deciderent, omnes sexanguli, villosis radiis. Eia mehercule rem quavis gutta minorem, figuratam tamen, eia strenam exoptatissimam Nihil amanti, et dignam quam det mathematicus, Nihil habens, Nihil accipiens, quia et de caelo descendit et stellarum gerit similitudinem.*

Redeatur ad patronum, dum durat strenula, ne corporis halitu tepido solvatur in nihilum.

Atque en fatale nomen. O rem Wackherio gratissimam Nihil amanti. Nam si a Germano[16] *quaeras nix quid sit, respondebit Nihil, siquidem Latine possit.*

Accipe igitur hanc Nihili[17] *accessionem sereno vultu, et si sapis, animam contine, ne denuo nihil accipias.*

Dicendum enim est Socrati de saltu pulicis:[18] *hoc est, quare nives primo casu, priusquam implicentur in maiores floccos, perpetuo cadant, sexangulae, villosis, ut pennulae, senis radiis.*

Immo facessat hinc popularis contemptus inscitiaeque leno Aristophanes. Quid enim mihi opus Socrate, ipsius fabulae materia? Ipse in Regium Psalten[19] *respicio, qui inter Dei laudes commemorat, quod det nivem sicut lanam, qua voce nisi fallor expressit villosos illos nivulae meae radios. Verisimile enim est, cum sederet fessus, aut staret innixus pedo, ad custodiam gregis, vidisse et notasse stellulas hasce nivales, in lanas ovium defluentes, ibique adhaerentes.*

Stellulae nivales.

Sed ad rem veniamus ioco misso. Cum perpetuum hoc sit, quoties ningere incipit, ut prima illa nivis elementa figuram prae se ferant asterisci sexanguli, causam certam esse necesse est. Nam si casu fit, cur non aeque quinquangula cadunt, aut septangula, cur semper sexangula, siquidem nondum confusa et glomerata multitudine, varioque impulsu, sed sparsa et distincta?

Qua de re, cum esset mihi sermo cum quodam nuper, primum

inter

In such anxious reflection as this, I crossed the bridge,[15] embarrassed by my discourtesy in having appeared before you without a New Year's present, except in so far as I harp ceaselessly on the same chord and repeatedly bring forward Nothing: vexed too at not finding what is next to Nothing, yet lends itself to sharpness of wit. Just then by a happy chance water-vapour was condensed by the cold into snow, and specks of down fell here and there on my coat, all with six corners and feathered radii. 'Pon my word, here was something smaller than any drop, yet with a pattern; here was the ideal New Year's gift for the devotee of Nothing, the very thing for a mathematician to give, who has Nothing and receives Nothing, since it comes down from heaven and looks like a star.

Back to our patron, while the New Year's gift lasts, for fear that the warm glow of my body should melt it into nothing.

And look, what an omen in the name! what a delight for Wacker, the devotee of Nothing! Ask a German[16] what Nix means, and he will answer 'nothing' (if he knows Latin).

So accept with unclouded brow this enrichment by Nothing,[17] and (if you have the sense) hold your breath for fear of once again receiving nothing.

Now Socrates has to say how far a flea can jump:[18] our question is, why snowflakes in their first falling, before they are entangled in larger plumes, always fall with six corners and with six rods, tufted like feathers.

Away with that panderer to vulgar scorn and ignorance, Aristophanes; what need have I of Socrates, the theme of his play? *I* look to the Royal Psalmist,[19] who records among the praises of God that He gives the snow like wool; by which phrase, if I am not mistaken, he meant those fleecy outcrops of my flakelet. Probably, when he was tired and sat down or stood leaning on his crook to guard his flock, he saw and observed these little stars of snow settling on the sheep's fleeces and clinging to them.

Starlets of snow.

But, joking apart, let us get down to business. There must be some definite cause why, whenever snow begins to fall, its initial formations invariably display the shape of a six-cornered starlet. For if it happens by chance, why do they not fall just as well with five corners or with seven? Why always with six, so long as they are not tumbled and tangled in masses by irregular drifting, but still remain widespread and scattered?

When I recently had a conversation with someone about this subject, *inter nos convenit, causam non in materia quaerendam, sed in agente. Materia enim nivis est vapor; is dum oritur ex terra, calore quodam suo subvectus, non alius quam continuus et quasi fluidus est: non igitur distinctus in singulares huiusmodi stellulas.*

Quaeras, unde hoc sciam? Cum si talis etiam sit vapor, id oculis cerni non possit, quia vapor pellucidus? Respondeo: vapor existit ex resolutione humoris subterranei, quod arguitur ex eius levitate et ascensu. In resolutione vero figurae non habent locum. Id enim habet figuram ex se, quod seipso terminatur, cum termini figuram constituant: vapor, resolutione facta, ex generibus humidorum est, et fluit, hoc est, seipso non terminatur, nullam igitur figuram retinet, donec condensetur in nivem vel guttam.

Cur igitur constaret, causam inditae figurae sexangulae esse penes agentem, dubitatum porro fuit, quodnam id esset agens, et quomodo ageret, num ut forma insita, an ut efficiens extrinsecus: num ex necessitate materiae efficeret figuram sexangulam, an ex sua natura, puta cui congenitus sit vel archetypus pulchritudinis quae est in sexangulo, vel finis notitia, ad quem ista forma conducat?

Ut pateat harum quaestionum discrimen, utamur exemplis nobilibus: sed geometrice descriptis. Nam ad quaestionem nostram plurimum faciet excursus iste.

Apum alveoli.

Si ex geometricis quaeras, quo ordine structi sint apum alveoli,[20] *respondebunt, ordine sexangulo. Simplex est responsio ex intuitu simplici foraminum seu portarum, laterumque, quibus efformantur alveoli. Circumstant enim alveos singulos sex alii, singulis lateribus de intermedio singuli communicantes. At ubi fundos alveorum fueris contemplatus singulos trinis planis in obtusum descendere videbis angulum. Fundum hunc (carinam potius nuncupes) cum senis alveoli lateribus copulant sex alii anguli tres altiores trilateri, planeque similes imo carinae angulo, tres humiliores quadrilateri interiecti. Praeterea considerandum est geminum esse alveolorum ordinem, portis aversis in contraria, posticis inter se contiguis et stipatis, angulis carinarum singularum ordinis unius, inter angulos tres trium carinarum ordinis alterius insertis, ea arte, ut alveus quilibet non tantum sex lateribus communicet cum senis al-*

we first agreed that the cause was not to be looked for in the material, but in an agent. For the stuff of snow is vapour, and when vapour arises from the earth and is wafted up by some inherent heat, it simply hangs together in an almost liquid state, and is not therefore broken up into single starlets of this kind.

You may ask how I know this, as, even if this is what vapour is like, its condition cannot be observed, since it is transparent. My reply is: vapour arises from moisture under the earth dissolving, as is proved by its light weight and tendency to rise. But shapes have no place in this dissolved condition. Only what is bounded by itself—when its boundaries define its shape—can be said to have a shape of its own. When moisture is dissolved, the vapour behaves like a kind of moisture, and *flows*; that is, is not bounded by itself, and so retains no shape, until it is condensed into flakes of snow or drops of water.

Since, then, we agreed that the cause of the imposed six-cornered shape lay with an agent, we of course wondered what that agent was, and how it acted: could it be as immanent form or as efficient cause from outside? did it stamp the six-cornered shape on the stuff as the stuff demanded, or out of its own nature—a nature, for instance, in which there is inborn either the idea of the beauty inherent in the hexagon or knowledge of the purpose which that form subserves?

To arrive at a clear decision on these questions let us take familiar examples, but set them out in geometrical fashion. A parenthesis of this kind will contribute a great deal to our problem.

Honeycombs.

If you ask geometers on what plan honeycombs[20] are built, their answer will be 'on a six-cornered plan'. The answer is obvious from a mere glance at the openings or gates, and at the walls of which the combs are built. Each cell is surrounded by six others, and each shares a party-wall between it and the next. But when you have examined the base of the cells, you will see that each ends downwards in an obtuse angle formed by three planes. Six other corners clinch this base—keel you might rather call it—each with the six walls of its cell; with the three upper corners inserted into a three-sided angle, exactly like the bottom angle of the keel; the three lower into a four-sided angle. Furthermore it is to be noted that the plan of the cells is a double one, with gates facing away in opposite directions, with posterns adjacent to each other and closely packed, and with the corners of each keel in one row inserted between the three corners of the three keels of the second row. The architecture is such that any cell shares not only six walls with the six cells that surround it in the same row, but *veis in eodem ordine circumstantibus, sed etiam trinis in fundo planis cum tribus aliis alveis ex ordine averso. Ita fit ut apes singulae novem habeant vicinas, a qualibet uno communi pariete distinctae. Plana carinarum trina, omnia inter se similia sunt, eius figurae, quam geometrae rhombum appellant. Quibus ego rhombis admonitus, coepi in geometria inquirere, num quod corpus simile regularibus quinque, et Archimedeis quattuordecim, ex rhombis meris constitui possit: invenique duo, quorum alterum cognatum sit cubo et octahedro, reliquum dodecahedro et icosahedro; (nam cubus ipse tertii vicem sustinet, cognatus duobus tetrahedris invicem coaptatis) primum duodecim rhombis clauditur, alterum triginta. Sed primo haec est communis proprietas cum cubo, quod ut anguli octo cuborum octonorum circa idem punctum coaptati locum omnem explent, nullo relicto vacuo, sic rhombici primi, obtusi seu trilateri anguli quaterni idem praestent,*[21] *et quadrilateri anguli seni similiter. Itaque strui potest locus solidus ex meris hisce rhombis, sic ut semper quattuor trilateri aut ut sex quadrilateri anguli ad unum et unum punctum concurrant, et ut summa quaedam fiat. Quando locus solidus per cubos aequales ordine recto impletur: tunc unum cubum contingunt alii 32*[22] *angulis singulis, et praeterea sex quaternis, itaque contingentium sunt octo et triginta. At quando impletur locus solidus per rhombica aequalia, tunc unum rhombicum contingunt alia sex angulis singulis quadrilateris, et praeterea duodecim, angulis quaternis, itaque contingentium quomodocunque sunt octodecim.*

Corpora regularia rhombica.

Haec igitur illa figura geometrica est, regularis quam proxime, impletrix loci solidi, ut sexangulum quadrangulum triangulum consummatores loci plani: haec inquam est quam apes effingunt in suis alvearibus. Nisi hoc tantum dempto, quod alveoli carent tectis carina similibus.

Si enim etiam haec adderent, et quaelibet apis intra alias duodecim seu octodecim abderetur; non pateret ipsi exitus, conclusae circumcirca. Itaque cum tectis non indiguerint, nihil obstitit quominus latera sena pro modulo corpusculi sui producerent ultra modulum rhomborum in carinis, efficerentque ea illorum altrinsecus dissimilia. *Porro*

17 aut ut] ut et *K* 18 concurrant. Et . . . fiat: quando *K* 34 attrinsecus *K*

also three plane surfaces in the base with three other cells from the contrary row. The result is that every bee has nine neighbours, separated from each and any other by one party-wall in common. The plane surfaces of the keel are always three. All are exactly alike and of the shape called the rhombus by geometers.

Regular rhomboid solids.

These rhombi put it into my head to embark on a problem of geometry: whether any body, similar to the five regular solids and to the fourteen Archimedean solids could be constructed with nothing but rhombi. I found two, one with affinities to the cube and octahedron, the other to the dodecahedron and icosahedron—the cube can itself serve to represent a third, owing to its affinity with two tetrahedrons coupled together. The first is bounded by twelve rhombi, the second by thirty. But the first has this property in common with the cube: the eight corners of eight cubes boxed together at one point fill the whole space and leave none empty, and with rhomboid solids of the first kind [rhomboid dodecahedra] every four corners formed by the obtuse [120°] angles of the rhombi on three planes and every six [formed by the acute (60°) angles of the rhombi] on four planes do the same [fill space]. Thus a solid space can be filled by using nothing but these rhombi, such that always four three-sided corners [formed by three intersecting planes] or six four-sided corners [formed by four intersecting planes] meet at one and the same point, and a certain volume is completed. When a solid space is filled by equal-sized cubes in straight rows [horizontally and vertically], then one cube is touched by one corner of each of the thirty-two others,[22] and further by four corners of each of six others, and so there are thirty-eight in contact. But when a solid space is filled by equal rhomboid solids, then one rhomboid is touched by six others at one four-sided corner each and, further, by twelve others at four corners, and so there are eighteen points of contact in one way or the other.

This, then, is the geometric figure, as near as possible to a regular solid, which fills space, just as the hexagon, square, and triangle are the fillers of a plane surface. This, I repeat, is the figure which bees form in their combs, with only this exception that the cells have not got roofs of the same kind as their keel.

For if they added roofs too and any bee were stowed away between twelve or eighteen others, it would have no way out, enclosed as it would be on every side. Accordingly, as they had no need of roofs, there was nothing to prevent their prolonging the six walls in each cell on the scale of their own body beyond the limit of the rhombi in the keels, and making those on top unlike those on the opposite side.

Quae sit figura grano mali punici.

Porro si quis grandius aliquod malum granatum aperiat, videbit acinos plerosque in eandem figuram expressos, nisi quantum impedit series radicum, per quas alimentum illis suum suppeditatur.

Quaeritur iam in his duobus exemplis, quis sit auctor figurae rhombicae in alveolo apum inque granis mali punici. Materia in causa non est. Nuspiam enim inveniunt apes huiusmodi foliola rhombica, in praeparato, quae colligant apes atque coaptent, ad effigiandas suas domunculas. Neque verisimile est, in solis malis punicis sponte excrescere acinos in angulos, cum in omnibus aliis fructibus rotundi evadant, qua non impediuntur, humore suggesto lentos cortices explente et referciente, ut turgescant, et, qua datur, protuberent.

Est igitur in acino quidem punici mali figurae causa in anima plantae, quae pomi procurat incrementum. Sed non est haec adaequata figurae causa, neque enim hoc praestat fructui ex formali proprietate, sed adiuvatur necessitate materiali. Nam cum acini inter initia dum parvi sunt, rotundi sint, quamdiu spatium ipsis intra corticem sufficit, tandem indurescente cortice, crescentibus vero continue acinis, fit eorum constipatio et compressio, ut et pisorum intra suos oblongos calices. Sed pisa non habent quorsum cedant: oblongis enim siliquis ex ordine sunt inserta: comprimuntur igitur a duobus tantum lateribus. Acini vero rotundi in malis punicis uberius spatium a principio nacti, facile sese singuli intra ternos ordine adverso protuberantes insinuant, rotunditate sua adiuti, humoremque inde unde urgentur, declinantes in spatia vacua. Quod si quis aliquam vim globulorum rotundorum, interque sese aequalium ex materia molli constantium, concludat in rotundo vase illudque circulis aereis incipiat coarctare undique a plagis omnibus: globuli plurimi exprimentur in schema rhombicum: praesertim si prius illos globulos succussione vasis diligenti, locum angustiorem libero rotatu capere permiseris. Nam directa globulorum dispositione, quae turbari non possit, compressione facta cubos etiam efficies.

Pisa in[23] quam figuram expriman-tur.

In universum enim duobus modis inter se ordinantur globuli aequales in vase aliquo collecti, pro duobus modis ordinationis eorum in aliqua planitie. Nam

8 effigiendas *K cf.* 22, 32 effigiavit; *hic inter* effigiandas *et* effingendas *haesisse videtur*
19 *in margine* Pisani *K*

What shape the loculus of a pomegranate has.

Again, if one opens up a rather large-sized pomegranate, one will see most of its loculi squeezed into the same shape, except in so far as the pattern of veins, by which their nourishment is supplied, gets in the way.

Now the question arises in these two examples: what agent creates the rhomboid shape in the cells of the comb and in the loculi of the pomegranate? There can be no question of matter as the cause. For nowhere do bees find rhomboid leaves of this sort ready made to collect and fit together for the construction of their little dwellings; and it is not probable that the loculi in pomegranates alone should swell into angles, whereas in all other fruits the berries turn out round, where nothing hampers them, as the rising sap fills the tough and pliant rind and its pressure makes them swell—and even bulge out where the rind gives.

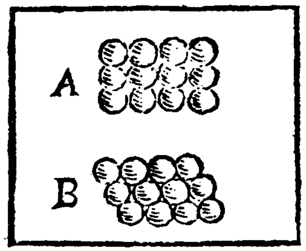
So in the loculus of the pomegranate, anyhow, the cause of its shape lies in the plant's 'soul' or life-principle, which sees to the growth of its fruit. But this is not a sufficient cause of its shape: for it is not from its formal properties that it induces this shape in its fruit, but it is assisted by material necessity. For the loculi to begin with, when they are small, are round, so long as there is enough space for them inside the rind. But later as the rind hardens, while the loculi continue to grow, they become packed and squeezed together, just as peas do too within their oblong pods. But peas have no direction in which to give way, as they are set in a row inside their oblong pods: so they are squeezed from only two sides. The round loculi in pomegranates, however, from the start get more freedom in space and easily work their way into place, each one between the three that swell out from the opposite row. They are helped by their roundness, and deflect their sap away from where they are under pressure into the empty spaces. Now if one encloses in a round vessel a number of round pellets of equal size and of soft stuff, and then begins to tighten it all round with metal rings from every quarter, a great many pellets will be pressed into a rhomboid shape, especially if you have previously by careful jogging of the vessel encouraged those pellets to roll freely and occupy a more restricted space. If you have a pattern of pellets at right angles that cannot be disrupted, you will by applying pressure even produce cubes.

Into what shape peas are squeezed.

For in general equal pellets when collected in any vessel, come to a mutual arrangement in two modes according to the two modes of arranging them in a plane.

Nam si errantes in eodem plano horizontali globulos aequales coegeris in angustum, ut se mutuo contingant; aut triangulari forma coeunt, aut quadrangulari; ibi sex unum circumstant, hic quattuor: utrimque eadem est ratio contactus per omnes globulos, demptis extremis. Quinquanguli forma nequit retineri aequalitas, sexangulum resolvitur in triangula: ut ita dicti duo ordines soli sint.

Iam si ad structuram solidorum quam potest fieri arctissimam progrediaris, ordinesque ordinibus superponas, in plano prius coaptatos, aut ii erunt quadrati A aut trigonici B: si quadrati aut singuli globi ordinis superioris singulis superstabunt ordinis inferioris aut contra singuli ordinis superioris sedebunt inter quaternos ordinis inferioris.

