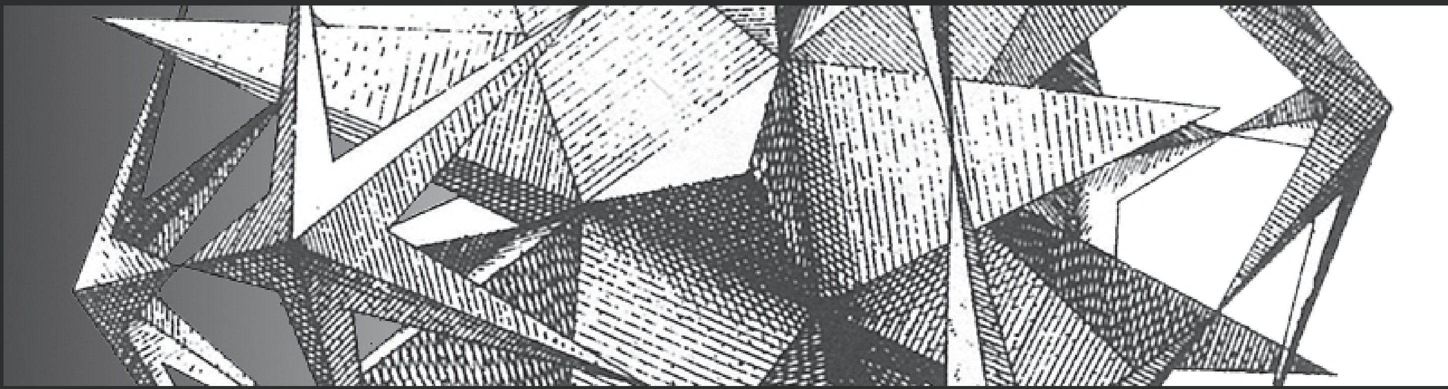

GEOMETRIAS'19: POLYHEDRA AND BEYOND

05, 06, 07 . September. 2019 | Faculdade de Ciências da Universidade do Porto

GEOMETRIAS'19: BOOK OF ABSTRACTS

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CENTRO DE MATEMÁTICA UNIVERSIDADE DO PORTO

FCT Fundação para a Ciência e a Tecnologia

APROGED ASSOCIAÇÃO DOS PROFESSORES DE GEOMETRIA E DE DESENHO

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Conference Venue: Faculdade de Ciências da Universidade do Porto, Portugal.

The International Conference Geometrias'19 was organized by Aproged, Faculdade de Ciências da Universidade do Porto and Centro de Matemática da Universidade do Porto.

Geometrias'19 Organizing Committee:
Helena Mena Matos, Faculdade de Ciências da Universidade do Porto
João Pedro Xavier, Faculdade de Arquitectura da Universidade do Porto
Vera Viana, Aproged and CEAU - Faculdade de Arquitectura da Universidade do Porto

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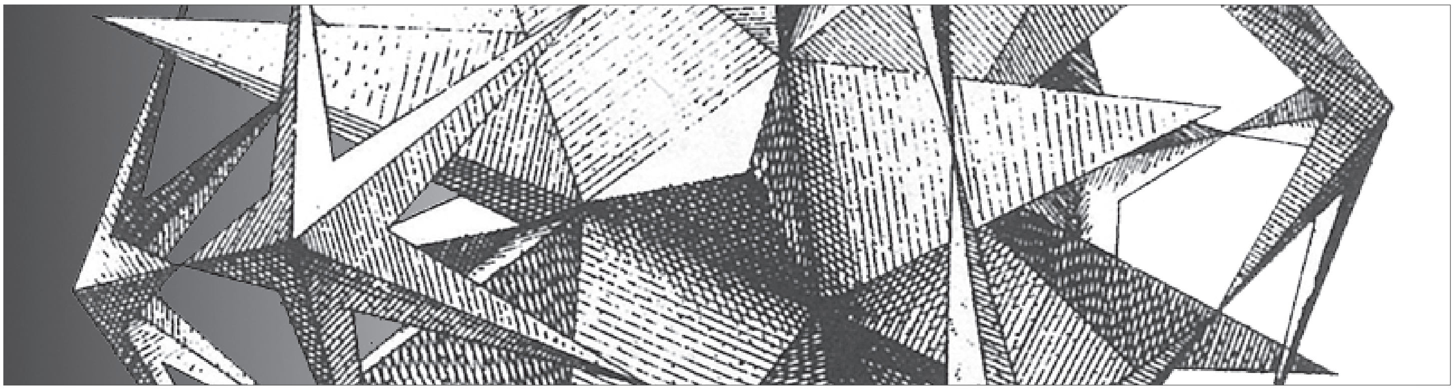

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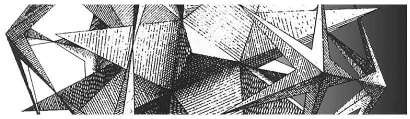

SATURDAY, 07 SEPTEMBER 2019

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FOREWORD

The Organizing Committee is exceptionally pleased to welcome all the participants, who have travelled from Austria, Belgium, Brazil, Germany, Israel, Italy, Japan, the Netherlands, Portugal, Russia, Serbia, Slovenia, Spain, and the United States of America to the city of Porto, to attend this unique conference held in the Faculty of Sciences of Porto's University, between the 5th and the 7th of September, 2019.

Geometrias'19 is the 5th edition of the International Conferences that have been organized, since 2013, by Aproged (the Portuguese Association of Geometry and Drawing Teachers), with the aim of expanding a network of common interests and knowledge between academics, artists, researchers, scientists and students, fostering opportunities for them to share their investigations and practical experiments related to the theme proposed for each edition, closely connected to Geometry through a broad panorama of topics.

The first conference, Geometrias'13, took place in 2013, in the Faculty of Architecture of Porto's University; and Geometrias'14 at ISCTE in the University Institute of Lisbon, with *New challenges on practice, researching and teaching Geometries and Drawing* as engaging theme. The 2015's edition, held in Lisbon's' Lusíada University, was a joint conference between Geometrias and the Brazilian *Graphica*, gathered under the theme *Trends on Graphic Thinking*. Geometrias'17 was held in the Department of Architecture of Coimbra's University with *Thinking, Drawing, Modelling* as leitmotif.