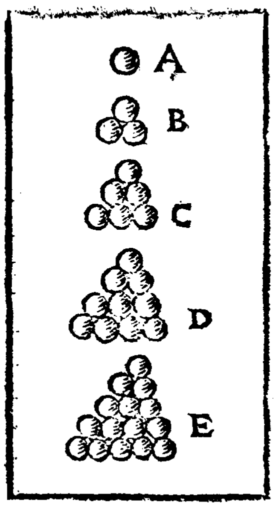


Priori modo tangitur quilibet globus a quattuor circumstantibus in eodem plano, ab uno supra se, et ab uno infra se: et sic in universum a sex aliis, eritque ordo cubicus, et compressione facta fient cubi: sed non erit arctissima coaptatio. Posteriori modo praeterquam quod quilibet globus a quattuor circumstantibus in eodem plano tangitur etiam a quattuor infra se, et a quattuor supra se, et sic in universum a duodecim tangetur; fientque compressione ex globosis rhombica. Ordo hic magis assimilabitur octahedro et pyramidi. Coaptatio fiet arctissima, ut nullo praeterea ordine plures globuli in idem vas compingi queant. Rursum si ordines in plano structi fuerint trigonici; tunc in coaptatione solida aut singuli globi ordinis superioris superstant singulis inferioris, coaptatione rursum laxa, aut singuli superioris sedent inter ternos inferioris. Priori modo tangitur quilibet globus a sex circumstantibus in eodem plano, ab uno supra, et ab uno infra se, et sic in universum ab octo aliis. Ordo assimilabitur prismati, et compressione facta fient pro globulis columnae senum laterum quadrangulorum, duarumque basium sexangularum. Posteriori modo fiet idem, quod prius posteriori modo in qua-

10 trigonici: B si *K* 31 assmilabitur *K*

If equal pellets are loose in the same horizontal plane and you drive them together so tightly that they touch each other, they come together either in a three-cornered or in a four-cornered pattern. In the former case six surround one; in the latter four. Throughout there is the same pattern of contact between all the pellets except the outermost. With a five-sided pattern uniformity cannot be maintained. A six-sided pattern breaks up into three-sided. Thus there are only the two patterns as described.

Now if you proceed to pack the solid bodies as tightly as possible, and set the files that are first arranged on the level on top of others, layer on layer, the pellets will be either squared (A in diagram), or in triangles (B in diagram). If squared, either each single pellet of the upper range will rest on a single pellet of the lower, or, on the other hand, each single pellet of the upper range will settle between every four of the lower. In the former mode any pellet is touched by four neighbours in the same plane, and by one above and one below, and so on throughout, each touched by six others. The arrangement will be cubic, and the pellets, when subjected to pressure, will become cubes. But this will not be the tightest pack. In the second mode not only is every pellet touched by its four neighbours in the same plane, but also by four in the plane above and by four below, and so throughout one will be touched by twelve, and under pressure spherical pellets will become rhomboid. This arrangement will be more comparable to the octahedron and pyramid. The packing will be the tightest possible, so that in no other arrangement could more pellets be stuffed into the same container. Again, if the files built up on the level have been triangular, then in solid packing either each pellet of the upper range rests on one of the lower, when the packing is again loose; or each one of the upper range will settle between every three of the lower. In the first mode any pellet is touched by six neighbours in the same plane, by one above and by one below its level, and thus throughout by eight others. The pattern will be comparable to the prism, and, when pressure is applied, columns of six four-cornered sides and of two six-cornered bases will be formed instead of the pellets. In the second mode the same result will occur as before in the second mode of the four-sided arrangement, *quadrilateris. Esto enim B copula trium globorum. Ei superpone A unum pro apice; esto et alia copula senum globorum C, et alia denum D et alia quindenum E. Impone semper angustiorem latiori, ut fiat figura pyramidis. Etsi igitur per hanc impositionem singuli superiores sederunt inter trinos inferiores: tamen iam versa figura, ut non apex sed integrum latus pyramidis sit loco superiori, quoties unum globulum deglu-beris e summis, infra stabunt quattuor ordine quadrato. Et rursum tangetur unus globus ut prius, a duodecim aliis, a sex nempe circumstantibus in eodem plano tribus supra et tribus infra. Ita in solida coaptatione arctissima non potest esse ordo triangularis sine quadrangulari, nec vicissim. Patet igitur, acinos punici mali, materiali necessitate concurrente cum rationibus incrementi acinorum, exprimi in figuram rhombici corporis: cum non infestis frontibus pertinaciter nitantur rotundi ex adverso acini, sed cedant expulsi, in spatia inter ternos vel quaternos oppositos interiecta.*



Causa figurae in acinis mali punici.

In alvearibus vero apum ratio est alia. Non enim conglobantur apes confuse, ut acini in malo, sed arbitrariam struunt aciem, omnes capitibus prominentes in unam vel adversam plagam; omnes alvorum extremis invicem obnitentes. Quod si ex conglobatione huiusmodi existeret figura haec, oporteret alveos apibus superindui ex consistentia exsudati lentoris, ut cochleis contortis solent supercrescere domunculae. At certum est, apes ipsas suos sibi fingere alveos, totamque a fundamentis contignationem exstruere.

Unde domunculis cochlearum sua figura?

Causae figurae in alveolis apiariis.

Quare ipsa apis natura hunc instinctum habet ex proprietate sua, ut hac potissimum figura aedificet: hic illi archetypus a creatore impressus est: nihil hic materia neque cerae neque corpusculi apis, nihil incrementa possunt.

Hoc animadverso quaeritur iam porro et de fine, non quem apis ipsa con-

2 demum *K* 12 eadem *K*

thus: let B be a group of three balls; set one, A, on it as apex; let there be also another group, C, of six balls; another, D, of ten; and another, E, of fifteen. Regularly superimpose the narrower on the wider to produce the shape of a pyramid. Now, although in this construction each one in an upper layer is seated between three in the lower, yet if you turn the figure round so that not the apex but a whole side of the pyramid is uppermost, you will find, whenever you peel off one ball from the top, four lying below it in square pattern. Again as before, one ball will be touched by twelve others, to wit, by six neighbours in the same plane, and by three above and three below. Thus in the closest pack in three dimensions, the triangular pattern cannot exist without the square, and vice versa. It is therefore obvious that the loculi of the pomegranate are squeezed into the shape of a solid rhomboid: the demands of their matter coincide with the proportions of their growth. The globular loculi opposite each other do not persist in pushing face to face, but are displaced and slip aside into the spaces left between three or four others in the confronting plane.

Why the loculi of the pomegranate have their shape.

But in honeycombs the reason is different. For bees are not rolled together pell-mell, like cells in a fruit, but marshall themselves as in line of battle, all with their heads projecting in one direction or its opposite, as they think best, all pushing in mutual support tail to tail. Now if this sort of rolling together were to make the shape, it would have to be that, when the viscous secretion of the bees set, the cells overlapped them, as shells grow over snails on their spiral. But it is certain that bees shape their own combs and themselves build up their many-storied blocks from the foundations.

Where does the shape of a snail-shell come from?

The causes of the shape in the cells of honeycombs.

The bee, therefore, by nature has this instinct as its property, to build in this shape rather than others. This original pattern has been imprinted on it by the Creator. The matter of the wax or of the bee's body can have nothing to do with it; the processes of growth nothing either.

This observation at once raises the further question of the purpose, not which the bee itself pursues in its business, but which God Himself, the *consectetur discursu suo, sed quem Deus ipse, apiculae creator propositum habuit, cum illi has architecturae suae leges praescripsisset.*

Atque hic iam demum rursum ingreditur finis destinationem, consideratio corporum materiaeque. Tria enim de hoc fine dici possunt. Primum vulgare est apud physicos, qui ad solam quidem sexangularem structuram respiciunt, ut illa cum hiatibus extrinsecus sese repraesentat. Cum enim locum planum impleant excluso vacuo tantum hae figurae, triangulum, quadrangulum, sexangulum: ex iis sexangulum capacissima est figura. Capacitatem autem sibi parant apes ad mella condenda.

Sexanguli figura capax.

Potestque ampliari haec ratio etiam ad solidorum considerationem, in hunc modum; quod cum solidum spatium non dividatur sine hiatu, nisi in cubos et rhombica: rhombica sunt cubis capaciora. Sed non sufficit haec ratio: nam si capacitatem quaerunt, cur non quaelibet sibi rotundum fingit nidum, quid opus est minutias loci consectari, quasi nullum in toto alveari restet spatium? Probabilior esset haec altera causa, quamvis nec illa sufficiens, ob rationes dictas, quod mollia apicularum corpuscula commodius locantur in nido figurae plurium et obtusorum angulorum, quaeque cognatior est sphaericae, quam in cubo, qui paucos et longe procurrentes habet angulos, fundum planum, a corpore tereti abhorrentem.

Finis proprius figurae rhombicae in apiariis alveolis.

Igitur tertiam causam necesse est addere, qua minuitur ipsis labor, si semper duae communem struant parietem et quod in rectitudine coassationum maior firmitudo, ad cratem integram sustinendam, quam si singulae domunculae teretes ideoque compressu faciles fuissent: denique figurae rotundae hiant cum maxime coniunctae sunt: itaque frigus se per hiatus insinuaret. Quibus omnibus providetur, quod 'consortia tecta Urbis habent'[24] *ut Virgilius canit.*

Has igitur rationes materialem necessitatem respicientes ita puto sufficere, ut hoc loco non existimem philosophandum de perfectione et pulchritudine vel nobilitate figurae rhombicae: neque satagendum, ut essentia animulae quae est in ape, ex contemplatione figurae, quam fabricatur, eliciatur: quale quid nobis fuisset inceptandum, si usus figurae nullus apparuisset.

Idem de malo punico intelligendum. Apparet necessitas materialis, quae acinos perducit ad rhombicum, succedentibus incrementis. Itaque vanum est de essentia animae in hac arbore cogitare, quae rhombicum potissimum efficiat. *Con-*

27 sunt] *melius* sint *si* cum, *ut videtur, concessive dicitur*

bee's creator, had in mind when He prescribed to it these canons of its architecture.

Now here again at long last a consideration of bodies and matter enters into the determination of purpose. Three things can be said about this purpose. The first is a matter of common knowledge among natural philosophers, who pay attention to the hexagonal structure only as it presents itself outwardly with its openings. A plane surface can be covered without gaps by only three shapes, the triangle, the square, the hexagon. Of these the hexagon is the roomiest, and it is room for storing honey that bees provide themselves with.

The roomy shape of the hexagon.

Moreover, this reasoning can be extended to three dimensions, thus: space in the round cannot be divided without remainder except into cubes and rhomboids; and rhomboids are roomier than cubes. But this reason does not go far enough: for if room is what they want, why does some one of them not build itself a round nest? What need is there to run after the tiniest remainders of space as if there were no room to spare in the whole hive? Our second cause would be more acceptable, though for the reasons already given it too is inadequate; namely this, that the tender bodies of young bees are more comfortably lodged in a nest, with more numerous obtuse angles, and so closer in shape to the circular, than in a cube which has fewer and more deeply recessed corners and a flat floor at odds with a streamlined body.

The essential purpose of the rhomboid shape in beehives.

Consequently it is necessary to supplement with a third cause. Labour is saved if there are always two bees to build the party-wall, and greater stability for the maintenance of the framework intact is given by the regular carpentry than if each apartment had been rounded off and was therefore easy to dislocate. Finally, rounded shapes leave gaps, however closely they are bunched together, and thus cold would seep in through the gaps. All this is provided for because, as Virgil says in his poem, 'they hold the houses of their city in common'.[24]

These then are the reasons which concern necessity in the material; they are in my opinion sufficient to excuse me from arguing at this point about the perfection, beauty, or dignity of the rhomboid, or from bestirring myself to conjure up, from a meditation on the shape that the bee builds, the inner nature of its diminutive soul. We would have had to embark on something of the kind, if no utility in the shape had come to light.

The pomegranate is to be understood in the same way. A material necessity that guides the loculi to the rhomboid as growth follows on growth, has come to light. It is therefore superfluous to think about the inner principle of soul in this tree that directs it to prefer the rhomboid.

Causa quinarii in foliis florum.

Contra si quaeratur, cur omnes adeo arbores et frutices, (aut certe pleraeque) florem explicent forma quinquangulari, numero scilicet foliorum quinario, quem florem in pomis et piris sequitur fructus dispositio, in eodem vel cognato numero, quinario vel denario: quini intus loculi continendis seminibus, dena filamenta:[25] *quod et obtinet in cucumeribus et id genus aliis: hic inquam locum habet speculatio pulchritudinis aut proprietatis figurae, quae animam harum plantarum characterisavit. Et detegam obiter cogitationes meas super hac re.*

Corpora regularia quinario utentia orta ex proportione divina.

Duo sunt corpora regularia, dodecahedron et icosahedron, quorum illud quinquangulis figuratur expresse, hoc triangularis quidem, sed in quinquanguli formam coaptatis. Utriusque horum corporum, ipsiusque adeo quinquanguli structura perfici non potest sine proportione illa, quam hodierni geometrae divinam appellant. Est autem sic comparata, ut duo minores proportionis continuae termini iuncti constituant tertium; semperque additi duo proximi, constituant immediate sequentem, eadem semper durante proportione, in infinitum usque. In numeris exemplum perfectum dare est impossibile. Quo longius tamen progredimur ab unitate, hoc fit exemplum perfectius. Sint minimi 1 et 1 quos imaginaberis inaequales. Adde, fient 2; cui adde maiorem 1, fient 3; cui adde 2, fient 5; cui adde 3, fient 8; cui adde 5, fient 13; cui adde 8, fient 21. Semper enim ut 5 ad 8, sic 8 ad 13, fere; et ut 8 ad 13, sic 13 ad 21, fere.

Ad huius proportionis seipsam propagantis similitudinem, puto effictam esse facultatem seminariam: itaque in flore praefertur seminariae facultatis γνήσιον vexillum quinquangulum. Mitto cetera quae ad huius rei confirmationem iucundissima contemplatione possent adduci. Sed proprius illis debetur locus. Nunc haec exempli tantum causa praemisimus; ut in rimanda figura nivis sexangula simus instructiores exercitatioresque.

An frigus causa figurae stellatae in nive.

Cum enim proposuissemus inquirere originem figurae huius in nive inter causas extrinsecas et intrinsecas: inter externas primum sese offerebat frigus. Condensatio sane est a frigore: per condensationem vero vapor coit in figuram stellae: videbatur igitur frigus illi figuram praestare stellae. Tunc itum est ad considerationem aliam, an frigus sit na- *tura*

25 *γνησίον* K

Cause of fivesidedness in flower petals.

On the other hand we may ask why all trees and bushes—or at least most of them—unfold a flower in a five-sided pattern, with five petals. In apple- and pear-trees this flower is followed by a fruit likewise divided into five or into the related number, ten. Inside there are always five compartments for the reception of the seeds, and ten veins.[25] This is true of cucumbers and others of that kind. It is here, I insist, that a consideration of the beauty or special quality of the shape that has characterized the soul of these plants, would be in place, and I will incidentally divulge my thoughts on this subject.

Regular bodies based on the number five arise from the divine proportion [golden section].

Of the two regular solids, the dodecahedron and the icosahedron, the former is made up precisely of pentagons, the latter of triangles but triangles that meet five at a point. Both of these solids, and indeed the structure of the pentagon itself, cannot be formed without the divine proportion [golden section] as modern geometers call it. It is so arranged that the two lesser terms of a progressive series together constitute the third, and the two last, when added, make the immediately subsequent term and so on to infinity, as the same proportion continues unbroken. It is impossible to provide a perfect example in round numbers. However, the further we advance from the number one, the more perfect the example becomes. Let the smallest numbers be 1 and 1, which you must imagine as unequal. Add them, and the sum will be 2; add to this the greater of the 1's, result 3; add 2 to this, and get 5; add 3, get 8; 5 to 8, 13; 8 to 13, 21. As 5 is to 8, so 8 is to 13, approximately, and as 8 to 13, so 13 is to 21, approximately.

It is in the likeness of this self-developing series that the faculty of propagation is, in my opinion, formed; and so in a flower the authentic flag of this faculty is flown, the pentagon. I pass over all the other arguments that a delightful rumination could adduce in proof of this. They deserve a place of their own. Here and now I provide this preamble by way of example only, so that we should be the better equipped and practised for research into the six-cornered shape of the snowflake.

Whether cold is the cause of the starry shape in snow.

When we had proposed to inquire into the origin of this shape in snowflakes and to decide between external and internal causes, as an external cause the first to present itself was cold. Cold certainly gives rise to condensation, and by condensation vapour shrinks into the shape of a star, and so it seemed that cold gave it that shape. Then we went on to another reflection, whether cold is a force of nature like the Heat of physicians.

tura quaedam ut medicorum calor? Videbatur enim esse mera privatio, cui neque mens, sexanguli fabricatrix, nec omnino operatio ulla propria.

Sed ne misceamus quaestiones, maneat frigori condensatio: potuit condensatio fieri, ut videtur, in formam globosam rectius. Immo si consideretur frigus late fusum, et vapor illi superficie tenus occurrens, magis est consentaneum, ut condensatio fiat in formam omnino planam, superficiei similem et eam quorumcunque terminorum. Ut si tota vaporis extima superficies ex frigore densitatem, ex densitate pondus, ex pondere casum, ex casu comminutionem in frustula seu bracteas nancisceretur: utique non omnes bracteae, quin immo paucissimae, ac nescio an ullae evadent sexangulae, praesertim radiis adeo concinne striatis.

Quae causa figurae in pruinosis consistentiis circa fenestras.

Admonebant istae striae rei illius quae contingit in hypocaustis vapidis, brumali rigore pertusas fenestras obsidente. Luctantur circa illas angustias frigidus aer et vapor. Quoties enim sese mutuo contingunt, calor superiora petit, frigus inferiora. Est enim in calido dilatatio materiae, in frigido densitas et pondus, pellitque calida sursum. Vapore igitur confertim exire nitente, fit fuga vacui, ut et frigidus aer confertim irrumpat, unde limbi patentis fenestrae vel rimulae frigidissimi efficiuntur. Ad eos limbos quicquid appellit vaporis, continuo gelatur. Succeditque in illam materiam frigus aeque magnum, ut quicquid porro vaporis ad hanc appellit pruinam, et ipsum geletur, appositione continua, intercedente tamen, seseque introrsum insinuante rectis lineis aere frigido: qua alternatione ingressus et egressus, illae pruinosae vaporis consistentiae strias sortiuntur et acutos radios.

Nihil ad hoc instar de figuratione nivulae nostrae dici potest. Nam quinam hic ingressus, qui exitus, quae angustiae, quae lucta in patentissimis aeris campis? Concessero inter cadendum ex alto per vapidum aerem fieri aliquam ad villos appositionem a contingentibus vaporibus. At quare sex locis, quodnam senarii principium? quis capitellam antequam caderet in sex effigiavit cornua frigida? Quae causa statuens in illa superficie iam iam condensanda sex puncta, ad quae seni circum radii connectantur? Cum

3 valla *K*   15 illos *K*   19 Cimbi *K*   20, 22 *an* appellitur?
30–31 contigentibus *K*

It had seemed a mere negation [absence of heat], without a mind to design the hexagon, and without any operation at all of its own.

But to keep the question distinct, let us leave it that condensation is the effect of cold. Condensation could have occurred more properly, so it would seem, into globular form. Indeed, if cold is imagined as widely spread and vapour as meeting it on its outer surface only, it is more plausible that condensation should assume a quite flat shape, like the surface it meets, and that too of unlimited extent. For instance, if the whole outermost surface of the vapour were to acquire from cold density, from density weight, from weight downward motion, and from downward motion disintegration into crumbs or wafers [like gold foil], not all the wafers anyhow, but rather very few, indeed I doubt if any at all, would turn out to be six-sided, still less with radii so symmetrically striped.

What the cause of the shapes in formations of hoar-frost round windows is.