In 2019, *Geometrias: Polyhedra and Beyond* is being held in the Department of Mathematics of the Faculty of Sciences of Porto's University, with the intent of fostering interdisciplinary discussions and connections between theoretical researches and practical studies on polyhedra and geometric structures and its applications in architecture, arts, computer science, crystallography, design, education, engineering, mathematics, materials' science and other related fields of study, all of which we believe might enrich each other and renew mutual interests and synergies through networks of common inspirations.

The program of Geometrias'19 includes 5 lectures from the outstanding Keynote Speakers Javier Barrallo, Henry Segerman, Manuel Arala Chaves, Michael Hansmeyer and Rinus Roelofs, who will share with us some of their remarkable researches related to the conference's leitmotif.

In addition to each of the Plenary Sessions, the authors that have travelled to Porto will have ample opportunity to orally present and discuss their researches (17 papers and 7 posters), during specific sessions in each of the days of the conference. These presentations were selected in accordance to the reviewing process that the Scientific Committee underwent in response to the Call for Abstracts, that ended in March 2019.

In the afternoon of the last day, Javier Barrallo and Rinus Roelofs will certainly give the participants of each of their Workshops, more enjoyable reasons to remember this conference.

Along with unique opportunities to learn and share knowledge, the additional activities of the social program and many enjoyable moments to socialize will certainly provide everyone great opportunities to learn and enhance fruitful personal connections and friendly moments that, hopefully, all the participants will pleasantly remember.

The Organizing Committee would like to express our deepest gratitude to The Faculty of Sciences and the Center of Mathematics of Porto's University, without whom this conference would not have been possible; to FCT - Fundação para a Ciência e a Tecnologia for their significant support; to every member of the Scientific Committee for their incomparable commitment in the evaluation of all the submissions; and to all the Keynote Speakers, Sessions Moderators, Speakers and Authors, for their exceptional contribution to the conference scientific program. A word of recognition still to all the collaborators who, in some way, contributed to the accomplishment of this conference.

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We hope that the conference program and the additional activities we prepared match the expectations of all the participants and that many productive opportunities occur to everyone enhance their personal, academic and professional development. We wish that everyone will be able to learn within and beyond their fields of interest, with noteworthy repercussions and positive memories of the people and events gathered in the beautiful city of Porto, that welcomes everyone, as much as the organizers, with wide open arms.
As a symbolic gesture of the fruitful and close connections that we wish to foster during these three days, we offer all the participants a printed model of the first stellation of the rhombic dodecahedron, the interesting space-filling polyhedron also known as Escher's solid.

A final remark to denote that this Book of Abstracts was conceived as a record of the events that will take place in the three days of the conference and also as a guide for each of the sessions, given that the sequence of chapters and abstracts follow closely the presentations scheduled in the conference program.
Following the conference, a number of presentations will be selected to develop into full-papers, to be included in a book entitled "*Polyhedra and Beyond. Outcomes of the Geometrias'19 Conference*", to be published in a near future.

The Organizing Committee wishes everybody a great time during the conference and a very pleasant stay in the city of Porto!

Helena Mena Matos, Faculdade de Ciências da Universidade do Porto
João Pedro Xavier, Faculdade de Arquitectura da Universidade do Porto
Vera Viana, Aproged and CEAU - Faculdade de Arquitectura da Universidade do Porto.

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ACKNOWLEDGMENT TO THE SCIENTIFIC COMMITTEE

Helena Mena Matos, João Pedro Xavier and Vera Viana express their highest gratitude and appreciation to all the members of the Scientific Committee (mentioned below), for the importance of their contribution during the reviewing process of all the submissions. We congratulate ourselves for the collaboration with every member of the Scientific Committee, without whom, we are sure, the quality of this international conference could not be the same.

Thank you!

The Organizing Committee,
Helena Mena Matos, Faculdade de Ciências da Universidade do Porto
João Pedro Xavier, Faculdade de Arquitectura da Universidade do Porto
Vera Viana, Aproged and CEAU - Faculdade de Arquitectura da Universidade do Porto

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USEFUL INFORMATION

The Scientific Program of the Geometrias'19 Conference will be held in the Department of Mathematics of Faculdade de Ciências da Universidade do Porto[1].
The participants' reception occurs on the 5th of September, from 09h00 to 13h00.
Each participant will be given a Conference ticket with his/her identification.
We ask everyone to wear it, at all times, during the conference.

All the presentations will take place in Room 2.19 in the second floor of Building FC1 (Departamento de Matemática).
Each Plenary Session (PS) has a slot of 60 minutes for presentation and debate.
Each of the Paper Presentations (PP) has a period of 20 minutes; Poster Presentations (PT), 07 minutes.
At the end of each Session of Paper and Poster Presentations, a short period is planned for debate and discussion.
The 5 minutes interval at the end of each presentation is scheduled for the preparation of the next presentation.
The workshops scheduled for the 7th of September will occur in Rooms 2.19 and 2.29, opposite to each other.

The Participants that have registered for the Social Program can take the Afterhours Tour to Torre dos Clérigos, any day between the 04th and the 07th of September, from 19h00 to 23h00.
On the 5th of September, the concert of *Orquestra XXI* at Casa da Música[2] begins at 21h00.
Founded in 2013, *Orquestra XXI* is a project that brings together young Portuguese musicians living abroad with the twofold objective of keeping a strong link between them and their native country, and providing outstanding musical moments to the most diverse audiences.
The Conference dinner will take place on the 6th of September, at Restaurante Terrella, near Casa da Música[3], after the guided tour to Casa da Música, scheduled for 19h00.

Following the conference, the authors of a limited number of presentations will be invited to develop their research into full papers, for publication in a contributed volume, to be proposed for publication to Springer.
This selection will be accomplished by the Organizing Committee on the basis of the extended abstract and its oral presentation at the conference.
More details about this publication will be announced in a near future.

CONFERENCE WEBPAGE:
http://www.aproged.pt/geometrias19.html

WI-FI ACCESS:
username: dmat.wifi.6@fc.up.pt
password: Geometrias19

[1] Rua do Campo Alegre, 687 4169-007 Porto.
[2] Avenida da Boavista, 604-610. 4149-071 Porto.
[3] Rua Ofélia Diogo da Costa 105 A. 4149-071 Porto.

GEOMETRIAS 19 - GENERAL SCHEDULE

THURSDAY 05 . SEPTEMBER . 2019

09h00 - 10h00	Reception and Welcoming Coffee
10h00 - 10h30	Opening Session
10h35 - 10h55	Paper Presentation 01
11h00 - 11h20	Paper Presentation 02
11h25 - 11h45	Paper Presentation 03
11h50 - 12h10	Paper Presentation 04
12h10 - 12h30	Debate (20’)
LUNCH	
14h00 - 15h00	Plenary Session 01 JAVIER BARRALLO
15h05 - 16h05	Posters' Presentations (PT 01 to PT 07)
16h05 - 16h30	Debate (25’)
16h30 - 17h00	Coffee Break
17h00 - 18h00	Plenary Session 02 MICHAEL HANSMEYER
21h00	Performance of “Orquestra XXI” at casa da Música

FRIDAY 06 . SEPTEMBER . 2019

09h00 - 10h00	Plenary Session 03 RINUS ROELOFS
10h00 - 10h35	Coffee Break
10h35 - 10h55	Paper Presentation 05
11h00 - 11h20	Paper Presentation 06
11h25 - 11h45	Paper Presentation 07
11h50 - 12h10	Paper Presentation 08
12h10 - 12h30	Debate (20’)
LUNCH	
14h00 - 14h20	Paper Presentation 09
14h25 - 14h45	Paper Presentation 10
14h50 - 15h10	Paper Presentation 11
15h15 - 16h05	Paper Presentation 12
16h05 - 16h30	Debate (25’)
16h30 - 17h00	Coffee Break
17h00 - 18h00	Plenary Session 04 HENRY SEGERMAN
19h30	Guided Tour to Casa da Música
21H00	Conference Dinner

SATURDAY 07 . SEPTEMBER . 2019

09h00 - 10h00	Plenary Session 05 MANUEL ARALA CHAVES	
10h00 - 10h35	Coffee Break	
10h35 - 10h55	Paper Presentation 13	
11h00 - 11h20	Paper Presentation 14	
11h25 - 11h45	Paper Presentation 15	
11h50 - 12h10	Paper Presentation 16	
12h15 - 12h35	Paper Presentation 17	
12h35 - 13h00	Debate (20’)	
LUNCH		
14h30 - 16h00	Workshop 01 with J. BARRALO	Workshop 02 with R. ROELOFS
16h00 - 16h30	Coffee Break	
16h30 - 18h00	Workshop 01 with J. BARRALO	Workshop 02 with R. ROELOFS

CONFERENCE PROGRAM
THURSDAY 05 . SEPTEMBER . 2019

09h00 - 10h00	RECEPTION and WELCOMING COFFEE	
10h00 - 10h30	OPENING SESSION	
10h35 - 10h55	Paper Presentation 01	Henriette-Sophie Lipschütz and Ulrich Reitebuch (Germany) *Creating Star Shaped Polyhedra Based on Catalan Solids Using Modular Origami*
11h00 - 11h20	Paper Presentation 02	Denise Ulivieri, Filippo Iardella and Marco Giorgio Bevilacqua (Italy) *Vittorio Giorgini's Architectural Experimentations at the Dawn of Parametric Modelling*
11h25 - 11h45	Paper Presentation 03	Gianluca Stasi (Italy) *Geodesic Structures: Double Curvature Triangulated Meshes*
11h50 - 12h10	Paper Presentation 04	Adolfo Pérez-Egea, Pedro Miguel Jiménez-Vicario, Pedro García-Martínez, Manuel Ródenas-López, Martino Peña Fernández-Serrano (Spain) *Geometry and Efficiency in the Joints of Deployable Structures*
12h10 - 12h30	Debate	
	LUNCH (Faculdade de Ciências da Universidade do Porto's Canteen)	
14h00 - 15h00	Plenary Session 01	JAVIER BARRALLO *Polyhedra: Didactic Experiences*
15h05 - 16h05	Posters Presentations	PT 01 | Emmanouil Vermisso (United States of America) *Gaudi Inverted:* *A Layered Design Protocol for Integrating Mathematical Inversion and Ruled Surfaces* PT 02 | Takashi Yoshino and Atsushi Matsuoka (Japan) *Straw Models Representing the Skeletal Structures of Radiolarian Pantanellium* PT 03 | R. Obradović, M. Vujanović, I. Kekeljević, I. Vasiljević, I. Đurić, L. Krstanović, B. Banjac (Serbia) *Polyhedral Characters as a Basis for Teaching Computer Animation* PT 04 | Luigi Corniello, Enrico Mirra, Lorenzo Giordano (Italy) *Architectural Applications of Geometric Structures.* *Case Studies of Survey In Southern of Albania* PT 05 | Raul Piepereit and Margitta Pries (Germany) *Automated Preprocessing of Virtual 3D Building Models for Flow Simulations* PT 06 | Samanta Teixeira, Thaís Yamada, Galdenoro Junior (Brazil) *Digital Folding Design and Deformation Test of Origami Tessellations* PT 07 | Teresa Pais (Portugal) *The Representation of Objects in the Observational Drawing*
16h05 - 16h30	Debate	
16h30 - 17h00	Coffee Break	
17h00 - 18h00	Plenary Session 02	MiCHAEL HANSMEYER *Tools of Imagination*

CONFERENCE PROGRAM
FRIDAY 06 . SEPTEMBER . 2019

09h00 - 10h00	Plenary Session 03	RINUS ROELOFS *Helices in Uniform Polyhedra*
10h00 - 10h30	Coffee Break	
10h35 - 10h55	Paper Presentation 05	Ulrich Reitebuch, Henriette-Sophie Lipschütz and Konrad Polthier (Germany) *Filling Space with Gyroid Symmetry*
11h00 - 11h20	Paper Presentation 06	Marija Obradović and Mišić Slobodan (Serbia) *Concave Deltahedral Rings Based on the Geometry of the Concave Antiprisms of the Second Sort*
11h25 - 11h45	Paper Presentation 07	Izidor Hafner and Mateja Budin (Slovenia) *Dissections of Cubes and Golden Rhombic Solids*
11h50 - 12h10	Paper Presentation. 08	Günter Weiss (Austria) *Equifaced Simplices and Polytopes*
12h10 - 12h30	Debate	
	LUNCH (Faculdade de Ciências da Universidade do Porto's Canteen)	
14h00 - 14h20	Paper Presentation 09	Alexei Kanel-Belov, A.V. Dyskin, E. Pasternak and Y. Estrin (Israel, Australia) *Topological Interlocking of Platonic Bodies: Geometry Enabling the Design of New Materials and Structures*
14h25 - 14h45	Paper Presentation 10	Dirk Huylebrouck (Belgium) *An Euler-Cayley Formula for General Kepler-Poinsot Polyhedra*
14h50 - 15h10	Paper Presentation 11	Andrés Martín-Pastor (Spain) *Polyhedral Transformation Based on Rotational Quadratic Surfaces Properties*
15h15 - 16h05	Paper Presentation 12	Leonardo Baglioni and Federico Fallavollita (Italy) *The Construction of Regular and Semiregular Polyhedra Through the Synthetic Method*
16h05 - 16h30	Debate	
16h30 - 17h00	Coffee Break	
17h00 - 18h00	Plenary Session 04	HENRY SEGERMAN *Artistic Mathematics: Truth and Beauty*