These stripes reminded me of what happens in Turkish baths, when the winter's frost lays siege to broken windows. Cold air and steam do battle at such frontier passes. Whenever they make mutual contact, heat rises and cold sinks; for there is expansion of matter in hot things, but density and weight in cold, and this drives the hot upwards. So when the steam makes a push to escape *en masse*, a vacuum is avoided when the cold air in turn breaks in *en masse*. Hence the edges of an open window or of a crack become the coldest. Whatever steam lands on these edges, at once freezes, and an equal degree of cold passes into the new deposit, so that whatever steam drifts against this ice, also freezes in uninterrupted accretion, until the cold air intervenes and makes its way inward undeflected. By this alternation of entry and exit those formations of hoar-frost acquire stripes and sharp prongs.

Nothing can be said in these terms about the shaping of our little snowflake. For what entry is there here in the wide 'champaigns of air', what exit, what narrow passes, what skirmishing? I will grant that, as flakes fall from above through steamy air, some incrustation on the plumes can occur from the vapour that comes in contact with them. But why at six points? What is the origin of the number six? Who carved the nucleus, before it fell, into six horns of ice? What cause is it that prescribes in that surface, which is now in the very act of condensing, six points in a circle for six prongs to be welded on to them?

Cum itaque causa externa, frigus, haec efficere nequeat: internam aliquam esse necesse est, vaporisque vel comitem vel alio quocumque modo propriam?

At haec perpendentem subiit admiratio, cur radii non potius in omnem ambitum sphaericum disponantur? Cur, si internus calor est huius rei auctor, in plana tantummodo superficie operatur? qui undiquaque aequaliter se habet, non vero in sola plana superficie vaporis inest.

Nihili opinio.[26]

Dum in his luctor meditationibus, dum ratio postulat radios in omnem ambitum distributos, incidit, quod alias saepe cum admiratione spectavi, stellulas huiusmodi non primo statim casus momento sterni super planitiem, sed particulis nonnullis sublimes teneri, deinque temporis mora subsidere in planitiem. Ex illa ratiocinatione veluti patre, et ex hac experientia veluti matre, nata est mihi opinio ista: stellulas istas inter cadendum trinis constare villosis diametris, decussatim ad unum punctum coaptatis, sex extremitatibus in orbem aequaliter distributis, ita ut tribus tantum villosis radiis incidant, reliquis trinis, (qui sunt incidentium oppositi in iisdem rectis diametris) in sublimi stent, donec flexis iis, quibus stellula sustinebatur, reliqui hactenus sublimes, in eandem planitiem cum prioribus, intermediis locis defluant.

Huius opinionis vim prosequar per totum, postea demum an vera sit examinabo: ne fortassis importuna vanitatis detectio me prohibeat, quod institui, verba de re Nihili facere.

Hoc igitur in causa[27] *positum esto, quaecunque causa sit horum sex radiorum, eam undique aequaliter fusam esse in omnes plagas: ut si frigus est causa senum radiorum, frigus igitur singulas vaporis portiunculas circumstare aequaliter, aut aequalibus certe intervallis undique: sin calor internus, et hunc in omnes sphaerae plagas ex uno et eodem centro operari.*

At sic nondum discussa sed translata est quaestio. Nam nondum patet quare non quinque vel septem, sed omnino sex villosi radii coaptentur ex eodem centro. *Et*

Since, then, the external cause, cold, cannot produce this result, there must be some internal cause, either concomitant or in some other way essential to vapour.

But as I weigh these ideas, it occurs to me to wonder why the prongs are not rather distributed over the whole spherical envelope. If internal heat is the author of this result, why does it take effect only on a plane surface? After all, heat is uniformly present throughout, and is not found only in an even layer of vapour.

A valueless notion.[26]

While I struggle with these thoughts, and while reason demands radii distributed over the whole circuit, I think of something which I have often otherwise watched with astonishment; that starlets of this kind are not at once spread over a plane surface in the first moment of falling, but are held up above it by some part of themselves, and then, after a moment's delay, settle down on to the level. To this reasoning, so to speak, as father, and to this observation as mother I owe the birth of the following notion: while these starlets are falling, they consist of three feathered diameters, joined crosswise at one point, with their six extremities equally distributed in a sphere; consequently they fall on only three of the feathered prongs, and tower aloft with the remaining three, opposite those on which they fall, on the same diameters prolonged, until those, on which they rested, buckle, and the remainder, until then upright, sag on to the level with the former in the gaps between them.

I shall push this notion as far as it will take me, and only afterwards shall I test its truth, for fear that the ill-timed detection of a groundless assumption may perhaps prevent me from fulfilling my engagement to discourse about a thing of Naught.

So let this issue be assumed as the ground of our submission,[27] that, whatever the cause of these six rays may be, it is everywhere equally diffused in all directions. Hence, if cold is the cause of the six rays, then cold surrounds each particle of vapour equally, or at least at equal intervals throughout. Alternatively, if internal heat is the cause, it too operates from one and the same centre in every dimension of a sphere.

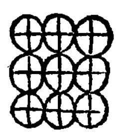
But in this way the question is not yet faced, but only shifted; for it is not yet clear why, not five or seven, but invariably six plumed rays are fastened to the same centre.

Et si quaeras a geometris, quanam in figura tres diametri sese orthogonaliter, seu in forma crucis duplicis, in eodem centro secent; is respondebit, in octahedro, connexis angulis oppositis. Octahedron enim habet sex angulos. Quare igitur fit, ut nix inter cadendum, priusquam complanetur, tribus villosis diametris se invicem orthogonaliter secantibus imitetur ipsum τὸ σκελετὸν *octahedri? Ut, si radiorum extrema vicina rectis duodenis connectas, integrum octahedri corpus repraesentabis?*[28]

Quae causa igitur, quod in hos tres villosos radios potius fit condensatio quam in globum integrum?

Possum quidem dicere modum unum quo ista fiant materiali necessitate: sed is assumit aliquid, quod rursum plus habet admirationis, quam hoc ipsum quod iam erat demonstrandum. Dicam tamen, si forte ex comparatione multorum falsorum eliciatur veritas. Esto ut vapor, quando primum frigus irruens sentiscit, coaguletur in sphaerulas certae quantitatis. Hoc est consentaneum. Nam ut in aqua, gutta minimum naturale est de fluido; propterea, quod aqua suo pondere non diffluit amplius, quando est infra guttae quantitatem: sic etiam facile concedi potest, inesse in vaporis materia tenacitatem aliquam, qua possit resistere frigori, in certa aliqua quantitate, puta guttae vapidae.

Secundo esto ut hae sphaerae vapidae se invicem contingant in certa dispositione, puta quadrangulari in plano, cubica in solido, qua de supra: sic enim tangetur sphaerula quaelibet ab aliis sex, quarum solae quattuor hic in plano depingi possunt; quinta et sexta intelligenda est superponi et supponi. His ita positis et assumptis, frigore vero per spatia irruente: sphaerulae a contactu uno ad oppositum erunt munitae contra frigus: itaque versus centra quidem sphaerularum fiet condensatio, sic tamen, ut etiam versus diametros contactuum, quibus scilicet locis tutae sunt a frigore.



Verum non immerito, ut praedixi, quaerat aliquis, qua vi sic disponantur sphaerulae in directum? Si

6 innitetur *K* *rectius* τὸν σκελετόν 8 repraesentassis *KC* 26 quinpta *K*

Now if you ask a geometer in what regular solid three diameters intersect at right angles at the same point (in other words, in the form of a double cross), he will reply 'in the octahedron', when lines are drawn from its opposite angles, of which it has six. Why then, does it happen that as a snowflake falls before it is flattened, it should imitate precisely the skeleton (so to say) of the octahedron with its three feathered diameters that intersect at right angles? Join up the neighbouring tips of the radii with twelve straight lines, and you will represent the complete solid of an octahedron.

What, then, is the cause why condensation takes place on these three feathered radii rather than in a perfect sphere?

I can point to one way for this to happen by material necessity; but this in its turn implies something that calls for more surprise than the very problem that it was to have solved. However, I will have my say, on the chance of coaxing the truth from the comparison of many false trails. Granted then that vapour coagulates into globules of a definite size, as soon as it begins to feel the onset of cold. This is reasonable: for the drop is the smallest natural unit of a liquid like water, because, when it is under the size of a drop, its weight does not make it spread more widely. Likewise it can be readily granted that the matter of vapour possesses a certain tenacity whereby to resist cold, assuming some definite limit of size in the drop of vapour.

Secondly, granted that these balls of vapour have a certain pattern of contact: for instance, square in a plane, cubic in a solid, as above. Thus any ball will be in contact with six others, only four of which can here be illustrated on a plane surface; the fifth and sixth must be understood as set above and below. On this assumption, when the balls are so arranged, the cold will force its way through the gaps, but the balls will be protected against it from one point of contact to its opposite. So condensation will occur towards the centres of the balls, but also at the same time towards the diameters between the points of contact, at which, of course, the balls are sheltered from the cold.

But, as I said before, somebody may well ask what force gets the balls into this four-square pattern.

Si materialiter fieri aliter non posset iam peractum esset negotium. At possunt materialiter duobus aliis modis disponi, ut supra dictum. At praeterea possunt omnes tres ordines ordinati confundi, ut fiat dispositio varia.

An hanc adsciscemus dispositionis huius causam, quod in hac sola dispositio est sibi ipsi undique similis, et puncta contactuum distribuuntur aequaliter, in ceteris nequaquam. Etsi enim ut supra dictum, globi singuli a duodenis aliis tanguntur, at spatia inter globos alternis triangula et quadrangula sunt: hic omnia undique quadrangula. Illic diametri quidem duo oppositorum contactuum sese secant orthogonaliter, reliqui quattuor non item: hic omnes tres diametri sese secant aequaliter et orthogonaliter. Illic connexis extremis diametrorum fit cuboctahedron, hic octahedron intra sphaerulam quamlibet.

Praestantia quidem hinc patet dispositionis directae prae obliqua: at causa nondum comparet, quae sphaeras hac potius quam illa ratione disponat. Num facit hoc frigus? At quomodo?

Nam si quid agit condensat aut penetrat materiam[29] *qua hiat illa, aut qua debiliter resistit. Et ut largus sim*[30] *directam quidem in profundum dispositionem causari possit descensu rectilineo, versus terram: at in transversum unde haec directio?*

Restat igitur, ut calor internus vaporis hanc guttarum dispositionem cubicam efficiat: si modo est cubica ipsarum dispositio, hoc est, si Nihil nostrum est aliquid.

Huc autem devolutare, iam perinde est, sive calor quamlibet guttam seipso in formam octahedri disponat: sive totam materiam in seriem stellarum ordinatam dispescat, atque ita singularum sphaerarum internam dispositionem per externam universarum seriem adiuvat. Neutrobique[31] *casu ordo existere potest tam constanter, ubi confusio, ut hic quidem, in proclivi est.*

Sed et argumenta sunt, ut potius credamus singulas guttas sine ope externi contactus seipsis disponi. Etenim, si figura singularum oriretur[32] *ab ordine et contactu mutuo plurium, necesse foret, omnes in vicem aequales esse stellulas. Iam vero magnum inter eas cernitur discrimen magnitudinis. Ipsa quin etiam multarum ordinatio, multum habet insolentiae.* *Nihil*

10 quidem] quidam *K;* quaedam *Caspar* 11 reliqua *K* 23 ipsorum *K*
29 neutrobique *K fortasse* neutroque in *vide notam* 32 *fortasse* se ipsas

Now if their material would not admit of their being otherwise, our task would be done. But, as was said above, the balls can be arranged in two other ways consonant with their material. Moreover, they can be duly stacked in all three orders, and then shuffled into hybrid patterns.

Shall we adopt as cause of the square pattern this: that in it alone the arrangement is identical in every direction and the points of contact uniformly distributed, whereas in the others quite the contrary? For, although in them each ball is touched by twelve others (as we noted above), the interstices, however, between balls are alternately three- and four-sided, whereas here [in the square pattern] all are four-sided throughout. In those others two diameters from opposite points of contact intersect at right angles, but the remaining four do not. In this pattern all three diameters intersect uniformly at right angles. In those a cuboctahedron results from joining up the ends of the diameters; in this an octahedron within every sphere.

This shows the superiority of the right-angled order over the oblique. But the cause which arranges the balls on this rather than that principle, is not yet in evidence. Surely cold cannot produce this effect? But how then is it produced?

For if cold produces any effect, it is that of condensation by penetrating the material either where there are gaps in it or where its resistance is feeble. And, to be generous, one could argue that the four-square pattern in depth came about owing to a vertical undeflected fall to earth. But then where could this sideways guidance of flakes into place come from?

The possibility therefore remains that the internal heat of the vapour causes this cubical distribution of drops, if indeed their distribution is cubical, that is, if our Nothing is Something.

Now that the question has come down to this, it is all one whether the heat on its own forms the drop into the shape of an octahedron or whether it parcels all the material into an ordered series of stars, and thus assists the internal organization of each individual ball by an external regimentation of all *en masse*. In neither case can an order remain so constant, where the slippery slope into chaos is as near as it is here.

But there are further arguments for believing rather that each drop is formed by itself without the help of an external contact. For if the shape of each arose from the organization and mutual contact of several more, it would follow that all starlets were equal to each other. But in fact we see a great difference of size between them. Moreover, the formation of many drops in a group has much that is abnormal about it.

Memineris praedictum, de nihilo esse opinionem de trina decussatione trium diametrorum.

Nihil itaque profecimus nisi pateat modus, quo calor internus guttam vapidam in tribus diametris, forma octahedrica vel certe sexangula firmet, ut ad eas fiat materiae per condensationem collectio.

Possit aliquis existimare, volitare villosa ista ramenta solitaria, interque cadendum decussatim concurrere fortuito. Verum id falsum est. Non enim perpetuo trina, non in punctis mediis, non ad unum punctum concurrerent. Adde quod villi omnes a centro seu stellae seu decussationis geminae aversi extrorsum porriguntur paene ut foliola in ramis abiegnis: quod argumento est, in centro nidulari vim formatricem, indeque in omnes plagas aequaliter sese didere.

Occasio sex plagarum in animalibus.

Sed fortassis haec causa est trium diametrorum, quod totidem sunt diametri plagarum in animalibus? Habent enim superas, inferas, anteriores, posteriores, dexteras, sinistras partes. Si quis hoc dixerit, meae is opinioni appropinquabit, sed praeter opinionem in paradoxa pertrahetur concessione sua. Primum enim consideret, quae natura sit huius caloris, quae similitudinem animalis architectetur in stellula nivis. Deinde videat, cui bono? Quid enim animali commune cum nive? Nix ad vitam, qua caret, plagis istis opus non habet. Tertio perpendat, ipsas animalis partes, non tam ad figuras geometricas, cubumque, primam solidarum figurarum, velut ad archetypum suum accommodatas: quam necessitate quadam ad finem obtinendum directas. Prima enim superi et inferi distinctio est a loco, quae est terrae superficies: pedes igitur deorsum vergunt ut contra pondus corporis nitantur, caput sursum est ut nervos imbre opportuno continue humectet, utque oculi et aures a planitie remotissimi plurimam eius circumferentiam in conspectu habeant, obstaculis remotis, denique ut cibus pondere, potus humore suo praecipitatus in suum locum descendat, neque continua ut in plantis uno loco fixis attractione opus haberet. Altera anticae et posticae distinctio tributa est animantibus ad motus exercendos, qui in recta linea super terrae superficiem tendit a loco ad locum. Itaque duae hae diametri necessario orthogonaliter se mutuo secant, signantque superficiem. At cum animalia non possint esse superficies, sed necessario corpora accipiant: tertiam diametrum dextri et sinistri ex ratione corpulentiae necesse fuit accedere, qua fit animal *quasi*

Remember what was said before: that the notion of the triple crossing of three diameters is based on nothing.

So we have made no progress, unless we bring to light a way for the internal heat to fix the drop of vapour on three diameters, in the shape of an octahedron, or at any rate in a six-sided shape, on which matter may accumulate by condensation.

Someone may think that these plumed particles drift about, mote by mote, and, as they fall, by chance meet crosswise. But this is a mistake: they would not always meet in threes, nor at their middle points, nor at one point. Furthermore all the plumes stretch outwards away from the centre of the star (or, in other words, the twin crosswise junction), almost like needles on branches of fir. This goes to prove that the formative power resides in the centre and from it disseminates itself equally in all dimensions.

The aptness of six directions in animals.

But is this perhaps the cause of the three diameters, that there is the same number of dimensions in animals? After all they have upper and lower parts, front and back, left and right. If anyone makes this suggestion, he will come close to my view, but his concession will involve him unexpectedly in self-contradictions. First let him consider, what the nature of this heat is that constructs a resemblance to an animal in a starlet of snow. Then he should ask, for whose benefit? What has an animal in common with a snowflake? A snowflake has no need of these dimensions for life, being lifeless. Thirdly let him reflect that the parts of an animal are less adapted to geometric figures and to the cube, the first of the regular solids, as their archetype, than guided by some necessity to achieve a purpose. For the first distinction, of upper and lower, depends on their habitat, which is the surface of the earth; their feet tend downwards so as to push against the weight of their body; the head is on top so as to lubricate the nerves continuously with suitable fluid, so that eyes and ears, lifted as far as possible from the level, should have the widest possible horizon in view, free from obstacles; and finally so that food delivered by its weight, and drink by its liquidity, should sink down to their proper place, and not require, as in plants tied to one place, continuous suction. The second distinction of front and back is conferred on animals to put in practice motions that tend from one place to another in a straight line over the surface of the earth. Consequently these two diameters cannot but intersect at right angles, and demarcate a surface. But since animals cannot be surfaces, but must have bodies given them, bodily existence entailed that the third diameter, of right and left, should be added, whereby an animal becomes so to speak doubled when the alternation of mover and moved is *quasi geminum: ut esset etiam in incessu, moventis et moti discrimen alternis. Non igitur, quae cubica sunt, hominis gerunt similitudinem propter aliquam figurae pulchritudinem: sed homo cubi acquisivit similitudinem, quasi concinnatam ex variis usibus ceu elementis.*

Principia ad eruendas causas figurae nivis non de Nihilo.

Itaque omnibus examinatis, quae occurrebant, sic ego sentio, causam figurae in nive sexangulae, non aliam esse, quam quae est figurarum in plantis ordinatarum numerorumque constantium. Ac cum in his nihil fiat sine ratione summa, non quidem quae discursu ratiocinationis inveniatur, sed quae primitus in creatoris fuerit consilio, et ab eo principio hucusque per mirabilem facultatum animalium naturam conservetur; ne in nive quidem hanc ordinatam figuram temere existere credo.

Est igitur facultas formatrix in corpore telluris, cuius vehiculum est vapor, ut humana anima spiritus: adeo ut nullus uspiam existat vapor, quin, ut calore quodam id effectus est quod esse dicitur, puta vapor, eodemque calore conservatur, ut id esse pergat: sic ratione etiam formatrice, quam alii calorem opificem dicunt, contineatur.

Sed duarum obiectionum solutione, quod reliquum est de opinione mea, declarabo. Etenim obiicere possis: in plantis finem subsequentem, qui est constitutio certi corporis naturalis, arguere, rationem formatricem in aliqua materia praecessisse; ubi enim media ad certum finem ordinata, ibi ordo, ibi nullus casus, ibi mera mens, mera ratio; in nivis vero formatione finem nullum spectari posse, neque fieri per figuram sexangulam, ut nix perduret, aut corpus naturale definitum certae et durabilis formae fiat. Respondeo, rationem formatricem non tantum agere propter finem sed et propter ornatum, nec solum tendere ad corpora naturalia efficienda sed et solere ludere in fluxis, quod multis fossilium[33] *exemplis patet. Quorum ego universorum rationem a ludicro (dum dicimus naturam ludere) ad hanc seriam intentionem transfero: quod puto calorem, qui hactenus tutabatur materiam, ubi a circumstanti frigore vincitur, ut hactenus ordine agebat (ratione quippe formatrice imbutus) ordine pugnabat, sic iam suo quodam ordine et fugae* *sese*

23 vero] vera *K*

distinguished in the process of walking. Whatever is cubic in shape does not therefore wear the likeness of a man by virtue of some beauty in its shape, but man has achieved likeness to a cube, a likeness that is a harmonious blend, with man's various functions as its constituents.

The principles needed to extricate the causes of a snowflake's shape do not arise from Nothing.

So after examining all the ideas that came into my head I conclude thus: the cause of the six-sided shape of a snowflake is none other than that of the ordered shapes of plants and of numerical constants; and since in them nothing occurs without supreme reason—not, to be sure, such as discursive reasoning discovers, but such as existed from the first in the Creator's design and is preserved from that origin to this day in the wonderful nature of animal faculties, I do not believe that even in a snowflake this ordered pattern exists at random.

There is then a formative faculty in the body of the Earth, and its carrier is vapour as the human soul is the carrier of spirit: so much so that no vapour ever exists without being bound by a formative principle, which others call the craftsman Heat, in the same way as it is by some form of heat that, being turned into what it is said to be, to wit, vapour, it exists and by the same heat is maintained so as to persist in being vapour.

But I will expound what ideas of mine still remain, by meeting two objections. You might argue thus: the ensuing purpose, which is the establishment of a definite natural body, points, in plants, to the pre-existence of a formative principle in some matter; for where the means are adapted to a definite purpose, there order exists, not chance; there is pure mind and pure Reason. But no purpose can be observed in the shaping of a snowflake; the six-cornered shape does not bring it about that the snowflake lasts, or that a definite natural body assumes a precise and durable shape. My reply is: formative reason does not act only for a purpose, but also to adorn. It does not strive to fashion only natural bodies, but is in the habit also of playing with the passing moment, as is shown by many ores from mines.[33] I transpose the meaning of all such from playfulness (in that we say that Nature *plays*) to this serious intention. I believe that the heat, which till then was protecting its matter, is now conquered by the surrounding cold; but just as previously, animated as it is by a formative principle, it had acted and fought in an orderly fashion, so now *sese comparare, pedemque referre; et diutius haerere in sparsis istis et ordinatim veluti per aciem distributis ramis, quam in tota reliqua materia; atque sic curae habere, ut (quod de Olympiade referunt historiae) non inhoneste nec inverecunde cadat.*

De Polyxena cum ad sepulcrum Achillis immolaretur hic apud Euripidem versus est *πολλὴν πρόνοιαν εἶχεν εὐσχήμων πεσεῖν.*[34] Eundem accommodat Plinius iunior in epistolis virgini cuidam Vestali quam Domitianus vivam defodit.[35]

Alius aliquis obiiciat, plantis singulis singulas esse facultates animales, cum seorsim etiam subsistant corporum plantarum singula: propterea que nil esse mirum, singulis etiam singulas aptari figuras. In nivis vero qualibet stellula peculiarem fingere animam, per esse ridiculum: quare ne figuras quidem nivis eodem modo ex animae opere, ut in plantis deducendas.

Respondeo, rem utrinque similiorem esse, quam, qui haec obiicit, credere possit. Demus plantis singulis singulas esse facultates: at eae omnes suboles sunt unius et eiusdem facultatis universalis, quae in terra inest, quaeque se habet ad plantas, ut facultas aquae ad pisces, facultas humani corporis ad pediculos, canini ad pulices, ovilli ad aliud genus pediculorum. Non enim omnes plantae ex semine, pleraque ἐξ αὐτομάτου primum ortae, etsi sese porro seminent. Facultas enim terrae, quae seipsa una est et eadem, dividit sese in corpora et cum corporibus,[36] *inque ea*[37] *inolescit et pro cuiusque materiae conditione interna externisve aliud atque aliud architectatur. Ita in vapore quoque, quem totum tota possederat anima, nihil mirum si frigore divisionem totius continui moliente, ob contractionem partium, circa partes ipsa, ut circa singula tota formando, occupetur.*[38]

O vere mortuam vitam sine philosophia! Hanc enim in nive formatricem facultatem si scivisset illa Aesopicae fabellae adultera,[39] *persuadere marito potuisset, se ex nive concepisse, spurioque suo non tam facile fuisset orbata, calliditate mariti.*

Animus imago creatoris.

Dixi de auctore figurae: restat ut inquiramus de figura ipsa, sive illa existat ex decussatione trium diametrorum, quod hactenus est inter supposita: sive inde ab origine sit sexangula, de quo postea. Nunc pergendum in tramite coepto. Causa igitur cur haec facultas octahedri dispositionem angulorum potius imitetur, haec esse possit. Primum universum genus animorum geometricis et regularibus sive cosmopoeeticis figuris cognatum est: quod multis documentis probari

6 corporum *K facilius* corpora 11–13 *in margine* πολλῶ προνοίαν εἴσχεν σύσχυατε πεοεῖν K^1: πολλὴν πρόνοιαν εἶχεν εὐσχήμως πεσεῖν K^2

it displays an order of its own in preparing for retreat and withdrawal, and holds out longer in the selected branches, or outposts, that are distributed in good order over the line of battle than in all the rest of its matter. Thus it takes care 'not to fall in an ugly and immodest fashion', as histories relate of Olympias.

The following verse about the sacrifice of Polyxena at Achilles' tomb is in Euripides: 'She took much forethought to fall decently.' Pliny the Younger in his *Letters* applies it to a Vestal Virgin whom Domitian buried alive.[35]

Someone else may object: each single plant has a single animating principle of its own, since each instance of a plant exists separately, and there is no cause to wonder that each should be equipped with its own peculiar shape. But to imagine an individual soul for each and any starlet of snow is utterly absurd, and therefore the shapes of snowflakes are by no means to be deduced from the operation of soul in the same way as with plants.

I reply: the likeness is much greater on either side than this objector could believe. Let us grant that each single plant has its own principle; but they are all offspring of one and the same universal principle, inherent in the earth and related to plants as the principle of water is to fish, of the human body to lice, of the bodies of dogs to fleas, and of sheep to some other kind of louse. Not all plants anyhow originated from seed, but most of them arose spontaneously, although they have since propagated themselves by seeding. The faculty of earth is in itself one and the same, but it imparts itself to different bodies and co-operates with them. It engrafts itself on to them, and builds now one design, now another, as the inner disposition of each matter or outer conditions allow. Thus in vapour too, which as a whole had been possessed by soul as a whole, there is nothing to wonder at, if, when cold engineers the break-up of the uniform whole, the soul should be busy with forming the parts, since it is the parts that contract, just as it had been busy, when whole, with forming wholes.[38]

How true it is that Life without philosophy is Death! If the celebrated adulteress of Æsop's fable[39] had known of this formative faculty in the snowflake, she could have convinced her husband that she had conceived from a snowflake, and would not have been so easily bereaved of her bastard by her husband's cunning.

Spirit is the image of the Creator.

I have spoken of the author of the shape. It remains to inquire about the shape itself. Either it arises from the crossing of three diameters, as has been one of our assumptions hitherto, or it is six-cornered right from the start. But of this later; meanwhile we must pursue our present line of argument further. The reason why this faculty prefers to imitate the arrangement of angles in the octahedron might be this: first the whole realm of spirits is akin to the regular geometrical or world-building figures, *bari potest. Cum enim animi sint quaedam exemplaria Dei Creatoris, certe in Dei Creatoris mente consistit Deo coaeterna figurarum harum veritas. Amplius cum certissimum sit, ipsos etiam animos penitissima sui essentia recipere quantitates, sine materia physica an cum ea, non disputo: consentaneum est, figuratas potius recipere quantitates, quam rudes: si figuratas, quare figuras regulares,*[40] *solidas, quia animi sunt non superficierum sed corporum solidorum.*

Reditur ad opinionem nihili.

Est autem inter regulares solidas prima, cubus, primogenita, parens ceterarum. Eius vero femina quasi quaedam est octahedron, habens tot angulos, quot cubus plana, eorumque centra, quibus singulis singuli respondent ex octahedro anguli.

Portiuncula itaque materiae vapidae deserenda, si figuram debet recipere, quod iam fecimus consentaneum, primum cubum arripiet, eiusque socium octahedron. Quorsum supra etiam materialis alludebat necessitas, globorum aequalium in unum acervum confusorum. Confundebantur enim et adumbrabantur in punctis contactuum rudimenta cubi octahedrique. At cur octahedri figuram potius quam cubi? An quia cubus est figura dilatationis, octahedron collectionis? Iam vero et materia et vis calorifica colliguntur impetita hostiliter a frigore. Unde vero certum sit, illam esse dilatationis figuram, hanc collectionis? Nempe quia octo anguli, quibus illa foris diditur, idem in hac intus centrum circumsistunt eodem numero. Etenim si cubo adimas angulos suos octonos resectos lateribus aequalibus, introrsumque componas plane constitues octahedron. Et cubus in plures, scilicet in octo angulos diffunditur, octahedron in pauciores, puta sex.

Aiunt gemmarii naturalia in adamantibus inveniri octahedra, perfectissimae et limatissimae formae. Id si est, multum nos confirmat. Nam facultas animalis, quae in terra indidit adamanti formam octahedri, ex penitissimo sinu suae naturae depromptam, eadem cum vapore progressa de terra, figuram eandem indidit et nivi ex vapore illo consistenti.

Quanquam, quod decussationem trium diametrorum attinet, in ea non magis inest octahedri quam cubi forma. Illic anguli, hic

and this can be demonstrated by many proofs. Spirits are, as it were, semblances of God, the Creator, and so without doubt the authentic type of these figures exists in the mind of God the Creator and shares His eternity. Further, since it is quite certain that even spirits admit of quantities in their innermost essence—whether with or without matter, I leave unargued—it is consistent that they should admit of shaped rather than raw quantities, and, if shaped, rather the shapes of the regular solids, since spirits are the spirits of solid bodies, not of surfaces.

Return to a conception of no value.

Now among the regular solids, the first, the firstborn and the father of all the rest, is the cube, and his wife, so to speak, is the octahedron, which has as many corners as the cube has sides and centres of these sides; and the corners of the octahedron correspond, one to one, to these centres.

Thus, when a fragment of vaporous matter is to be left to itself, if it must assume some shape, as we have already made plausible, it will seize first on the cube and its associate, the octahedron. It was in this direction that material necessity pointed above, when balls of equal size were tumbled together in one heap. For in the heap the outlines of cubes and octahedrons were sketched by their points of contact. But why the shape of the octahedron rather than of the cube? Is it because the cube is the shape of spreading out and the octahedron of gathering in? And gathered in is precisely what both matter and heat-engendering force are, under hostile attack from cold. But how can it be certain that the one is the shape of spreading out and the other of gathering in? Why, because the same eight corners by which the one is partitioned on the outside, in the other stand inside round the centre in equal number. Moreover, if you remove from a cube its eight corners on equal sections and rearrange them to face inwards, you will produce precisely an octahedron. Also the cube spreads out into more corners, to wit, eight, but the octahedron into fewer, to wit, six.

Jewellers say that natural octahedra of the most perfect and exquisite form are found in diamonds. If this is so, it goes far to support us. For the same faculty of soul which clothed the diamond within the earth in the form, which it furnishes from the innermost treasury of its nature, that of the octahedron, when it emerged from the earth in vapour, clothed with the same shape the snowflake which arises from that vapour.

As for the crossing of three diameters, however, the octahedron is no more involved in it than the cube is. In the octahedron the corners and *hic centra planorum connectuntur huiusmodi tribus diametris; illic anguli diametrorum ad centrum, hic angulus qui corpus finit exprimitur. Frustra igitur de totius figurae electione satagimus, ubi est utriusque rudimentum saltem.*[41]

Quo vero abripior stultus ego, qui dum paene Nihil donare affecto, paene etiam Nihil ago: quia ex hoc paene Nihilo paene mundum ipsum, in quo omnia, efformavi: cumque ab animula minutissimi animalculi supra refugerim, iam ter maximi animalis, globi telluris, animam in nivis atomo exhibeo?

Itaque pedem referam, et sedulo dabo operam, ut quod donavi, quodque dixi, id Nihil sit. Fiet autem id, si quam cito nivula mea liquescit, tam cito ratiunculas istas ego contrariis ratiunculis profligavero atque annihilavero.

Denique serio de figura nivis stellata.

Dum enim ista scribo, rursum ninxit, et confertius, quam nuper. Contemplatus sum sedulo corpuscula nivis, cadebant igitur omnia radiosa, sed duorum generum: quaedam minuta valde, radiis circumcirca insitis, incerto numero, et simplicibus, sine villis, sine striis; erantque subtilissimi, in centro vero colligati ad grandiusculum globulum: atque horum erat maxima pars. Inter spargebantur autem secundi generis rariores sexangulae stellulae earumque nulla aliter nisi plana neque volitabat neque cadebat, villis etiam in eandem planitiem cum caule suo compositis. Vergebat autem inferius deorsum radiolus septimus, quasi radix aliqua, in quam cadentes incumbebant, eaque sustinebantur sublimes aliquandiu: quod me supra non fugit, sed sinistre exceptum est, ac si terni diametri non essent in eodem plano. Itaque non minus quod hactenus dixi, quam de quo dixi,[42] *a Nihilo quam proxime abest.*

Primum genus grumosum, puto esse ex vapore iam paene deserto a calore, et iam iam in guttas aqueas condensando. Itaque et rotunda sunt, et figuram pulchram non sortiuntur, deserta iam ab architecto, et radiosa sunt undique, iis principiis, quae supra ad contemplationem pruinosae consistentiae in fenestris sunt adhibita.

In secundo vero genere, quod est stellularum, locum nullum habet contemplatio cubi vel octahedri, neque ullus guttarum contactus: cum plana

26–27 quam quo dedixi *K* quam quo de dixi *Caspar*

in the cube the centres of the sides are linked by three such diameters. There the angles between the diameters are drawn at the centre, here the crossing is expressed in the angles that bound the cube externally. So we have busied ourselves in vain over the choice of the complete figure, where both have at least the makings of the pattern.[41]

But this is folly, to be so carried away. Why, my endeavour to give almost Nothing almost comes to nothing! From this almost Nothing I have almost formed the all-embracing Universe itself! Although above I fought shy of the diminutive soul of the tiniest mite, here I am exhibiting the soul of the 'thrice-greatest' Animal, the globe of the Earth, in the mote of a snowflake!

So I shall beat a retreat and take pains to see that what I have given and said should be Nothing. This will come about if, as quickly as my snowflake melts, I rebut these trivial arguments with as trivial counter-arguments and reduce them to—nothing.

At last the starred shape of the snowflake is taken *seriously*.

For as I write it has again begun to snow, and more thickly than a moment ago. I have been busily examining the little flakes. Well, they have been falling, all of them, in radial pattern, but of two kinds: some very small with prongs inserted all the way round, indefinite in number, but of simple shapes without plumes or stripes, and very fine, but gathered at the centre into a slightly bigger globule. These formed the majority. But scattered among them were the rarer six-cornered starlets of the second kind, and not one of them was anything but flat, whether it was floating or coming to earth, with the plumes set in the same plane as their stem. Furthermore, under the flake a seventh prong inclined downwards like a root, and, as they fell, they rested on it and were held up by it for some time. This had not escaped me above, but I took it in a mistaken sense as though the three diameters were not in the same plane. So what I have said hitherto, no less than what I have had my say about, is as little removed from Nothing as may be.

The first, lumpy, kind is formed, I think, from vapour that has almost lost its heat and is on the point of condensing into watery drops. So they are round, and no beautiful shape comes their way either, abandoned as they are by the master builder [heat], and they stretch out radially in all directions on the principles that were applied above to the examination of hoar-frost formations on windows.

But in the second kind, that is, of starlets, observation of cube or octahedron has no place. There is no contact of drops, since they *plana incidant, non ut supra sum opinatus, decussata trinis diametris.*

Etsi igitur formatrix anima hic quoque locum suum tuetur, manetque in causa:[27] *de electione tamen figurae quaestio est redintegranda. Primum cur plana? An quia non recte supra ademi planas formatricibus corporum? Nam in omnibus floribus inest quinquangulum planum: non dodecahedrum solidum. Tunc causa planae figurae vere haec esset: quod frigus calidum vaporem in aliqua planitie tangit nec ita totum vaporem aequaliter circumstat cum stellulae gignuntur, ut cum grumi cadunt.*

Cur figura potissimum sexangula.

Cur autem sexangula? An quia ex regularibus haec prima est vere plana, et quae in nullum corpus secum colligatur? Nam trigonus tetragonus pentagonus corpora efficiunt. An quia sexangulum sternit planitiem, excluso vacuo? At idem facit triangulum quadrangulum? An quia proxima haec circulo ex iis quae planitiem sternunt, excluso vacuo. An hoc discriminis inter facultatem sterilia figurantem, et alteram illam, quae fecunda figurat, ut illa triangula vel sexangula faciat, haec quinquangula? An denique ipsa huius formatricis natura in intimo sinu suae essentiae particeps est sexanguli?

Ex quinque adductis causis prima, secunda, et tertia hoc sibi usurpant, facultatem formatricem, e re nata consilium capere, et pro opportunitate campi aciem instruere: ut quia pugna calidi vaporis et frigidi aeris in planitie existit, non per corpulentiam, ipsa quoque figuram eligat quae planitierum est potius quam corporum. Itaque et materialis necessitatis rationem haberet in secunda et tertia. Nam prima causa sola sexanguli proprietate freta est, respiciens decentem congruentiam huius figurae ad hanc pugnam. In plano pugna, necessario igitur figura plana, at non necessario figura talis, quae ad nullum corpus secum ipsa coeat, sed ideo solum talis quia ut corporibus physicis figurae respondent, quae solidum ambeunt, sic planitiebus figurae quae solidum non ambeunt: hic decentia formalis spectatur, non necessitas materialis.

At in secunda et tertia hoc dicendum esset, necessitate etiam material ieligi a formatrice sexangulum, ne quid scilicet relinquatur vacuum, et

18 *an melius* ⟨facultatis⟩ formatricis? *at vide ll. 5 et 34*

settle as flat objects and not, as I thought above, with three diameters crossed.

So, although here too the formative soul maintains its place and remains in play as a cause,[27] the question of the choice of shape must be taken up again. First, why flat? Is it because I was wrong to remove, as I did, plane surfaces from among the builders of bodies? There is, after all, in all flowers a flat pentagon, not a solid dodecahedron. If so, the cause of flatness would really be this: that cold touches warm vapour on a plane and does not surround all the vapour uniformly when starlets are produced as it does when it falls in lumps.

Why the six-cornered shape is preferred.