CONFERENCE PROGRAM
SATURDAY 07 . SEPTEMBER . 2019

09h00 - 10h00	Plenary Session 05	MANUEL ARALA CHAVES *Symmetries in Portuguese Azulejos and what this has to do with Geometries)*	
10h00 - 10h30	Coffee Break		
10h35 - 10h55	Paper Presentation 13	João Pedro Xavier, José Pedro Sousa, Alexandra Castro and Vera Viana (Portugal) *An Introduction to Solid Tessellations with Students of Architecture*	
11h00 - 11h20	Paper Presentation 14	Cristina Candito and Ilenio Celoria (Italy) *Applications of Polyhedra and Geometric Structures Sources and Features of the Small Stellated Dodecahedrons Painted in Genoa*	
11h25 - 11h45	Paper Presentation 15	Joan Carles Oliver (Spain) *The "Hele" Module of Rafael Leoz.* *Research and Photographic Dissemination of Modular Combination in Architecture*	
11h50 - 12h10	Paper Presentation 16	Joana Maia and Vitor Murtinho (Portugal) *Invention and Order*	
12h15 - 12h35	Paper Presentation 17	Olga Melnikova (Russia) *Polyhedrons in the Wooden Temple Architecture of Ancient Russia*	
12h35 - 13h00	Debate		
	LUNCH (Faculdade de Letras da Universidade do Porto's Canteen)		
14h30 - 16h00	Workshops	JAVIER BARRALO *Didactic Experiences with Polyhedra*	RINUS ROELOFS *Making Paper Polyhedra Models*
16h00 - 16h30	Coffee Break		
16h30 - 18h00	Workshops	JAVIER BARRALO *Didactic Experiences with Polyhedra*	RINUS ROELOFS *Making Paper Polyhedra Models*

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THURSDAY, 05 SEPTEMBER 2019

PAPERS' SESSION: COMPUTATIONAL AND CONSTRUCTIVE METHODS
SESSION MODERATOR: FEDERICO FALLAVOLLITA

PAPERS' SESSION: ARCHITECTURAL AND ENGINEERING APPLICATIONS OF GEOMETRIC STRUCTURES
SESSION MODERATOR: FEDERICO FALLAVOLLITA

PLENARY SESSION 01: KEYNOTE SPEAKER

POSTERS' SESSION
SESSION MODERATOR: GÜNTER WEISS

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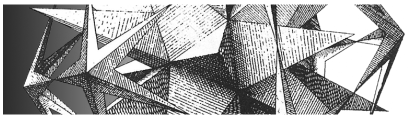

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PAPERS' SESSION
PAPER 01

CREATING POLYHEDRA IN SHAPE OF A STAR BASED ON CATALAN SOLIDS USING MODULAR *ORIGAMI*

Henriette-Sophie Lipschütz[1], Ulrich Reitebuch[2] and Konrad Polthier[3]

KEYWORDS: *Origami*, Platonic Solids, Archimedean Solids, Catalan Solids.

INTRODUCTION

Every polytope can be stacked, i.e. every face of the polytope can be equipped by a pyramid. Especially polytopes consisting of regular faces, such as the Platonic solids or the Archimedean solids, yield beautiful star-shaped polyhedra after placing a a pyramid on each face. The regularity resp. the symmetry of the faces refers to choosing regular triangles or isosceles triangles as faces of the pyramids. In modular *origami*, there are several modules presented that can be used to create such star-shaped objects as P. Bascetta did, with a module simple to fold and easily to put up [01]. Since the faces of the Platonic solids and the faces of the Archimedean solids are regular, it is possible to construct a regular pyramid having a face of the solid as base and isosceles triangles as faces. This is not true for the Archimedean duals or Catalan solids. In the abstract, we investigate the Catalan solids due to the pyramids the faces can be equipped with and present different *origami* modules to create a star-shaped polyhedron.

RESEARCH

P. Bascetta created a module in the shape of a rhombus, that is subdivided into two isosceles triangles by folding along the shorter diagonal [01]. The shorter diagonal replaces one of the edges of the chosen Platonic solid. Every edge lies in two neighboring faces. Therefore, each of the triangles belongs to one of the pyramids over these faces. Hence, the number of edges of the solid is equal to the number of modules needed to create a star-shaped polyhedron. All the Platonic solids can be turned into star-shaped polyhedra using Bascetta's module. Considering the dodecahedron, it stands out that the faces of the pyramids lie in the same plane as the pentagon they are adjacent to. This is not the case for the other Platonic solids - on one hand, because the dihedral angles of the tetrahedron and the cube are smaller or equal to π. Therefore, their faces do not give a proper pyramid. On the other hand, the angles that arise by putting up several Bascetta modules are fixed, because the angle opposite to the shorter diagonal is equal to $\pi/4$. But the folding principle of Bascetta can be used to create a module resulting in pyramids fulfilling the observation made above. As discussed in [02], the Bascetta module is not suitable for all Archimedean solids, since joining eight modules yields a flat octagon or, in the case of a decagon, the joined modules over the decagon give some hyperbolically shaped region. A module by M. Mukhopadhyay has the shape of a rhombus and is folded along its shorter diagonal, too [03]. This module yields an angle opposite the shorter diagonal which is smaller than the one in Bascetta's module, it can be used to build a pyramid based on an octagon, but not to build a pyramid based on a decagon. The *Mukho-padhyay* module does not fulfil the property of extending the original faces.