Next, why six-cornered? Is it because this is the first of the regular figures to be essentially flat, incapable, that is, of combining with itself to form a solid body? For triangle, square, pentagon, all form bodies. Is it because the hexagon lays a flat surface without a gap? But triangle and square do the same. Or because the hexagon comes nearest to the circle of those figures which lay a flat surface without a gap? Or does this make the difference between a faculty that builds sterile shapes, triangles, and hexagons, and that second faculty that builds fruitful shapes, pentagons? Or, finally, does the nature of this formative faculty partake of six-corneredness in the inmost recess of its being?

Five causes have now been adduced; the first, second, and third of them assert this claim for themselves, that the formative faculty makes its plan of action in accordance with what is there for it and draws up its line of battle as the battlefield offers. The battle between hot vapour and cold air takes place in a plane and not in bodily depth, and so the faculty conforms in choosing a shape that belongs to planes rather than bodies. Thus in the second and third causes it would take material necessity into consideration. The first cause above relied on the hexagon, with an eye to the beautiful aptness of *this* shape to *this* battle. The battle is on the flat, and so necessarily the shape is flat, but not necessarily such as not to combine with others of its kind to produce a body; flat, then, for this reason alone, that shapes unable to enclose a solid correspond to planes, just as shapes enclosing a solid correspond to physical bodies. Here aptness and beauty of shape, not material necessity, is in view.

But with the second and third causes we should have to say that six-corneredness is chosen by the formative faculty from material necessity *cuum, et ut commodius fieri possit collectio vaporis in nivis consistentiam.*

In circulo enim commodissime fieret, at quia circelli vacua spatia relinquunt, ideo circuli similior eligitur figura. Verum huic causae iam supra fuit opposita inaequalitas stellularum, quarum aliquae minutissimae sunt, radiis etiam exilissimis et simplicibus, sine villis. Quod est argumento, non magnam aliquam vaporis planitiem simul coire in nivem. Sed seorsim planitiunculas, minimas, alias post alias, easque inaequales. Non habet ergo locum consideratio exclusionis vacui, quae regnat tantum in divisione integra superficiei in sexangula aequalia. Ita fiet ut secunda et tertia causa e numero deleantur, nisi quatenus ad primam redigi possunt, ut formatrix facultas sexangulum eligat, nulla materiae spatiorumque necessitate coacta, sed solum decentia hac invitata, quod alias sexangulum struat planitiem excluso vacuo, sitque (ex iis figuris, quae idem possunt) circuli simillima.

Qui flores senarium numerum habeant et ternarium?

Quarta quidem causa sic nuda consistere nequit. Nam alba lilia trinis senisque effigiantur foliis, et sterilia non sunt: eodem modo multi calices florum fere silvestrium. Nisi forte hoc discriminis sit, quod fructus sub flore quinquangulo enascitur carnosus ut in pomis pirisque, aut pulpaceus, ut in rosa cucumeribusque,[43] *seminibus intra carnem vel pulpam abditis. At sub flore sexangulo nil enascitur, nisi semen in sicco loculo, estque velut in flore fructus. Aut est hoc forte discrimen, quod nullus flos sexangulus in arboribus et fruticibus, sed in herbis et fere bulbaceis. Vel consideret alius ipsos succos an aliquod in iis discrimen secundum figuras florum.*

Considerandа botanicis.

Figurae fossilium crystalli sexangulae.

Res mihi nondum comperta est, itaque sufficiat leviter admonuisse alios de hac quarta causa.

Pro quinta causa faciunt opera huius formatricis facultatis alia, ut crystalli, omnes sexangulae, cum adamantes octahedrici sint rarissimi. Sed formatrix telluris facultas non unam amplectitur figuram, gnara totius geometriae, et in ea exercita. Vidi enim Dresdae in aede Regia cui Stabulo nomen, exornatum abacum[44] *aere argentoso, ex quo quasi efflorescebat dodecahedron avellanae parvae magnitudine, dimidia parte extans. Extat et in descriptione thermarum Bollensium I-*[45] *cosahedri*

8 planiticulas *K* 12 formatriis *K* 19 sylnestrium *K* 21 in rosa, cucumeribus seminibus *K*: in rosa cucumeribus seminibus *C*

as well, so that no gap should be left and the gathering of vapour into formations of snow should take place more smoothly.

It is in a circle that it would be smoothest, but, as rings leave empty spaces, a shape rather like a circle is chosen. Now this cause has already above met with the objection that the starlets are of unequal size; some are quite tiny and their radii too are very slender and simple, without plumes. This goes to show that it is no large flat surface of vapour that condenses simultaneously into snowflakes, but small planes of minute size, separately, one after another, and of different sizes at that. Consequently the avoidance of empty space can play no part in the argument, since it is the rule only where a whole surface is to be divided into equal hexagons. The result will be to delete the second and third causes from the list, except in so far as they can be reduced to the first, namely that the formative faculty chooses six-corneredness, not under duress of any material or spatial necessity, but solely because it is allured by this aptness whereby the hexagon elsewhere can form a plane without remainder and more than any other shape with the same capacity resembles a circle.

What flowers exhibit sixes and threes?

The fourth cause too cannot stand on its own without some qualification. For white lilies are formed with petals in threes and sixes, and are not sterile; likewise many calices of flowers, for the most part wild—unless perhaps there is this difference, that the fruit from a five-cornered flower grows fleshy as with apples or pears, or pulpy as with roses and cucumbers, in which the seeds are tucked away inside the flesh or pulp. But from a six-sided flower nothing is born but a seed in a dry satchel, and the fruit is virtually in the flower. Or this is perhaps the difference, that among trees and bushes there is no six-cornered flower, but only among vegetables and as a rule the ones with bulbs. Some botanist might well examine the saps of plants to see if any difference there corresponds to the shapes of their flowers.

For the attention of botanists.

I have not yet got to the bottom of this. Enough if I have given others some slight notice of this fourth cause.

Shapes of geological specimens from mines: six-cornered crystals.

For our fifth cause other products of this formative faculty can speak. Crystals for instance, all six-cornered, whereas octahedral diamonds are exceedingly rare. But the formative faculty of Earth does not take to her heart only one shape; she knows and is practised in the whole of geometry. I have seen in the Royal Palace at Dresden, in the Stables, a panel[44] inlaid with silver ore; from it a dodecahedron, like a small hazelnut in size, projected to half its depth, as if in flower. In an illustration of the hot baths at Boll[45] the front part of an icosahedron is shown among the exhibits

Consideranda metallariis et alchimistis.

cosahedri pars anterior inter fossilia. Itaque verisimile est hanc facultatem formatricem pro diverso humore diversam fieri. In vitriolo crebra est figura cubica rhombica. In nitro sua est figura. Dicant igitur chymici, an in nive sit aliquid salis, et quodnam salis genus, et quam illud alias induat figuram. Ego namque, pulsatis chymiae foribus, cum videam quantum restet dicendum ut causa rei habeatur, malo abs te, vir solertissime, quid sentias audire quam disserendo amplius fatigari.

Nihil sequitur

FINIS

For the attention of metallurgists and chemists.

from its mines. It is probable, therefore, that our formative faculty varies with variations in the liquid. In sulphates of metals the rhomboid cubic shape is common, and saltpetre has its own shape. So let the chemists tell us whether there is any salt in a snowflake and what kind of salt, and what shape it assumes otherwise! Now that I have knocked at the door of chemistry and see how much remains to be said before we can get hold of our cause, I prefer to hear what a man of your great acumen thinks rather than to tire myself with further discourse.

Nothing to follow

The End

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ON THE SHAPES OF SNOW CRYSTALS

A commentary on
Kepler's essay 'On the Six-Cornered Snowflake'

B. J. MASON

THE remarkable beauty of snow crystals, revealed in the classical elegance of the simple geometrical shapes and the delicate tracery of the more intricate forms, has long been recognized and recorded by the naturalist, the scientist, the artist, and, recently, by the industrial designer. Composed of ice, and formed by the condensation of water vapour about some minute nucleus, they appear in a wide variety of shapes and forms that fall broadly into four main classes: the thin hexagonal plate, the hexagonal prismatic column, the long, slender needle, and the six-pointed star, typical examples being shown in Fig. 1. These four principal themes are subject to almost infinite variation and it is very difficult to find two crystals that appear identical in their fine structure. Combinations of two or more basic types are not uncommon; the prismatic column with end-plates (Fig. 2 (*a*)), and the transition from plate to stellar form (Fig. 2 (*b*)), being particularly good examples.

Snowflakes are agglomerates of individual crystals in which the star-shaped crystals are generally prominent (see Fig. 3), but in which needle and plate forms may also appear. The largest snowflakes are as large as the palm of the hand and contain hundreds of individual crystals.

The outstanding common feature of the various forms of snow crystal is their hexagonal symmetry and this was pointed out, perhaps for the first time in Europe, in Kepler's remarkable essay of 1611. The reason for this macroscopic six-fold symmetry, which so exercised Kepler, is now readily explained in terms of the atomic architecture of ice, but the variety and change of crystal shape has always been a puzzle and still awaits a complete explanation.

Early Observations on the Structure of Snow Crystals

Although Kepler's essay marks the beginning of the scientific examination of snowflakes in Europe, the subject has its origins in ancient China, where the six-sidedness of the crystals was noted in the second century B.C. In a most interesting article on 'The Earliest Snow Crystal Observations', Joseph Needham and Lu Gwei-Djen[1] draw attention to several early Chinese references, the following being of particular interest.

In his book *Han Shih Wai Chuan* (Moral Discourses Illustrating the Han Text of the Book of Odes) published about 135 B.C., Han Ying makes the statement: 'Flowers of plants and trees are generally five-pointed, but those of snow are always six-pointed.'

A poem by Hsiao Thung (A.D. 501–531) contains the following lines:

The ruddy clouds float in the four quarters of the caerulean sky,
And the white snowflakes show forth their six-petalled flowers.

In the twelfth-century writings of the great Chinese philosopher Chu Hsi, there appears: 'Six generated from Earth is the perfected number of Water, so as snow is condensed into crystal flowers, these are always six-pointed.' In many of the classical Chinese writings it seems that six was the symbolic number for the element Water, while five was associated with Earth.

The European literature on snowflakes begins with the writings of Albertus Magnus (*circa* A.D. 1260), who thought the crystals were star-shaped. The next reference did not appear until A.D. 1555, when in a book by Olaus Magnus, Archbishop of Uppsala, we find the first sketches of snow crystals. The very poor woodcut in this book shows, among twenty-three forms, only one recognizable stellar shape. The hexagonal symmetry was not clearly recognized until sixty years later, first by Kepler, and re-emphasized by Descartes in his *Meteorologia* of 1635.

Descartes made detailed and quite accurate drawings, some of which are reproduced in Fig. 4, and these may be regarded as the first scientific records of snow crystals. It is surprising to find here a drawing of a rather rare type of crystal consisting of a hexagonal column with plates on both ends. Descartes was obviously impressed by the hexagonal symmetry of the crystals but apparently believed that this resulted from the uniform packing of initially irregular crystals. He imagined the exposed, untidy arms of the crystals melting, the resulting liquid spreading over the surface, filling in the irregularities, and leaving a flat, polished surface. Composite snowflakes are rarely, if ever, as regular as depicted by Descartes, and nowadays his explanation seems curiously inverted. Nevertheless, he sensed, like Kepler, a relationship between the hexagonal form and the packing of uniform particles in two dimensions.

In 1661, there appeared a little essay, *De Figura Nivis*, by Erasmus Bartholinus, which contains a perceptive discussion of the geometrical organization of crystals. Rather as Descartes had done, Bartholinus

[1] *Weather* **16** (1961) 319.

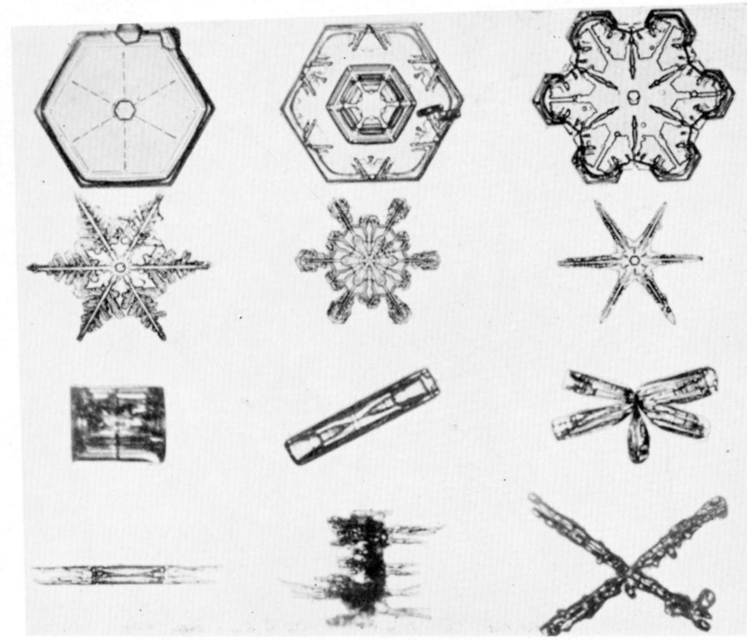

FIG. 1. A representative sample of natural snow crystals showing hexagonal plates, stellar crystals, six-sided columns, and needles

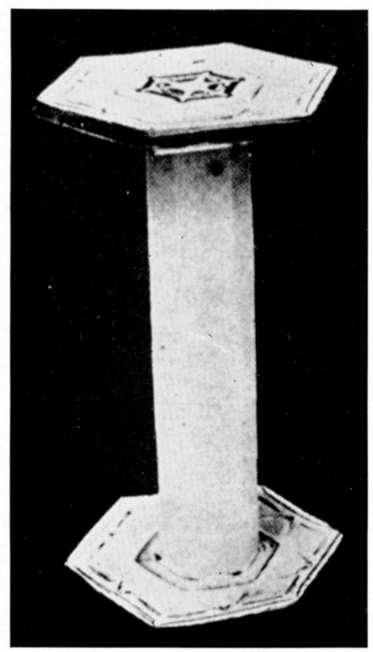

FIG. 2(*a*). A six-sided hexagonal column with end-plates

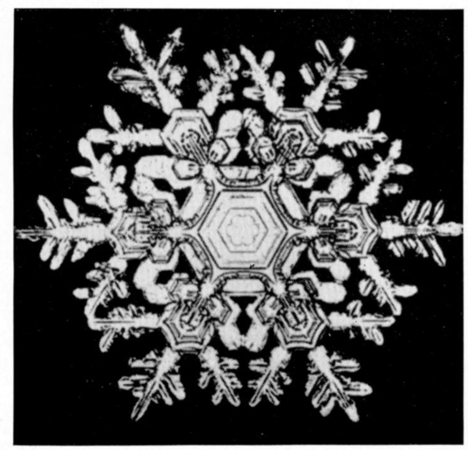

FIG. 2(*b*). A hexagonal plate which has developed into a richly branched (dendritic) stellar crystal. (From Bentley and Humphreys, *Snow Crystals*, McGraw Hill, 1931)

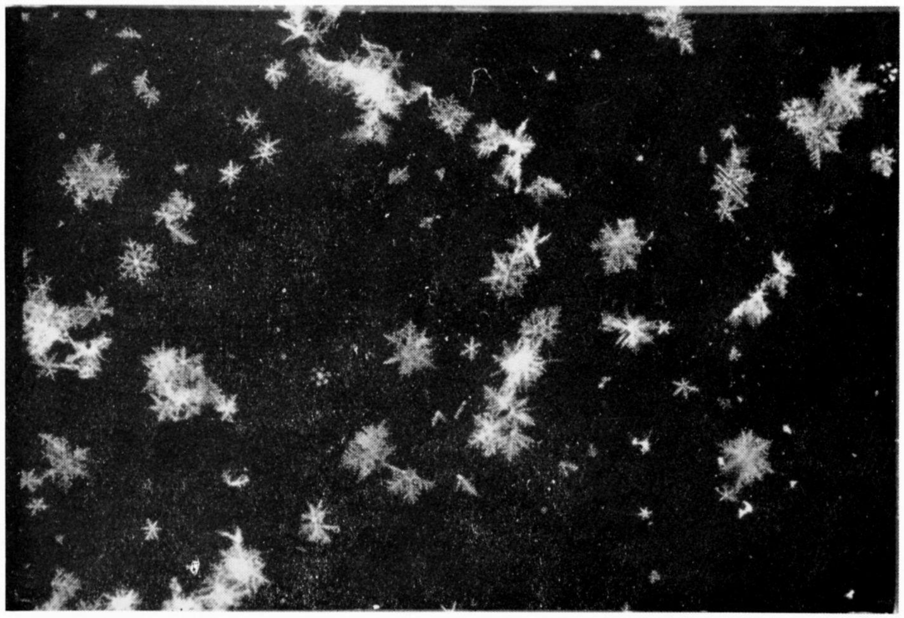

FIG. 3. Photograph of snowflakes consisting of clusters of dendritic stellar crystals

thought that the shaping of six-pointed stars resulted from contacts between an organized array of globules.

In his *Micrographia*, published in 1665, Robert Hooke reproduces several sketches of snow and frost crystals made with the aid of his microscope. He particularly emphasized that, in the branching star-shaped crystals, all the twigs extending from a main branch were parallel to the

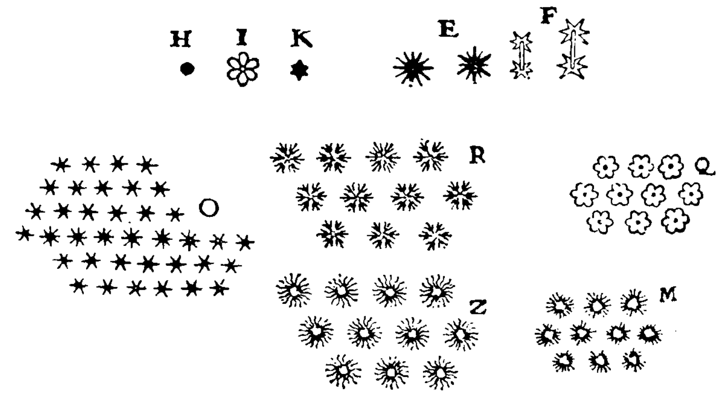

FIG. 4. Sketches of snow crystals by Descartes (from *Snow Crystals* by U. Nakaya, Harvard Univ. Press, 1954).

neighbouring main branch and so revealed the errors in the sketches of earlier workers.

A few years later, we find the first discussion of the correlation between snow-crystal forms and weather conditions. This appeared in a report, by Friedrich Martens, of his journey from Spitzbergen to Greenland, during which he made many observations of snow crystals in arctic regions.

In 1681, Donato Rossetti, an Italian priest and mathematician, collected drawings of sixty snow crystals and made the first attempt to classify them into (five) distinct forms. On the basis of careful observations under the microscope, he refuted Descartes's idea that snowflakes were composed of myriads of fine threads, the aggregation and consolidation of which formed the basis of Descartes's theory of hexagonality.

These early observers laid the foundations for a scientific study of snow crystals but, with the exception of Descartes, they confined their attention almost exclusively to the hexagonal plane crystals. Detailed descriptions of columnar and pyramidal crystals, and of hybrid forms, did not appear until 1820 with the publication of William Scoresby's book describing whale fishing in the Arctic.