Both ways of folding can be transferred to other paper formats. In case of the Bascetta module, the theoretical construction of dividing a given angle between 0 and $\pi/2$ twice into halves is applied, to create the border of the module and the flaps that are used to join the modules. In practice, the revised Bascetta module is not well-suitable to build pyramids based on octagons, since the modules are prone to fall apart. Here, the Mukhopadhyay module is better suitable to work with. The Mukhopadhyay module can be revised in such a way that all Archimedean solids

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can be equipped with pyramids over their faces. Since the edges of an Archimedean solid are, in general, contained in two different types of faces, it depends on the faces how the original face can be extended (as in case of the truncated cube, for instance). Neighboring faces make a dihedral angle. Considering the plane containing one of the faces, the plane and the second face make an angle such that the sum is equal to π. The angles made by neighboring bases will determine if extending the faces results in a pyramid (for instance, in case of the truncated dodecahedron - building the appropriate pyramid over a triangle does not lead to a pyramid over the neighboring decagon). In general, it is possible to construct polyhedra that are star-shaped from a more artistic point of view by decreasing the angle opposite to the shorter diagonal, as done in case of the so-called Herrnhuter star (see Fig. 1) or the so called Marienberger star which is a stacked dodecahedron.

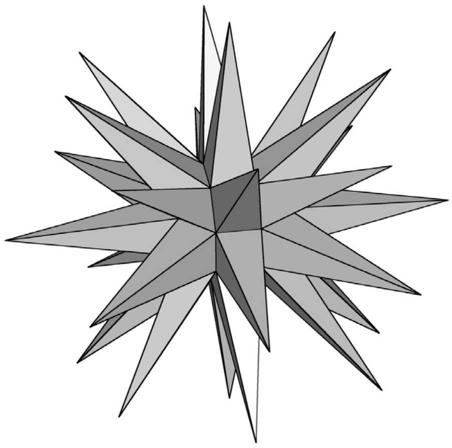

Fig. 1 - A Herrnhuter star
(a stacked rhombicuboctahedron).

The basic idea described above also fits to the Johnson solids. Of course, the application of the described procedure yields stacked polytopes, but one might be more interested in more spherical-shaped results, since some of the Johnson solids are not (the pentagonal rotunda for instance). Hence, the natural question arises if it is possible to handle the Archimedean duals resp. the Catalan solids. Opposite to the so far named families of polytopes, every Catalan solid consists of exactly one type of face. The faces are not regular, neither due to the side length, nor to the angles. Extending the faces of a Catalan solid until they meet results in a pyramid, since the faces arising intersect in exactly one point - the apex of the pyramid (see Fig. 2). We are going to present different *origami* modules based on the abovementioned Mukhopadhyay module and a module created by M. Mukerji that gives a quadrilateral pyramid over a rhombical base [04].

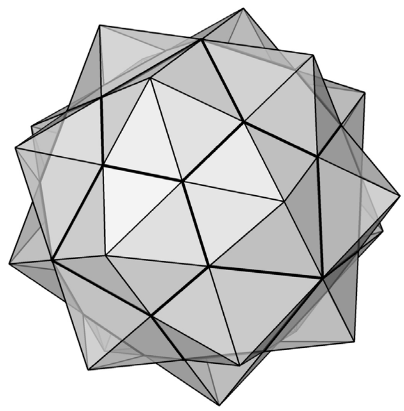

Fig. 2 - The pentagonal icositetrahedron equipped with pyramids providing the original faces.

Our investigation contains also the consideration of the constructability of the achieved pyramid faces with respect to the *origami* axioms as listed in [05]: we are evolving a folding routine starting with a chosen edge length, so all folding steps depend solely on the edge length of the chosen solid. For instance, considering the rhombic dodecahedron, the Mukerji module can be modified in a suitable way to a pyramid having faces with edge length a and $2\sqrt{3}a$ (Fig. 3).

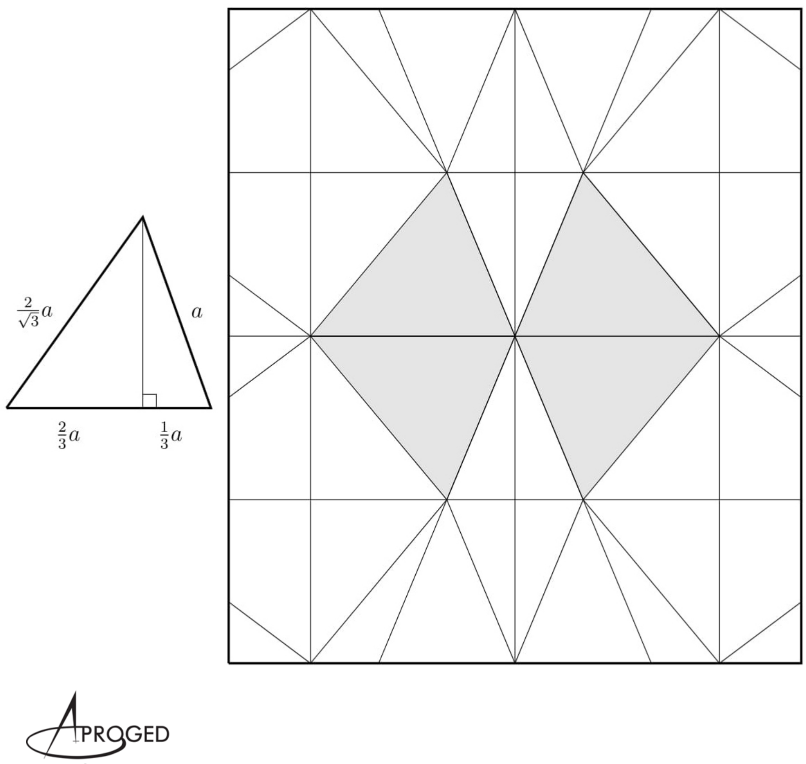

Fig. 3 - Sketch of unfolded module of the modified Mukerji module, left: face of the pyramid, right: shaded regions show the faces of the pyramid.

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CONCLUSIONS

In this extended abstract, we described a family of star-shaped polyhedra based on the Catalan solids. Due to the close relation of the Catalan solids and the Archimedean solids and, therefore, to the Platonic solids as well, an approach similar to one based on recent research ([02]) was chosen. Inspired by the observations made by considering the Platonic solids and the Archimedean solids, we investigated if the faces of the Catalan solids can be provided into pyramids over neighboring faces. Since this is true, we constructed and presented different approaches by modifying known *origami* modules to create models of the star-shaped Catalan solids. Furthermore, we made sure that the presented modules can be folded given the edge length of the chosen solid.

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PAPERS' SESSION
PAPER 02

VITTORIO GIORGINI'S ARCHITECTURAL EXPERIMENTATIONS AT THE DAWN OF PARAMETRIC MODELLING

Denise Ulivieri[1], Filippo Iardella[2] and Marco Giorgio Bevilacqua[3]

KEYWORDS: Vittorio Giorgini, Symmetrical and Asymmetrical Meshes.

INTRODUCTION

Vittorio Giorgini (1926-2010), a space-morphologist architect, grew in Florence, where he attended the School of Architecture. From the earliest years of his academic studies, he showed interest in developing a research on natural models with the aim of applying them to architecture, in order to obtain more efficient complex systems. His studies focused, therefore, on the analysis of membrane structures, tensile structures and on the elaboration of tetrahedral and octahedral structural meshes. He experimented spatial meshes which contract, expand, recompose and adapt to tensions, as it happens in the nervous, circulatory or muscular systems.

Vittorio Giorgini committed to the systemic vision of contemporary scientific thought - "the structure of a system is the order in which the elements are organized" [01: 211] - to develop a dynamic, articulated, sophisticated architecture, open in all the directions, where geometric principles, structural and functional needs are perfectly integrated.

RESEARCH

Vittorio Giorgini could be considered as a pioneer, as he fully understood, along with a few others, what Thomas S. Kuhn [02] defines as a "new paradigm", that is, the transition to a new vision of the physical universe, in which instability and fluctuations are at the origin of the incredible variety and richness of forms, and structures that could be seen around us in the world [03].

He thought about a different way of understanding architecture, based on the search for integration with nature, which, however, is not solved in the simple imitation of the forms of the organic world, but in the design of spaces suitable for Man's life.

Until the first half of the 19th century, Euclidean geometry was the only instrument used for describing nature, but the advent of non-Euclidean geometry led to a "fundamental rethinking of the meaning of space-time, matter and energy, information and noise" [04], which inevitably led to the study of new ways of conceiving and realizing architecture.

According to Giorgini, geometry is "an analysis, verification and operational tool" [01: 193]. He considered the study of geometry as the basis of static and structure. "Geometry is the basic order from which the models are developed" and his attempt was to approach models of nature that have "efficient mechanisms of self-control".

Giorgini's world is, therefore, "post-Euclidean", a complex and continuous reality where natural evolution proceeds "systematically with dynamic transformations, adaptations and continuous retroactions" [05: 39], a dynamic and interrelated reality that, since the 1950s, he investigated also through topological geometry. He chose the term "Spatiology" for defining his studies on morphology, "the study of geometry as a mathematical discipline and 'backbone' of statics, (systemic) taxonomy, and technology" [01: 193]. As a morphologist-spatiologist architect, he

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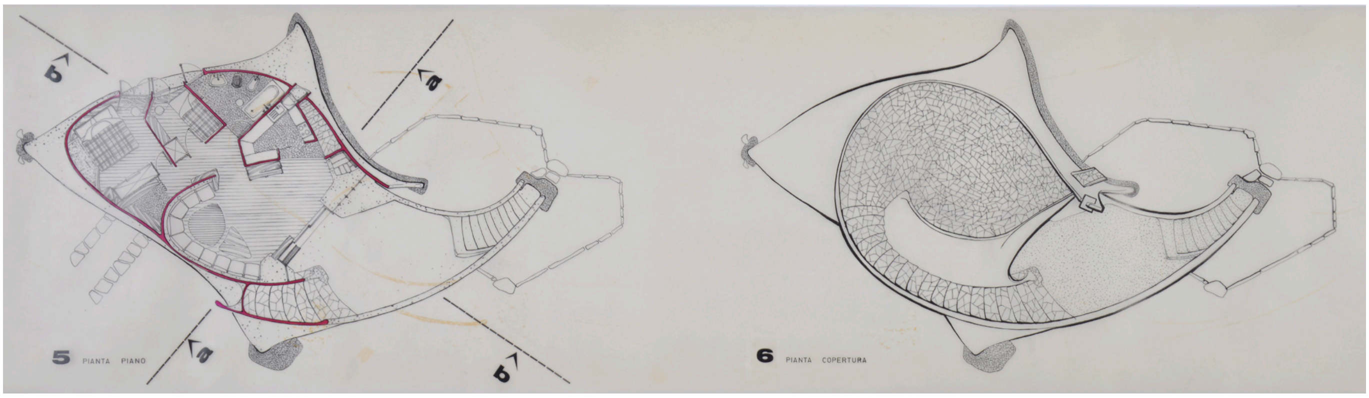

Fig. 1 - Plan drawings of the Saldarini House (Courtesy B.A.Co. - Vittorio Giorgini Archive).

created in 1962 the Saldarini House in the Gulf of Baratti (Tuscany), also known as 'Casa Balena' (Whale House) or 'Casa Dinosauro' (Dinosaur House) (Fig. 1), a "fanciful morphology" [05] where the "topological notions of transformation and continuity intertwine with the architectural topics of flexibility, fluidity and dynamism" [06: 130]. Giorgini analyzed "the transitions from the linear (the straight line), to the bent (broken) up to the curved, both for lines and for surfaces" and "meshes generated by different geometries and transformed, symmetrically and asymmetrically, following the forces action" [01: 199] (Fig. 2). On the other hand, from the topological point of view, "forms are the product of the continuous transformation of other forms", and, as such, the topology "considers a brick and a billiard ball as if they had the same shape, and even a teacup or a disk".