In 1832, shortly after Scoresby's book was published, there appeared in Japan the *Sekka Zusetsu* (Illustration of Snow Blossoms) by Toshitsura Doi containing eighty-six sketches of snow crystals made with a 'Dutch glass'. These drawings, and ninety-seven similar ones published in a second volume in 1839, are superior to anything published previously and are probably the most accurate observations made before the development of microphotography. It is curious that, with the publication of *Sekka Zusetsu*, the scientific study of snow crystals apparently ceased in Japan for nearly one hundred years. According to Professor Nakaya, even the importation of modern Western science after the Meiji Restoration left no trace of snow-crystal studies in the Japanese scientific literature until the subject was taken up again in the early 1930's.

The development of microphotography led to a renewed interest in Europe, and many English, German, and Russian meteorologists took very good photographs of snow crystals. The paper of Nordenskiöld, published in Stockholm in 1893, and that of Hellman of Berlin appearing in 1894 are of particular interest in that they classified snow crystals into three kinds: planar, columnar, and combinations of the two. The basic idea of this system remains to the present day.

Another important, but little-known, work is Dobrowolski's extensive study reported in his *Historja Naturalna Lodu* published in Warsaw in 1922. But the most outstanding collection of snow-crystal photographs from the artistic point of view is undoubtedly that built up over fifty years by W. A. Bentley. Taken in Vermont, U.S.A., in the early days of photography, out-of-doors, in sub-freezing temperatures, these pictures have never been surpassed. In 1931, the American Meteorological Society gathered together a permanent collection of Bentley's work. Accompanied by a brief text, written by W. J. Humphreys, over two thousand photographs were published in one of the most handsome of scientific books ever printed.[1]

The systematic and sustained scientific study of snow crystals, of their growth and form in relation to environmental conditions, really started in 1932 with the work of Ukichiro Nakaya and his Japanese colleagues, which has continued almost uninterrupted for thirty years. Most insight into the mechanisms of snow-crystal growth has arisen from experiments in which artificial crystals are grown under carefully controlled conditions in the laboratory.

[1] *Snow Crystals*, by W. A. Bentley and W. J. Humphreys. McGraw-Hill (1931). Reprinted as paperback by Dover Press (1963).

Kepler's Hypotheses on the Formation of Six-Sided Crystals

Until very recently, the literature on snow crystals was concerned almost exclusively with description and classification of the main crystal forms and very little with the mechanisms of growth and the factors that might determine shape and form. In that he clearly posed the problem, and proposed several alternative explanations of the hexagonal symmetry of snow crystals, Kepler's contribution remained almost unique for 300 years.

Kepler recognized that snowflakes are composed of several individual units brought together, as he thought, by irregular drifting and that each individual always possesses six corners and not five or seven. In this he differed (correctly) from Descartes, who held roughly the inverse view, that the hexagonal unit was evolved by a partial melting of an agglomeration of irregular particles. Kepler realized that the six-sidedness could not be inherent either in the water vapour or in the coldness of the air, but discussed whether it was imposed by an external factor and arose perhaps because the hexagon was a particularly suitable or efficient form. This led him to a consideration of ways of filling space with repetitive figures and to the idea that natural structures such as bees' honey-combs and pomegranates achieve their shape from necessity. He discusses, for the first time, cubical and hexagonal close-packing of equal spheres, pointing out that, in the cubical arrangement, each sphere is touched by six others and, in the hexagonal arrangment, by twelve others. Hexagonal close-packing was found to give the tightest packing and to produce rhomboidal aggregates. Kepler suggested that a pomegranate needs to store the maximum number of seeds in the smallest possible space, so that hexagonal close-packing is not only the most efficient method, but is also a material necessity. But he saw also that such arguments were not necessarily applicable to a *flat* snow crystal[1] and, having argued convincingly that such a form is highly unlikely to result from the reduction of a regular three-dimensional aggregate of snow, even if this were composed of regularly packed rows of globules, Kepler went on to discuss space-filling in two, rather than three, dimensions. He thought that the flatness of the snow crystal might be explained in terms of the cold meeting the warm vapour at a plane interface, but then asked why should it be six-cornered. Regular hexagons can fill two-dimensional space without gaps but so can equilateral triangles and squares. In any case, it is possible to fill space completely with hexagons only if they are of the same size and, as Kepler points out, snowflakes are not of uniform size.

[1] We must remember that Kepler did not recognize that snow was crystalline but thought of it, rather, as an aggregate of globules of condensed moisture.

In short, after considering a number of possible explanations for preference of the hexagon, Kepler remains unconvinced by any of them, and finally ponders on the possibility that it may arise from a *formative faculty* in the body of the Earth, and be formed as a part of the Creator's design because of its aptness and beauty. But, even so, he admits that Nature produces other shapes in other crystals and so is finally unable to arrive at a satisfactory explanation for the hexagonality of snow.

The Importance of Kepler's Essay in the History of Crystallography

Although Kepler was unable to offer a satisfactory explanation of the six-sidedness of the snowflake, his discussions of space-filling and symmetry laid the early foundations of crystallography. He discusses, in print, for the first time, the ordered symmetrical shapes that arise from the packing together of similar bodies, which may themselves lack symmetry. The Pyramids bear witness to the fact that man has long been aware of the superstructures that can be built from the stacking of repeated geometrical units. This awareness was revealed later by the three-dimensional stacking of spheres in the work of the early goldsmiths and in the piling of cannon balls, but the relationship of these models to the minute geometry of matter was not recognized until Kepler's time.

Although Kepler was the first to publish his speculations on the relationship between the symmetry of the snowflake and the packing of bodies, Thomas Harriot,[1] an Englishman, really preceded him. As early as 1599, he saw the relationship between certain decorators' patterns and the corpuscular theory of matter. He studied two- and three-dimensional arrays of circles and spheres and was apparently the first to postulate that closest packing was achieved when one ball was surrounded by twelve similar neighbours. He saw the difference between hexagonal and cubic close-packing and, altogether, seems to have had all the insights into crystalline order that appear in Kepler's essay.

Space can be filled with any arrangement of contiguous cells, regardless of their shape, size, and symmetry, provided that they conform to a law, enunciated by Euler, that the numbers of

$$\text{vertices} - \text{edges} + \text{faces} = 2,$$

or, more generally,

$$n_0 - n_1 + n_2 - n_3 = 1,$$

where n_0, n_1, n_2, n_3 denote respectively the number of features in zero, one, two, and three dimensions. This applies to any array whatever. Symmetry

[1] MSS. in the British Museum and Petworth House, Sussex.

arises from a specification of a relationship between the number of edges, faces, or corners in the units and from a uniformity in their size.

In two dimensions, space can be filled with identical polygons in only three ways: by triangles meeting six at each corner; by quadrangles meeting four at each corner; or by hexagons joining in groups of three at every point in the array. This was clearly recognized by Kepler. The most economical sub-division of two-dimensional space—that which minimizes the total length of the boundaries between a given number of regular cells—is the hexagon.

In three-dimensional space, however, the simplest symmetrical unit, the cube, is not the simplest nor the most economical space-filling unit. The cell with the minimum number of junctions is the truncated octahedron. That which gives the minimum interfacial area is obtained from a truncated octahedron by introducing double curvatures into the hexagonal faces so that all faces meet at a dihedral angle of 120°.

Kepler's essay contains both aspects of the space-filling problem. The stacking of spheres is a case of symmetrical groups arising through point contacts. The structure of the pomegranate and the bees' honeycomb may be discussed in terms of stacked polyhedra. Kepler's was the first explicit statement of the fact that an aggregate of uniform spheres in contact is symmetrical only along certain directions, the units themselves having no directional properties.

But Kepler did not think of the spheres as atoms. Indeed, it was not until the end of the nineteenth century, long after the concept of the atom had been accepted, that the packing units in crystals were identified with the constituent atoms. It seems that the crystal and the atom developed as quite separate concepts. While mineralogists had long identified and classified crystals by their angles, it was a long time before shape was associated with chemical composition and before the importance of internal order was realized. The chemists completely failed to recognize that constancy of composition would result from atoms arranged in continuous array until this was demonstrated conclusively by X-ray diffraction.

Modern Explanation of the Hexagonal Symmetry and Shapes of Snow Crystals

The hexagonal symmetry of a snow crystal is a macroscopic, outward manifestation of the internal arrangment of the atoms in ice. At ordinary temperatures and pressures, X-rays show the molecules (or at least the oxygen atoms) of ice to be arranged in an open lattice with a hexagonally symmetrical structure—see Fig. 5. The molecules are arranged in puckered layers which lie parallel to the paper in Fig. 5 (*a*), each molecule being surrounded by four nearest neighbours, so that each group has one molecule at the centre and the other four at the corners of a tetrahedron, all four being the same distance (2·76 ångströms) away. This open arrangement is far removed from spheres in hexagonal close-packed array and the latter concept is therefore irrelevant to an explanation of the hexagonal shape of snow crystals.

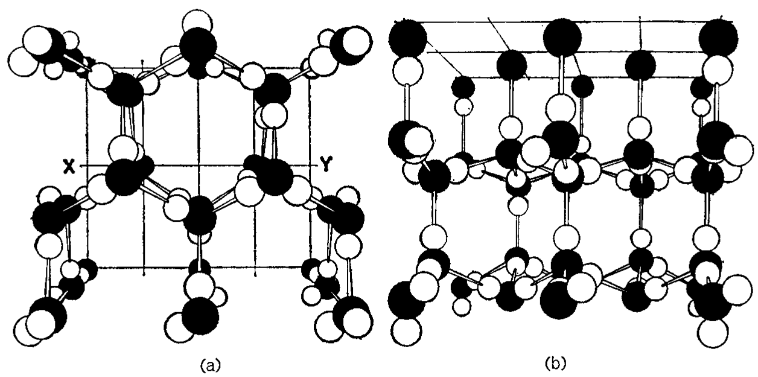

FIG. 5. The arrangement of the oxygen atoms (black) and the hydrogen atoms (white) in the ice lattice. The left-hand picture shows a model of ice looking at right angles to the basal plane of the crystal, and reveals the hexagonal arrangement of the molecules. The right-hand drawing shows the same model turned forward through 90° about the axis *XY*. The hydrogen bonds are represented by rods.

The mean positions of the hydrogen atoms in ice, which are not easily found because of their low scattering power for X-rays, have recently been located by the diffraction of neutrons. It appears that a hydrogen atom lies on the line joining the centres of each pair of adjacent oxygen atoms, being 1·01 ångströms from one and 1·75 ångströms from the other. The ice molecule is thus V-shaped with an included angle of 109°. The forces between the molecules are due mainly to electrostatic attraction between the positively-charged side of the hydrogen atom and the electrons of a neighbouring oxygen atom, to form what is known as a *hydrogen* bond. It is to these hydrogen bonds, which bind the molecules of both liquid water and ice into a united association of molecules, that many of the peculiar properties of water can be attributed.

Whether a snow crystal grows in the form of a hexagonal plate, a prism, or a star-shaped crystal is determined basically by the air temperature. This has been demonstrated conclusively by the present author and his

colleagues[1] in a series of experiments in which snow crystals are grown on a thin fibre, which is suspended in a cloud chamber, and along which the temperature varies from 0° C to −50° C. The crystal shape varies with temperature along the length of the fibre in the following manner:

0° C to −3° C	thin hexagonal plates,
−3° C to −5° C	needles,
−5° C to −8° C	hollow prismatic columns,
−8° C to −12° C	hexagonal plates,
−12° C to −16° C	fern-like stellar crystals,
−16° C to −25° C	plates,
−25° C to −50° C	hollow prismatic columns.

Except at −25° C, the transitions between one crystal form and the next are very sharp and occur within a temperature range of one degree Centigrade. The effect of a sudden change of temperature on the growth form of the crystal may be demonstrated rather dramatically by raising or lowering the fibre in the chamber. Whenever a crystal is thus transferred to a new environment, its further growth takes the form characteristic of the new temperature régime. In this way, it is easy to grow needles with either plates or stars on their ends, and, in fact, to reproduce all the hybrid forms of snow crystal found in Nature.

The really intriguing question arising from this work is, of course, the nature of the growth mechanism by which only a degree or two variation in temperature can change completely the crystal shape and which, furthermore, allows the habit to change five times in a temperature range of only 25° C.

Very recently we have obtained some experimental data that suggest an explanation. We have observed that snow crystals grow as layer upon layer of new material sweeps across the surface. Careful measurements reveal that, at constant temperature and humidity, the layers travel with speeds inversely proportional to their thickness, and this is consistent with the notion that they progress by molecules landing on the crystal surface from the vapour and diffusing across it into the growing step. Furthermore, it seems that the rate at which molecules migrate across the surface varies with temperature in a remarkable manner that is rather different for the basal and prism faces of the crystal. The suggestion is, that for crystals growing in the temperature ranges 0° C to −3° C and −8° C to −25° C, there is a net surface migration of molecules from the basal to the prism faces, and that this biases the early development of the crystal

1 See, for example, J. Hallett and B. J. Mason, *Proc. R. Soc.* A, **247** (1958) 440; B. J. Mason, G. W. Bryant, and A. P. van den Heuval, *Phil. Mag.* **8** (1963) 505.

towards a plate-like habit. With temperatures between $-3°$ C and $-8°$ C and between $-25°$ C and $-50°$ C, the situation is reversed; there is a net flux of material from the prism to the basal faces and the crystal grows as a prismatic column or as a needle.

Once the basic shape of the embryonic crystal has been determined by migration of molecules on its surfaces, the vapour field around the crystal takes notice and orients itself to conform to the crystal geometry and tends to maintain it. For example, the lines of diffusive flux will tend to concentrate toward the edges and corners of a hexagonal plate and accentuate its development. The fact that at moderate supersaturation crystals continue to develop as polyhedra suggests that the excess material arriving at the edges and corners is distributed over the crystal faces by surface diffusion. However, if the supersaturation of the vapour is very high, surface diffusion is unable to cope with the non-uniform deposition of material, so that the corners and edges begin to sprout and give rise to stellar plates, hollow prismatic columns, and other skeletal forms. Under these conditions, new growth layers are formed in rapid succession and tend to bunch together to form steps that are discernible under a low-power microscope. The formation of layers and steps gives a rhythmic character to the growth that may explain such features as the origin and fairly regular spacing of the side-branches on star-shaped crystals.

We see then, that although the basic hexagonal structure of snow crystals is inherent in the atomic architecture, their shape and growth rate are governed by the environmental conditions such as temperature and humidity, and partly by conditions at the crystal surface that control the rate at which material becomes built into the crystal lattice.

The broad features of snow-crystal design seem now to be understood but there are many fine points of detail that still require explanation before we can claim to have dealt satisfactorily with the question posed so acutely by Kepler more than 350 years ago.

ACKNOWLEDGEMENT

Some of the historical facts in this article were brought to my attention in an unpublished essay on 'The History of Crystallography' by Professor C. S. Smith. I am indebted to Professor Smith for making his manuscript available to me.

KEPLER'S UNSOLVED PROBLEM AND THE *FACULTAS FORMATRIX*

LANCELOT LAW WHYTE

KEPLER leaves the reader of his *New Year's Gift* in no doubt that he identified a clear problem in the mathematical physics of small systems: why are certain snowflakes hexagonal? or that he knew that it was beyond his power to solve it. Not only did the hypotheses which he tried lead to difficulties, but he suspected that he had not the facts on which to base an answer. He even indicated matters which chemists should investigate in order to elucidate the problem. In 1611 such scientific clarity on an issue bordering on the micro-realm was without precedent, though from around 1690 it was to become common owing mainly to the use of the microscope, to interest in the atomic ideas which Kepler rejected, and to the achievements of the new mathematical physics.

Kepler's particular question: 'Why six?' left him baffled. But he was convinced that he knew the answer to the general problem: How do visible forms arise? The genesis of forms was everywhere due to a *facultas formatrix* (formative capacity or faculty) of the Earth itself, a universal spirit pervading and shaping everything. For him this capacity was the *anima terrae*, or soul of the living Earth—echoing Lucretius' *daedala tellus* or shaping Earth—just as a *vis formatrix* or *matrix formativa* constituted the individual human soul. This formative tendency in all its manifestations was the kernel of Kepler's animistic cosmology, lying behind and justifying his geometrical mysticism.

In retrospect we see that Kepler stood at the historical threshold when the magic of words was surrendering to the newly revealed power of mathematics. In the *New Year's Gift* we observe a mind capable both of defining a mathematical problem in physics centuries before it became ripe for solution and of experiencing a divine magic in two words: *facultas formatrix*. There is no contradiction here. Kepler's attitude is fundamentally natural and truly fertile. He displays that empirical mysticism which is indispensable to science: the passionate a-rational search for a rigorously objective single order. He believes without question that a universal formative tendency is at work in the universe, though he cannot, even in his chosen example, the snowflake, discover its mathematical expression.

My aim here is briefly to examine the sources of Kepler's *facultas formatrix* and to consider how far the physics of 1965 can provide a substitute and answer his special question.

It is a characteristic of the human mind, when well-defined forms are observed, sooner or later to ask how they came into existence, and to start by regarding them as the products of a divine activity. For in seeking to understand the genesis of forms, man has the sense of touching a singularly deep issue, affecting everything that exists including himself. It is as though to ask why forms exist were to probe the meaning of existence itself. Yet science has learnt to distil from ultimate mysteries tangible features posing problems which are capable of solution.

In Plato's *Timaeus* we see the first clear shift from a purely verbal or religious description towards a geometrical approach to natural forms. Aristotle's idea of existence as the realization of potential form, though non-geometrical, marks a step of equal importance. This Aristotelian tendency towards form is in several respects distinct from our conception of a temporal process of the genesis of visual forms, and any attempt to render his idea precise in twentieth-century scientific language must be misleading. Yet Aristotle was deeply interested in biological and other kinds of development, and his conception of a movement towards form, though abstract and metaphysical, has influenced all subsequent thought regarding the temporal genesis of natural forms.

I am advised by specialists that Aristotle did not use any particular Greek term with the meaning which I consider *facultas formatrix* had for Kepler. That would scarcely be possible, on account of Kepler's Pythagorean–Platonic sense of a divine geometry pervading all natural processes, which Aristotle did not share.

Pauli has linked Kepler's *facultas formatrix* with a similar idea expressed in Paracelsus's *Archaeus*, and that thinker's *Iliaster*, a formative spirit immanent in the universe, may be even closer.

But Paracelsus's archetypes are obscure, and I suspect that Kepler's use of the term derives, directly or indirectly, from Galen (*c.* A.D. 130–200), the Greek physician who is regarded as the first experimental physiologist. Galen was a whole-hearted Aristotelian, like Kepler an opponent of atomism, and he was actively concerned with many problems in medicine and biology. He was a close observer of organisms, from our point of view a more scientific thinker than was Aristotle, and he exerted great influence. In his embryological works Galen ascribes the emergence of new visible forms in the developing egg to a 'formative faculty' (*δύναμις διαπλαστική*), and he explains that by 'faculty' he means 'some unknown cause of change'. A particular faculty is the unknown cause that leads to a process which results in some final state. Galen's meaning may seem clear, but we must not read scientific precision into the mind of a second-century Aristotelian embryologist.

Kepler probably read Galen in Latin, four sixteenth-century translations of Galen's essay being available. If so he blended this neo-Aristotelian idea with a Neoplatonic geometrical mysticism inherited from Nicholas of Cusa (1401–64) and others, and the Pythagorean view of the importance of number. Aristotle's thought deeply influenced even those thinkers whose task was to transform and to overcome it.