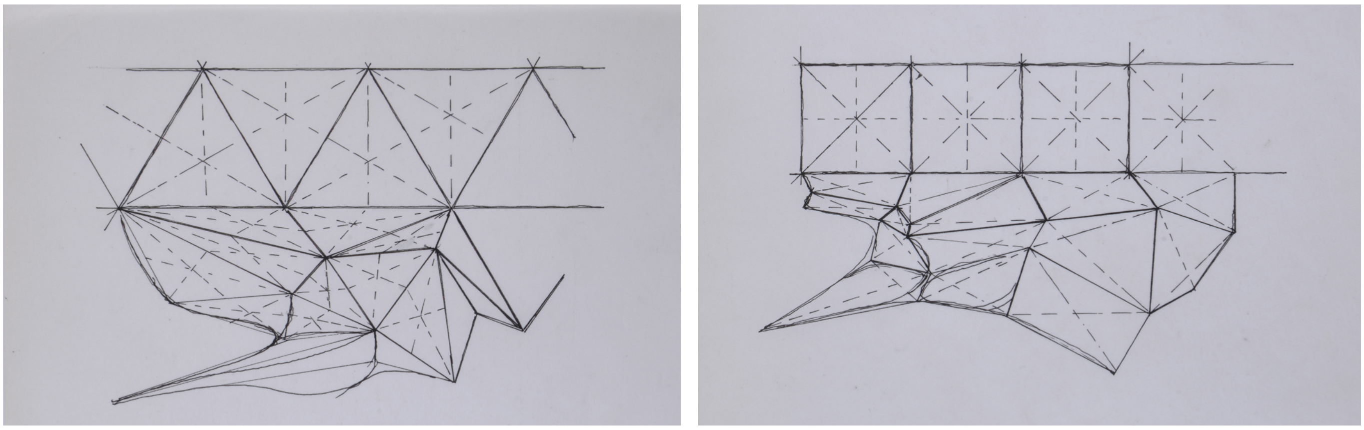

Fig. 2 - Study for the deformation of a mesh (Courtesy B.A.Co. - Vittorio Giorgini Archive).

Giorgini agreed with the theory by the British biologist and mathematician D'Arcy Wentworth Thompson, that "the shape of an object is a diagram of forces"; he then applied Thompson's "theory of transformations" [07: 290-349] to symmetrical and asymmetrical meshes, analysing the structural behaviour and quantifying the forces that modify the original model. Vittorio Giorgini's studies, such as Le Corbusier's research on hyperbolic geometry, the technological and formal solutions of Richard Buckminster Fuller and Frei Otto, "came out of the renaissance static perspective mental framework and joined Einstein's curved space, the folded space by Gilles Deleuze or the topologically deformed space described by René Thom" [08: 55].
Leaving aside the orthogonality of the usual space, Giorgini developed a "*ante-litteram*" morphing process, based on the rectification of curved lines. Giorgini's pioneering and unrealized projects of his American period (1969-1996), mostly characterized by the use of tetrahedral and octahedral structures, were branded as utopian and absurd. However, he firmly believed in innovation and technology as tools for reducing the distance between Man and nature.

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At the *Biennale of Venice* in 1978, he declared, in fact, that: "New instruments will allow for incremental and infinite spatial variants with possibilities and economies never thought before". The architectural archive of Giorgini still saves many of his projects, models, sketches, notebooks, design objects, photographs which tell us his tormented life of research.

Nowadays, the advent of the electronic paradigm makes representing Giorgini's themes possible. Following what was presented at the *Nexus Conference* in Pisa and has now been published in the issue of the *Nexus Network Journal* [09], the aim of this research is to demonstrate how pioneering Giorgini was, anticipating several years of recent experimentation in the field of parametric modelling and computational design. In particular, the research focuses on the case study of symmetrical geometric meshes which, through dynamic transformation, i.e. the application of forces, change into asymmetric meshes as it happens in nature.

Fig. 3 - Liberty Community Centre. On the left, concept drawing; on the right, study model (Courtesy B.A.Co. - Vittorio Giorgini Archive).

Giorgini, in fact, tried to define static diagrams in order to quantify the forces and the reasons that transformed the original symmetrical model, with the aim of experimenting more efficient and economic design techniques.

Giorgini's meshes contract, expand, reform, move in every direction, composing different symmetrical and asymmetrical geometries, adapting themselves to tensions such as the nervous or circulatory systems tissues.

However, he pointed out that "while in nature it is simple for a mesh to become asymmetrical, with our techniques it is difficult and expensive" [01: 214]. Giorgini modelled his meshes in an intuitive and experimental way, as in the case of Casa Saldarini or the Liberty project, where meshes were modelled manually, in order to obtain the desired curvature. Giorgini understood that only with the help of technology and electronic instruments, which in those years saw the first slight beginnings, it is possible to obtain a mathematical control of meshes.

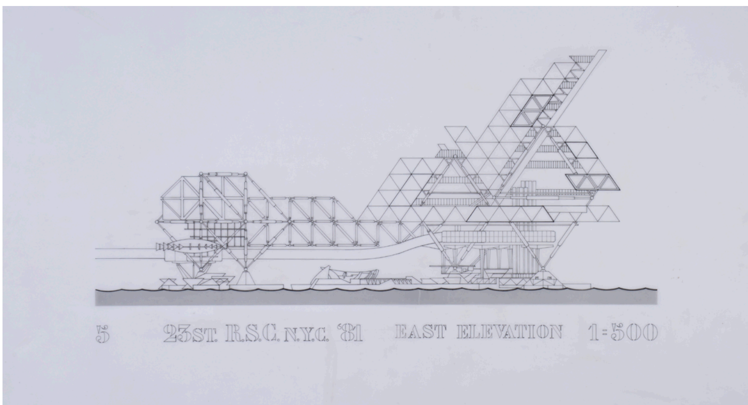

Fig. 4 - Octa-frame System. Hydropolis. East Elevation drawing (Courtesy B.A.Co. - Vittorio Giorgini Archive).

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Giorgini's experimentations have, therefore, been developed in current software of parametric modelling. In particular, the experimentation will focus on the modelling of double-curved asymmetric surface systems with topological morphological characteristics, such as the Saldarini House and the unfinished project of the Liberty Community Centre (State of New York, 1976-1979) (Fig. 3), up to the unrealized projects designed for Manhattan, Hydropolis (1981-1982) (Fig. 4) and Genesis (1984) (Fig. 5), based on Giorgini's Octa Frame System, a self-bearing octahedral-tetrahedral base module. Nowadays, modelling in a digital environment allows us to develop a critical analysis of Giorgini's work, verifying in particular the limits induced by the lack of specific software and, at the same time, a better understanding of the peculiar characteristics within the context of his cultural references.

Fig. 5 - Octa-frame System. Genesis. Model (Courtesy B.A.Co. - Vittorio Giorgini Archive).

CONCLUSIONS

The aim of this research is to give back to Vittorio Giorgini the role of pioneer that he deserves. In this sense, a strong reproposition of the actuality of his work and the different sensibility from the general mainstream of his time, through dynamic models and methods of investigations and design, means re-reading, analysing and critically evaluating his design thinking.

Giorgini surprises us for his cultural and social relationships: he was a friend of André Bloc and of the sculptor Isamu Noguchi; he met Richard Buckminster Fuller for discussing about architecture; he was also a good friend of Sebastián Matta and of his son Gordon Matta-Clark, a colleague and a friend of John M. Johansen. Giorgini met Peter Eisenman at the Pratt Institute where Giorgini teached until 1996, before coming back to Italy.