As I understand it, Kepler's *facultas formatrix* is close to Galen's 'formative faculty', for both were observers of visual forms, interested in temporal processes in which new forms are generated. Galen was mainly concerned with embryology and the developing egg; Kepler, in this work and several others, with the growth of plants and the formation of crystals. The main difference is that Galen was an experimentalist exploring highly complex organic processes where nothing was logically clear or geometrically precise, while Kepler was a theoretician inspired by the divine geometry of mathematically simple forms. Thus Galen's unknown general cause of the genesis of forms was revealed to Kepler as a divine geometry seeking perfect realization; it was only its particular mathematical and physical expression, for example in the snowflake, that was obscure to him and invited mathematical and chemical research.

Whatever the sources of this term, there is no question of Kepler's attachment to the adjective *formatrix*. It occurs twelve times in the twenty-four pages of the *New Year's Gift*, usually with *facultas*, but sometimes with *vis*, *ratio*, and *anima*, and repeatedly in other works. It becomes one of his favourite ideas when he passes from the geometry of stationary forms, such as planetary orbits or light rays, to the genesis in course of time of visible objects in any realm, cosmic or terrestrial, organic or inorganic. Here Kepler anticipated what was to become a popular idea as interest in natural processes developed. For between 1650 and 1850 thinkers of many kinds concerned with visual forms referred to a universal formative virtue, energy, or process, and to a plastic or forming power.

Kepler did not state that there must exist one general mathematical expression of which all processes of the generation of visible forms were special cases. From the point of view of mathematical physics the term *formative process* (to give it its appropriate contemporary formulation) remains for us what *facultas formatrix* was for Galen, a convenient label for a somewhat vague principle, or class of processes whose scientific definition and theoretical status are still obscure. For even in 1965 physical theory does not include any general principles covering all, or even most, cases of the genesis of well-defined visual forms. This would imply the existence of a comprehensive theory of complex partly-ordered systems showing how and when they move towards equilibrium states. It is remarkable that in 1611 Kepler could recognize a class of problems in mathematical physics which still awaits a general solution, or set of solutions.

To clarify this issue I shall trace in outline the history of Kepler's problem of the six.

Kepler was the first to publish the recognition that the hexagonal form of snowflakes presents an interesting scientific problem. But many did so shortly after him. Descartes read Kepler's *New Year's Gift* during the winter of 1629–30 and in his own essay 'On Meteors' (1637) explained the hexagonal form as produced by the packing in a plane of spherical water globules, thus adopting one of Kepler's suggestions. Letters between Descartes and Gassendi in 1639 show that they, and probably Mersenne also, were considering the problem. From 1660 there was increased interest in snowflakes, as is shown, for example, by E. Bartholinus's *De figura nivis* (1661) and D. Rossetti's *La figura della neve* (1681). As early as 1673 Nathaniel Grew ascribed the persisting failure to solve the problem of the six to the tendency of philosophers to speculate excessively, to treat the six as universal, and to neglect the study of other materials which crystallize in contrasted shapes. The increasing use of the microscope, by Hooke and others, and the interest in atomic ideas soon overshadowed the speculations of Kepler and Descartes made without the microscope or a belief in true physical atomism.

No positive advance on the problem of the six was achieved until Hessel (1830) and Bravais (1849) showed that certain characteristics of natural crystals (mainly the law of rational indices) implied that only thirty-two classes of crystal forms were possible, and that if a crystal possesses an n-fold rotation axis, i.e. n-fold symmetry in a plane with respect to a perpendicular axis, then n can only possess the values 2, 3, 4, and 6. To Kepler's 'Why six?', mathematical theory had now given a partial answer: 'if a crystal is to have rotational symmetry, n can only be 2, 3, 4, or 6.' Kepler's problem had been transformed. The issue was now 'Under what conditions do particular materials crystallize with a rotation axis, for what classes of materials is n 2, 3, 4, and 6, and why?' This is a generalization and sharpening of the original question which is directly relevant since it is now known that at low temperatures ice crystallizes in cubic form.

During the late nineteenth century it became increasingly probable, and by 1912 no doubt was left, that the symmetry properties and structure of crystals were due to three-dimensional arrangements of atoms. Kepler's question could now be reformulated: 'Can the hexagonal form of certain snowflakes be explained as due to a particular stable arrangement of the oxygen and hydrogen atoms?'

In 1933 Bernal and Fowler, using a model of the water molecule derived from experiment, showed that a hexagonal form of ice crystals may be expected to be most stable and therefore to be dominant within certain ranges of temperature, pressure, etc. Assuming that the hydrogen atoms remain in fixed positions, they found that the simplest structure having hexagonal symmetry was composed of repeatable cells each containing twelve molecules. The configuration of the electric charge in the water molecules required each molecule to be surrounded by four equidistant neighbours at the corners of a tetrahedron, with the result that the molecules are arranged in hexagonal puckered layers.

Kepler might be satisfied with this answer, in which chemical studies helped to trace the symmetrical form of the snowflake to a regular arrangement of parts. He would be shocked that atomism had proved so important, but pleased that symmetry played a fundamental role.

But that is not the end of the story. During the last thirty years physics has looked deeper into the matter and has disclosed another feature which Kepler would have found difficult to accept: in an ice crystal at finite temperatures there is an essential irregularity in the arrangement of the hydrogen atoms.

For not only are the hydrogen atoms less accessible to standard methods of observation, but they have been shown to be essentially mobile and at any one moment irregularly distributed, in crystals at normal temperatures. The oxygen lattice is definite and stable, but the position of the hydrogen atoms is only statistically regular, on the average over a period of time.

This demands a change in the natural philosophy of visual symmetry. We must not ask 'What is the characteristic form of structural order in a snowflake?' but 'Under the given conditions what particular blend of order and disorder is most stable and therefore dominant?' For disorder is now known to play a basic role in stabilizing particular kinds of visual forms at normal temperatures. It would be impossible for Kepler to like this interpretation, which is not only atomic, but introduces disorder into the divine regulation of the universe. The visually perfect hexagon starlet is the progeny of a union of order and disorder.

Kepler might counter: 'That is only the result of your belief in atomism; the disorder comes in with your atomism. As I see it, my snowflake is regular; there is no visible disorder.' But for us atomism, with the accompanying disorder, is justified by its empirical success.

Yet Kepler might retaliate: 'I looked for a rational derivation of the six. How far can the six be theoretically predicted, by pure logic and mathematics, from the basic principles of physics as far as they are known now?'

Here Kepler would gain a point. For the six cannot be predicted from fundamental theory alone. It cannot predict under what conditions particular complex groupings of water molecules will be stable in the solid state. Snowflakes are known to be much more complex than Kepler, who repudiated atomism, could have imagined, and no theory yet exists which covers all the observable properties of such complex systems. Visually simple facts are often too complex to be treated theoretically.

One question is still unanswered. Kepler asked himself how the snowflake came into being. What do we know about this? What is the present scientific status of his *facultas formatrix*?

The answer is instructive as regards the present frontier of science. For though we know very little for certain, important advances are now being made and genuine understanding may be close. Nakaya, the Japanese scientist who has devoted a lifetime to the study of natural and artificial production of snowflakes, has written recently of 'the untrodden field of the physics of form'. But to know that it is untrodden is evidence that one is already there, and much research has in fact been carried out in the last thirty years on the growth of crystals, i.e. on the conditions under which the most regular visible forms are generated.

Two points have emerged from this work: the importance of exceedingly small structures such as minute solid nuclei in initiating growth by sublimation and structural dislocations facilitating growth; and the inadequacy of external parameters such as supersaturation and temperature in controlling the shape and detailed pattern of symmetrical visual forms.

If one wished to report to Kepler what is known today regarding the processes by which crystals are formed, it could be summarized thus: The constitution of matter is atomic, and if a minute solid nucleus is already present, if the necessary additional materials are available close at hand, and if excess heat can be dispersed, the neighbouring atoms or molecules tend to aggregate around the nucleus in arrangements of lowest free energy, these possessing high structural symmetry. But the genesis of the original nucleus and the precise conditions under which macroscopic symmetry results during crystal growth are not understood. (See Mason's essay for recent studies on this.) We certainly have not yet a comprehensive scientific substitute for Kepler's *facultas formatrix*.

Kepler's *New Year's Gift* had little influence, but it defined a new realm of inquiry for exact science: the mathematics of the genesis of form. We can honour him for this and yet not hesitate to recognize that his scientific philosophy was mistaken. He not only rejected atomism but he treated simplicity and perfection as basic. We know that this universe is essentially complex and is disturbed by pervasive interactions. Apparent

perfection always conceals an element of disorder which is not a mere deviation but plays an essential role in generating visual form. We have to accept the paradoxical fact that imperfection and irregularity appear to be indispensable factors in the genesis even of the most perfect known forms: regular crystals. What Kepler would have rejected passionately holds the clue to his problem. He saw the need for a mathematics of morphogenesis, but not the kind of calculus which it would require.

Yet a puzzle remains. For the fact which so impressed Kepler still constitutes a challenge: three-dimensional geometrical symmetry plays an important role in the visual world and yet it is not clear how it is generated in a universe supposedly dominated by random interactions and disturbances. We should not expect complete knowledge of highly complex systems, but it is reasonable to require of science a simple explanation of simple observations. If the hexagonal snowflake is highly complex, is there no short cut from the postulates of physics to our visual observations? What in the ultimate laws produces visually perfect patterns?

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NOTES

1. *p.* 2, *l.* 4. WACKER. Johann Matthäus Wacker von Wackenfels. The fullest account of him is that of Theodor Lindner, *Zeitschrift des Vereins für Geschichte und Alterthum Schlesiens* **8** (1867), 319–51, who used an account of 139 letters, addressed to Wacker, in the Kasperlik Collection, by G. Biermann in *Gymnasialprogram von Teschen* (1860). There are, or were, many more letters to him in the Stadtbibliothek at Breslau (now Wrocław), and in the Provinzial-archiv of the Bishops of Breslau. *Allgemeine deutsche Biographie* **40** (1895), 448–9, gives a summary of Lindner.

Wacker was born in March 1550 at Constance to a family of the 'reformed' (Calvinist) Church. He studied law at Strassburg and Geneva. In 1575 he took his doctorate in Padua and for some years travelled widely in Italy and France. From 1580 to 1597 he worked under the Bishop of Breslau in the imperial administration and was employed on diplomatic missions, especially to Poland. In 1592 he became a Catholic and remarried. In 1592 too he was ennobled and in 1597 called to Prague as Reichshofrat. The Emperor Rudolf rewarded his long and devoted services by making him *comes Palatinus* in 1616. When Bohemia revolted in 1618, he fled to Breslau and then Vienna, where he died on 7 September 1619.

He was a man of wide interests, in touch with most of the intellectually distinguished Germans of his time, with literary tastes. He tried his hand at drama in his youth, and twelve pieces of his Latin verse are preserved in *Delitiae Poetarum Germanorum collectore AFGG*, Frankfurt (1612), vol. vi, 1057–65, including the ode to the special Breslau brew of beer, called Scheps, one of the seven odes that all parody Horace, *Odes* iv. 3. This shows his taste for elegant trifles.

l. 9. Some private joke must be behind Kepler's reference to him as a 'lover of Nothing'. But it cannot be traced. He perhaps wrote an epigram or an essay on 'nothing' or negation or *privatio boni* or the like.

l. 10. Equally obscure is the reference to the sparrow. If it was a real one, and is not a nickname, was it perhaps called Nix or Nixe (water-sprite)? Conjecture is vain. If a real sparrow is meant, *argutissimum* will mean 'shrill, clear-toned, musical'.

The *New Year's Gift* was probably written for New Year's Day 1610 and not 1611, as the date of publication might suggest. In a letter of 16 April 1610 Georg Kepler writes from Venice asking for the *libellus latinus de nive sexangula*.

2. *p.* 2, *l.* 17. *Nihil vero a me habes antea.* This probably means simply 'you have never had anything from me before'. But it might mean 'you have already had a similar trifle, a Nothing, from me before', and that the *Snowflake* is not the first New Year's gift.

3. *p.* 2, *l.* 20. Archimedes, born *c.* 287, died 212 B.C., of Syracuse. Kepler refers to his Ψαμμίτης, *Arenarius*, or *Sand-reckoner*, first published by Gechauff at Basel in 1544; modern critical text with Latin translation, J. L. Heiberg, Teubner

(1913), ii. 216–59; English translation, T. L. Heath, *The Works of Archimedes* (1897).

The *Sand-reckoner* is addressed to King Gelon of Syracuse, born *c.* 268/7, son of and co-ruler with King Hieron (306–215 B.C.) whom he predeceased, to show that 'the number of the sand' is not 'infinite', as the proverb had it, but that a notation can be devised to express the number of grains of sand in the universe on the supposition that the whole sphere of the fixed stars is filled with sand and that one poppy-seed (0·6 mm. in diameter) contains 10,000 grains. Archimedes estimates the diameter of the universe as less than 10^9 stades (1/8 of a mile), and the total of grains of sand as less than '1,000 Myriads (tens of thousands) of the eighth order'. If one myriad, 10,000, is of the first order, its square is of the second, the square of the second is of the third, and so on. The total expressed in modern notation is 10^{63} (*productum fore sexagesumum quartum ab unitate numerum: is autem octavus est numerorum octavorum, qui est mille myriades numerorum octavorum*, in Heiberg's Latin).

4. *p.* 4, *l.* 7. Plato's pyramids. The reference is to Plato, *Timaeus* 56 b, where the regular solids are correlated with the elements, the tetrahedron (pyramid) being the form or seed of fire, the cube of earth, the octahedron of air, the icosahedron of water. See A. E. Taylor, *A Commentary on Plato's Timaeus*, Oxford (1928), 380–1. The word *pyramid* was, by the arbitrary method of etymologizing exhibited in Plato's *Cratylus*, derived from the word for fire, cf. Ammianus Marcellinus, XXII. xv. 29: *quae figura* (*pyramis*) *apud geometras ideo sic appellatur quod ad ignis speciem, τοῦ πυρός, ut nos dicimus, extenuatur in conum*. If it is not of Egyptian origin (pr-m-wś, the height of a pyramid), it is more probably derived from πυρός wheat, since both πυραμίς and πυραμοῦς mean a cake of wheat and honey, possibly of pyramidal shape. The ῡ's of πῡραμίς and πῡρός are long, while that of πῠρός, fire, is short.

5. *p.* 4, *l.* 10. *utribus*, skins made into bags. This is explained by reference to Kepler's posthumous work, published by his son Ludwig, the *Somnium sive Astronomia Lunaris* (1634), p. 2, note 15: *trita est haec traditio, vera an falsa, in geographicis, gubernatores navium ex Islandia navigantes, ventorum quem velint, aperto venti utre excire*. ('Among writers on geography this is a familiar tradition, whether it be true or false, that the pilots of ships on the voyage from Iceland summon whatever wind they want by opening the skin of the wind.')

Compare the bag of the winds that Aeolus, King of the Winds, gives to Odysseus in Homer's *Odyssey* 10. 19–20. On the northern origin of such stories see A. D. Fraser, *Classical Journal* (1933), 364–6: 'The Origin of Aeolus': also, Stith Thompson, *Motif-Index of Folk-Literature* (1958), C 322, 1.

6. *p.* 4, *l.* 14. *sacrosancti vates*, venerable bards or prophets. (Klug translates 'heilige Seher'; Rossmann 'hochheilige Sänger'; Strunz–Borm 'hochheilige Priester', but all leave them unidentified.)

Vates can mean prophet, seer, bard, poet, but *sacrosancti* suggests prophet rather than poet, biblical rather than pagan.

In the first place Kepler probably refers to Isaiah xl. 15: 'Behold, the nations are as a drop of a bucket, and are counted as the small dust of the balance: behold, He taketh up the isles as a very little thing.' The passage is the origin of the English phrase 'a drop in (*or* of) the (*or* a) bucket'.

In the Apocrypha, 2 Esdras vi. 56 reads: 'As for the other people, which also came of Adam, Thou hast said that they are nothing, but be like unto spittle: and hast likened the abundance of them unto a drop that falleth from a vessel.'

Kepler may also have had in mind Job xxxvi. 27: 'For He maketh small the drops of water: they pour down rain according to the vapour thereof.' But the Vulgate reads: 'qui aufert stillas pluviae'. Luther, however, has: 'Er macht das Wasser zu kleinen Tropfen, und treibt seine Wolken zusammen zum Regen.' The Septuagint has: 'the drops of rain are numbered by him', and the Hebrew probably means: 'he draws up drops of water from the sea, and they are distilled as rain from the mist that he has made.'

These passages, however, do not explain the *last* drop *that clings*. Kepler may have had in mind the Greek game of Kottabos, as well as the German custom to which he goes on in the next sentence. The Greeks jerked the dregs of a cup at a mark which rang when hit.

7. *p.* 4, *l.* 14. *Germani*, our Germans. The German translators do not discuss this. It is a German custom, referred to by K. F. W. Wander, *Deutsches Sprichwörter Lexicon*, Leipzig (1867–76), 5 vols., vol. iii (1873), p. 865: Nagelprobe:

'Dadurch bezeichnet man das völlige Austrinken eines Glases oder Bechers, wobei man zuletzt das Gefäß umkehrt, und mit seinem Rande schief auf den Daumennagel der linken Hand setzt, um zu beweisen daß kaum noch ein Tropfen darin geblieben ist . . . [a German court where the custom prevailed of "no heel-taps" is mentioned] . . . im Latein des Mittelalters hat man den Germanismus „Super nagulum" dafür gebildet, ein Ausdruck der sammt der Sitte auch zu den Briten und Franzosen übergewandert ist; denn bei jenen findet man die Redensart „to drink super nagulum" und bei diesen „boire rubis sur l'ongle", wie es denn auch in einem Liede heißt: Ils faisoient en les reversant un super nagle Allemand.'

Wander quotes J. Eiselein, *Die Sprichwörter und Sinnreden des deutschen Volks*, Freiburg (1840), whose no. 486 quotes the Latin:

Ebibe vas totum si vis cognoscere potum.

F. von Lipperheide, *Spruchwörterbuch* (1907) quotes 880b Klopstock, *Oden der Frühlingsfeier* (1759): Tropfen am Eimer; and 648b Nagelproben:

Schenk' voll ein
Trink' aus rein
Daß man das Glas von oben
Kann auf den Nagel proben
Das ist zu loben.
Spruch an einem Hausgerät.

The custom can be traced in France, see de V. Payen-Payne, *French Idioms and Proverbs* (7th ed.), Oxford University Press (1924), p. 240.

Rubis. Faire (*or* payer) rubis sur l'ongle = to pay to the last farthing. (This expression means literally to drain a tumbler so completely that there remains in it just one drop of wine, which being put on the nail looks like a ruby.) Reynard, *Folies Amoureux* iii. 4.

8. *p*. 4, *l*. 17. *Choaspem*. Rossmann and Strunz–Borm quote Herodotus, *Histories*, i. 188: *The Great King, when he goes to the wars, is always supplied with provisions carefully prepared at home, and with cattle of his own. Water too from the river Choaspes, which flows by Susa, is taken with him for his drink, as that is the only water which the Kings of Persia taste. Wherever he travels, he is attended by a number of four-wheeled cars drawn by mules, in which the Choaspes water, ready boiled for use, and stored in flagons of silver, is moved with him from place to place* (Trans. G. Rawlinson, 1858).

But Herodotus says nothing of a particular Persian who poured the water before the King from the hollow of his hand. Athenaeus, *Deipnosophistae* II, 45*b*, quotes Herodotus and also Ctesias, the Greek physician at the Persian court of King Artaxerxes and author of a history of Persia in 23 books (*c*. 400 B.C.), that the water was boiled, decanted into vessels, and brought to the King. Nor do other references to the royal water from the Choaspes say anything about the hollow of the hand, Plutarch, *de exilio* 610 D, Quintus Curtius v. ii. 9; Pliny, *Nat. Hist*. 31. 35 (copied by Solinus 38. 4), Isidore, *Etymol*. XIII. xxi. 15, etc. Kepler's precise source for these details escapes us.

The Choaspes, Persian *huvaspa* 'with fine horses', is the modern Karkheh, which flows SSE. from Luristan, past the site of Susa, to join the Dez or Dizful-Rud (ancient Copratas) which flows from the north, and the Khersan (Pasitigris), which flows from the east.

9. *p*. 4, *l*. 20. The Italian who gnawed off a sliver from his finger-nail and, presumably, spat it in the face of his enemy, saying 'I won't give you even that', remains unidentified despite research. 'The paring of a nail' is quoted in the *Oxford Dictionary of English Proverbs* for miserliness, 1639: 'he'll not lose the paring of 's nails.'

10. *p*. 4, *l*. 23. *noctuas Athenas*, owls to Athens, a familiar proverb, first in Aristophanes, *Birds*, 301. Athenian coins regularly bore Athena's owl. Cf. T. Gaisford, *Paroemiographi Graeci*, Oxford (1836), B 276; C 78; D 3, 81; Z 3, 6.

11. *p*. 4, *l*. 25. Parmenides of Elea in South Italy, born *c*. 515 B.C., opponent of Heraclitus' doctrine that 'everything is in flux, nothing stays', argued that what is is without beginning or end, single, motionless, continuous, determinate, like a sphere. (Fragments and evidence for his life and teaching in Diels–Kranz, *Fragmente der Vorsokratiker* (8th ed.), (1956), i. 22, 139–90; J. Burnet, *Early Greek Philosophy* (4th ed.), (1930), 169–96; W. Jaeger, *The Theology of the Early Greek Philosophers* (1947) 90–108.)

12. *p*. 4, *l*. 29. Julius Caesar Scaliger, born 1484 at Riva del Garda, died 1525 at Agen on the Garonne, notable in medicine and literary criticism. He studied at Bologna from 1514 to 1519. His encyclopedic work, *Exotericarum Exercitationum Liber quintus decimus de subtilitate ad Hieronymum Cardanum*, was published in 1557.

In the *Exercitatio* cxciiii, Num. 7, he discussed small insects such as lice and mites. According to Rossmann, the mite described by Kepler is the *Acaris scabiei*, which he mentions also in his *De stella nova in Pede Serpendarii* (1606), ch. 16 (*Gesammelte Werke* i. 236).

13. *p*. 4, *l*. 29. Hieronymus Cardanus, born 24 September 1501 at Pavia, died 21 September 1576 at Rome, notable in mathematics, especially for the so-called *formula cardanica* for cubic equations, and in medicine, in which he took his doctorate at Padua in 1534. His *de Subtilitate* was published at Basel in 1547. His Aristotelian, yet critical and observational, outlook has much in common with Kepler's. He believed in universal animation, in the two principles of warmth and humidity, and in matter as act and not mere potentiality. He refuted perpetual motion and explained fossils. His *Opera Omnia* in ten volumes including an autobiography, were published at Lyons, in 1663.

14. *p*. 4, *ll*. 34–35. Johannes Jessen (Jessensky), born 1566 at Breslau, executed in June 1621 by the Emperor Ferdinand in Vienna. He studied in Leipzig and at Italian universities, and took his doctorate in medicine at Wittenberg in 1596. He became Physician to the Prince Elector of Saxony and in 1601 Professor in the University of Prague, where he was noted for his teaching of anatomy. He became Rector and Chancellor of the University, and supported the Bohemian rising against the Emperor. On its defeat he was imprisoned and put to death with some twenty other leaders of the movement. See A. Hirsch, in *Allgemeine deutsche Biographie* **13** (1881), 785–6.

15. *p*. 6, *l*. 1. *pontem* Not, as Strunz–Borm have it, 'over *a* bridge', but the famous Karlsbrücke, begun in 1352 by Charles IV (1346–78), the founder of Prague University, on the design of the architect Peter Parler, over the Moldau (Vltava), 500 m. long, 10 m. wide, with sixteen Gothic arches, joining the lower city on the east, right, bank, where presumably Kepler lived, to the Hrad or Burg on the west, where Wacker as Imperial Counsellor lived. An engraving of 1606 (reproduced in Oskar Schürer, *Prag, Kultur Kunst Geschichte* (2nd ed., 1940), p. 128) gives a panorama of Prague with one bridge only; and another of 1757 (ibid. p. 256) again shows one bridge. No other bridge was built until the nineteenth century.

16. *p*. 6, *l*. 14. *nix*. German *nichts* (nothing) is colloquially pronounced *nix* in north-eastern Germany and in Austria. Kepler puns on the meaning of the Latin *nix*, snowflake, in German.

17. *p.* 6, *l.* 16. *Nihili accessionem*. Klug translates: 'Nimm also diese Bestimmung des „Nichts"' (this definition or designation of nothing); Rossmann: 'Nimm also diesen Anflug von Nichts' (sudden approach, or smattering); Strunz–Borm: 'Nimm daher das Ding, das dem Nichts nahe kommt' (this approximation to nothing). *Accessio* more often means 'addition' than 'approach to': *hanc ad nihil accessionem* would be more natural for 'approximation to nothing'. So 'this addition of nothing', i.e. 'this enrichment of you by a present which amounts to —nothing'.

18. *p.* 6, *l.* 18. *Socrati . . . pulicis*. The reference is to Aristophanes, *Clouds* (423 B.C.) 144–53; where Socrates is described as measuring how many feet of its own a flea can jump.

19. *p.* 6, *l.* 23. The Psalmist. Psalm cxlvii. 16: 'He giveth snow like wool: He scattereth the hoarfrost like ashes.'

20. *p.* 8, *l.* 22. Kepler uses alvei, alveoli, alvearia, words that mean 'beehives', but it is clear that he means honeycombs, although he does not use the common Latin word 'favus'; except perhaps on *p.* 18, [K 11], *l.* 16, where 'hive' seems better.

21. *p.* 10, *l.* 15. The subjunctive *praestent* does not seem to be required or justified by anything. But the 1611 edition has it. *Praestant* would be expected. But Kepler writes quickly and allows himself some incoherences, *p.* 22, *ll.* 10–11 (sequence of tenses); *p.* 26, *l.* 1: *a geometris* (plural) . . . *l.* 3 *is respondebit* (singular); *p.* 30, *ll.* 31–32: *Motus* (plural) . . . *qui tendit* (singular).

22. *p.* 10, *l.* 20. Kepler is in error. A cube needs 26 others ($3^3 - 1$) to cover it. Of these 26, 6 are in contact plane to plane, 12 line to line, and 8 in contact point to point: total 26.

23. *p.* 12, *l.* 19, in margin. Rubric. Pisani, the people of Pisa, have nothing to do with the case. It is a misprint for *pisa in quam*; IN has been read as NI.

24. *p.* 18, *l.* 28. *consortia tecta / Urbis habent*, Virgil, *Georgics* iv. 153–4.

25. *p.* 20, *l.* 5. *loculi*, cavities, or ovaries; elsewhere coffin, casket, purse, pocket, satchel (Horace, *Sat.* I. vi. 74). *filamenta*, literally, threads or fibres.

26. *p.* 24, *ll.* 13–14, in margin. Rubric: *Nihili opinio* 'Die Meinung vom Nichts' (Strunz–Borm); 'Ansicht vom Nichts' (Rossmann); rather 'an opinion of no value', cf. *p.* 36, Rubric, *ll.* 8–10, *reditur ad opinionem nihili*; *p.* 30 *l.* 3, Rubric: *de nihilo*, derived from, based on, nothing, groundless.

27. *p.* 24, *l.* 25. Here and in *p.* 40, *l.* 4 Kepler plays on the senses of *causa*, as cause and as law-suit or plea. *In causa positum* means 'alleged as cause' and included in the legal action, i.e. the discussion *pro* and *con* of his problem which is *sub judice*. Perhaps 'our plea in court', 'our submission'.

28. *p.* 26, *l.* 8: *repraesentabis*. Kepler's text has repraesentaßis with the German digraph for sz which Kepler's printer uses occasionally and at random instead of the more common ſſ (he uses ſ except at the end of words, where s is always printed). The digraph ß is used in printing German for the unvoiced sibilant, otherwise represented by double s, at the ends of words, before a consonant and after long vowels in other positions (e.g. dem Māße, or Masze, as opposed to die Măsse); see examples in notes 7 and 30. Kepler's manuscript B has, it seems, been mistaken by the printer for ß. There is no such form as *repraesentassis*. We should, however, expect the subjunctive *repraesentes*, parallel to *imitetur*. The certainly wrong reading *innitetur* makes no sense but produces a real word, the future of innitor. Has the subjunctive *repraesentes* been altered to the future *repraesentabis* to agree with this mistaken future? or did Kepler write in *l.* 6 *et, si radiorum*? If so, *repraesentabis* would not be parallel to *imitetur*, but an independent sentence.

29. *p.* 28, *l.* 18. *Nam si quid agit condensat aut penetrat materiam qua . . . aut qua*. Something has gone wrong here. The first *aut* is misplaced, and we require: *penetrat aut qua hiat aut qua*, etc. But in *si quid agit condensat penetrat* the asyndeton *condensat penetrat* is impossible, and we require either *condensat et penetrat* or *condensat penetrans*. But I incline to cut out *agit* and read *si quid condensat, penetrat*: 'if it condenses anything, it penetrates either, etc.'. *Si quid agit, condensat*, 'if it does anything, it condenses it and penetrates it', does not make good sense.

30. *p.* 28, *l.* 19. *ut largus sim*. Klug (p. 19) translates: 'und um noch eine Möglichkeit zuzulassen, kann bei dem geradlinigem Fall gegen die Erde wohl noch der Tiefe die direckte Anordnung möglich sein, aber woher kommt die nach der Breite?' Rossmann (p. 23): 'und um alles gründlich zu überlegen: es könnte zwar die senkrechte Anordnung nach der Tiefenrichtung durch den geradlinigen Fall zur Erde bewirkt werden, doch woher dann diese Ausrichtung quer dazu?' Strunz–Borm (p. 13): 'Und um gründlich zu sein: es könnte schließlich zumindest die senkrechte Anordnung in dem geradlinigen Fall zur Erde gegründet sein, aber woher kommt dann diese entgegengesetzte Anordnung?'

Klug seems right about *largus*, which means 'generous'. But it is not quite clear to whom Kepler proposes to make a further concession; to himself or to his imaginary opponent? As the text stands, *possit* has no subject; either (*aliquis*) must be supplied or *possit* changed to *possim*; which seems the smaller change.

Causari is a deponent verb, and means 'to allege, plead, pretend', and cannot be passive (*bewirkt werden, gegründet sein*); if *causari* were passive, the subject would have to be in the nominative case.

31. *p.* 28, *l.* 29. *Neutrobique*, a non-existent word. *neutro casu* makes sense. *bique* looks like part of *ubique*, everywhere, but this makes no sense; *bique* may be a misprint for *utique* or some such word.

32. *p.* 28, *l.* 33. *oriretur,* so 1611, and often in classical manuscripts, but the correct form is *oreretur.*

33. *p.* 32, *l.* 28. *ludere in fluxis, quod multis fossilium exemplis patet.* Klug: 'wie aus vielen Versteinerungen (fossilia) hervorgeht'; Rossmann: 'was aus vielen Beispielen von Mineralen hervorgeht'; Strunz–Borm: 'was sich an vielen Beispielen von Fossilien erweist'.

Fossils in the modern sense of the word cannot be right, since they are of extreme antiquity and could hardly be called *fluxa,* momentary. In classical Latin, as in English until the late seventeenth century when the modern meaning begins, the word *fossile* means simply 'excavated'. 'Minerals' seems best, what are extracted from mines.

34. *p.* 34, *l.* 13, in margin. The line of Greek Iambic verse is quoted from Euripides, *Hecuba* 569. The reading *εὐσχημως* is found in several manuscripts and in authors who quote the line, but modern editions read *εὐσχήμων,* nom. sing. in agreement with the subject of the verb. *εὐσχημως* is an unattested form, a compromise between the adverb *εὐσχημόνως* (which Clement of Alexandria, *Strom.* ii. 182 reads, though it does not scan) and *εὐσχήμων.* Difficulty was felt about the adjective *εὐσχήμων* since the adverb would be more natural, and about the case of the adjective, which is attracted from accusative to nominative.

The Bodleian copy of the *Snowflake* has *εὐσχήμως,* but the British Museum copy *πολλῶ προνοίαν εἴσχεν σύσχυατε πεοεῖν,* clearly the printer's first attempt at Kepler's Greek.

35. *p.* 34, *l.* 23, in margin. The story of the Vestal Virgin Cornelia comes from Pliny the Younger, *Epp.* IV. xi. 9.

Olympias, the mother of Alexander the Great, is said by Justin, *Historiae Philippicae Epitome* (after Pompeius Trogus) XIV. vi. 12, to have faced the assassins sent to kill her bravely and *compsisse insuper exspirans capillos et veste crura contexisse fertur, ne quid posset in corpore eius indecorum videri.* Cf. Suetonius, *Divus Iulius* 82: *toga caput obvolvit, simul sinistra manu sinum ad ima crura deduxit, quo honestius caderet etiam inferiore corporis parte velata.*

36. *p.* 34, *l.* 19. *dividit sese . . . cum corporibus,* an odd and difficult phrase. Has a verb perhaps dropped out after *corporibus,* e.g. *operatur?*

37. *p.* 34, *l.* 19. *inque ea,* acc. neut. plur. presumably, understanding *corpora,* and not abl. fem. sing. with *terrâ* or *facultate* understood, i.e. *in eâ* (*terrâ*). But the dative (or ablative) *eis* would be more normal, cf. Virgil, *Georgics* ii. 77: *udoque docent inolescere libro.*

38. *p.* 34, *l.* 24. A difficult passage; perhaps: 'The same applies to the vapour which had been wholly possessed by the whole "soul"; and there is nothing to wonder at, if, while the cold is engineering the break-up of the uniform whole by the contraction of its parts, the whole soul should itself (in person so to speak) be busy with the formation of the parts into wholes, just as it had been before with the formation of each complete whole.

Or, more fully, 'with the formation of these same contracting parts into new wholes'.

39. *p.* 34, *l.* 26. *Aesopicae fabellae adultera,* the adulteress of Aesop's fable.

This fable, of the adulteress who told her husband on her return from a journey that she had conceived a child from a snowflake, and of her husband's revenge by later taking the child on a journey to the south and returning without the child, saying that the snow-child had melted in the sun, is not found in the collections of fables called Aesop's and derived from the Greek of Babrius or the Latin of Phaedrus, but comes from the *Esopus* of Burchard Waldis, a German who versified 'Aesop' and added new fables from other sources. The title of his book is *Esopus Gantz New gemacht und in Reimen Gefaßt mit sampt hundert Newen Fabeln durch Bucardum Waldis MDXLVIII.* His collection consisted of four books, each of a hundred fables, of which this is iv. 71 (it is *not* included in the reprint, ed. Julius Tittmann, Brockhaus, Leipzig (1882), *Deutsche Dichter des sechzehnten Jahrhunderts,* xvi).

The motif of conception from a snowflake is widespread. Cf. D. P. Rotunda, *Motif-Index of the Italian Novella,* Indiana (1942), and Stith Thompson, *Motif-Index of Folk Literature,* Copenhagen (1958), J 1532, 1. A tenth-century Latin version is the *Modus Liebinc,* no. 14 of the *Carmina Cantabrigiensia* ed. Karl Strecker, Berlin, 1926, reprinted 1955, pp. 41–43 (*Mon. Germ. Hist.* xl). Cf. F. J. E. Raby, *Secular Latin Poetry,* Oxford (1934), i. 295–7.

40. *p.* 36, *l.* 6. *si figuratas, quare figuras regulares.*

Klug (pp. 23–24): 'So ist es auch vernunftgemäß, daß sie lieber schön gestaltete Formen als andere annehmen und daß sie in diesem Fall regelmäßige Körper nachahmen.'

Rossmann (p. 28): 'So ist ganz natürlich, daß sie eher gestaltete als rohe Quantitäten annehmen, und zwar wenn geformt, dann nach Art der regelmäßigen Körper.'

Strunz–Borm p. 16): 'Und wenn sie geformt sind, dann sind sie aus diesem Grunde regelmäßige Figuren.'

Quare is either relative or interrogative; it cannot mean 'therefore', *dann, ob eam rem.* If we had *quare, si figuratas, figuras regulares, quare,* relative, could mean 'and for this reason', but we should expect a fresh sentence to be introduced by *quare.* What the sense demands is *et, si figuratas, figuras regulares,* or *et si figuratas, potius regulares figuras. Quare* is perhaps *quaere,* query, a note in the margin which has got into the text, when the printer raised some point, e.g.: insert '*et*', *quaere?*

41. *p.* 38, *l.* 4. *rudimentum saltem.*

Klug (p. 25 top): 'wo von beiden wenigstens die Anfangsform vorhanden ist'.

Rossmann (p. 29 middle): 'wo von beiden gerade ein Bruchstück ist'.

Strunz–Borm (p. 17 middle): 'wo von beiden nur ein Bruchstück da ist'.

'Bruchstück' emphasizes the shortcomings of both, but Kepler's point is that the skeleton of decussation is to be found in both, and he stresses how far they get towards his pattern and not how far they fall short. Klug is right, and *saltem* means 'at least'.

42. *p.* 38, *l.* 27. Caspar's *quo de dixi* is an unusual inversion for *id de quo dixi*; other prepositions, e.g. *per, circum,* are inverted, but not *de*. On *p.* 26, *l.* 24 Kepler writes 'qua de supra'. The 1611 edition has *dedixi,* one word, which does not make sense. *De* was perhaps added in proof in the margin and attached to *dixi* instead of put before *quo.*

43. *p.* 42, *l.* 21. *K*'s *in rosa, cucumeribus seminibus* as it stands, looks like a list oi three parallel things, in asyndeton without connectives. But as we have *pomis piris*que, I read *cucumeribus*que with a comma after it, as *seminibus* is not parallel to *cucumeribus* but ablative absolute.

44. *p.* 42, *l.* 33. *abacus* can mean a sideboard or chess-board or calculating board. The German translators all have 'eine Verzierung'.

45. *p.* 42, *l.* 36. Boll is a small spa near Göppingen in Württemberg. Kepler refers to Johannes Bauhinus, *Historia novi et admirabilis fontis balneique Bollensis in Ducatu Wirtembergico ad acidulas Goepingenses,* Montisbeligardi (1598), and its illustrations. But Kepler also visited Boll in person: *Gesammelte Werke* ii. 200.

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