Giorgini foresaw the enormous creative possibilities offered by the digital language, in a certain sense, demonstrating to have a 'parametric mentality', which he couldn't develop, due to the lack of adequate tools and a series of other personal reasons: the first personal computers arrived in the 1980s and, in 1995, a serious eye disease profoundly left

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a mark in the last part of the life of Giorgini, who declared, with profound bitterness, that he had been left very alone and that his work had never generated great interest from critics.

> "*My research has remained fruitless to this day. What remains is only an intention, a concept, a supposition, but no confirmation.*" [10].

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PAPERS' SESSION
PAPER 03

GEODESIC STRUCTURES: DOUBLE CURVATURE TRIANGULATED MESHES

Gianluca Stasi[1]

KEYWORDS: Geodesic Structures, Spherical Geometry, Triangulated Meshes.

INTRODUCTION

Geodesic lines represent the shortest distance between two points on a curved surface. Often, however, the components of geodesic structures don't correspond with the arcs drawn on the surface (generically spherical) by the geodesic lines between their vertexes. Instead, with different approximations, they follow the chords - straight lines that pass inside the generator solid - forming triangular faces.

For this reason, geodesic structures are considered triangulated structures. Nevertheless these triangles do not belong to a planar system, but to a spherical geometric system.

Working on them with the axioms of Euclidean planar geometry leads to the appearance of geometric incongruities, which result in assembly problems and construction pathologies. It is necessary to use specific tools to work on "double curvature triangulated meshes" geometry and adapt them to the peculiarities of the different construction methods with which geodesic structures can be implemented.

RESEARCH

In the first part of the 20th century, Walther Bauersfeld and Richard Buckminster Fuller introduced to the world the geodesic technology for dome construction. Geodesic structures make possible to cover large spans without intermediate supports and with a high robustness, compared to the weight and characteristics of the constituent materials. However, the history of the application of those technologies is plagued by recurrent pathologies, due to approaches and work methods that, today still, continue to produce them.

As Popko [01] warns: "It is easy to evenly divide the circumference of a circle on a computer to any level of precision. It's not easy to evenly subdivide spheres, computer or not". There are, therefore, a great variety of strategies or methods of subdivision that, starting from different premises, can be used for the distribution of points on a sphere. The values thus obtained were disseminated in a context in which there was widespread belief that only one geodesic geometry that provides fixed values for each frequency of subdivision exists.

By presenting different datasets without explaining the different methods used, the emergence of a discussion, still active today, on the accuracy of these calculations was promoted. To this accuracy were attributed, and are today attributed, the assembly problems and recurrent construction pathologies in these systems. In reality, the different data sets correspond to the application of different subdivision strategies, and the origin of the inconsistencies must be sought in other factors.

Geodesic structures are generally defined as "triangulated meshes". In fact, three chords joining three adjacent vertexes form a triangle. The illustrations used in the existing bibliography contributed to this idea, by representing the geodesic systems from the chords, and not from the arches which define the corresponding spherical triangles. However, in the case of geodesic meshes, the measurement of the three sides is not sufficient to characterize those triangles, since they do not belong to an Euclidean plane. Although three points always define a plane, the triangles defined by the chords still belong to a spherical system and, therefore, for their geometric definition, other data is needed.

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Some of these data refer to the geometrical definition of a "double curvature triangulated meshes", others must be defined in a second stage for the specific configuration of each construction system.
A given geodesic geometry cannot be applied to different construction methods without adapting it to their characteristics and peculiarities: each of them configures its own geometry. For this reason, many of the data and concepts necessary for the definition of specific construction methods are not found in the existing bibliography.
During the last ten years, the author has integrated the development of theoretical research on the subject, with the realization of practical experiences of participative, real scale selfconstruction of the proposed models. Their realization provided different communities with equipment and infrastructure they needed, and, at the same time, provided an opportunity to check the developed models, collect data for further improvements, and endorse the technology transfering protocols used for the inclusion of the local community in the involved processes. These experiences highlighted the need to consider and define a wide range of new concepts and refinements, both for the analysis of construction methods applicable to geodesic structures, and for the different geodesic geometries that generate them.
The constructive method popularly known as *GoodKarma* is one of the most widespread for the self-construction of geodesic structures, due to the existence of several informal manuals, that, starting with self-edited booklets in the 1970s, are today disseminated in different formats through the Internet. They are extremely popular among dome enthusiasts.
This method can be used as an example to explain the problems that arise when adapting generic geodesic geometry to specific methods. As shown in Fig. 1, the *GoodKarma* method organizes wooden modules around each vertex.

Fig. 1 - *GoodKarma* constructive system. Hemispheric structure, module and vertex detail.

In the author's researches, for each triangle of a given geodesic subdivision mesh, the triangle formed by the three chords is named Principal Tetrahedron Triangle [PTT], and the triangles that each one of the chords forms with the center of the spherical system to which they belong, is named Axial Tetrahedron Triangle [ATT].
Fig. 2 shows how to configure each of the modules. Its components are aligned with the three ATT, so that their top faces do not belong to the PTT, but to a plane containing the chord and perpendicular to its corresponding ATT. To execute the cuts of component's ends, compound angles will be used. One of the constituent angles will be the Axial Angle, while the other is not the Internal Angle [IA], but the Principal OrthoDihedral Angle [POA] that is formed

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on this new plane. Fig. 2 also describes how, by definition, each PTT has three IA and six POA. A set of two of these six angles is assigned to each of the three components of each module. Being oriented toward a spherical and, thus, convex surface, the value of the POA is always higher than that of the AI that is formed by the same vertex. Using the IA instead of the POA results in the introduction of structural tensions that are, without any doubt, the main cause of the building and maintenance problems imputed to this construction system.

As the preparation of each triangular module requires six angles, and in each vertex of the system converge five to six modules, the introduced deviations can have significant consequences on the geometry of the whole structure. The rotation of the components upper face, with respect to the PTT, depends of another angle that is not referenced in the bibliography. The Dihedral Angle formed between the PTT being analyzed, and those with which it shares one side, isn't, in fact, a functional data to this calculation. It must be noted that the ATT plane that each chord forms with the center of the spherical system does not divide the corresponding Dihedral Angle in equal parts.

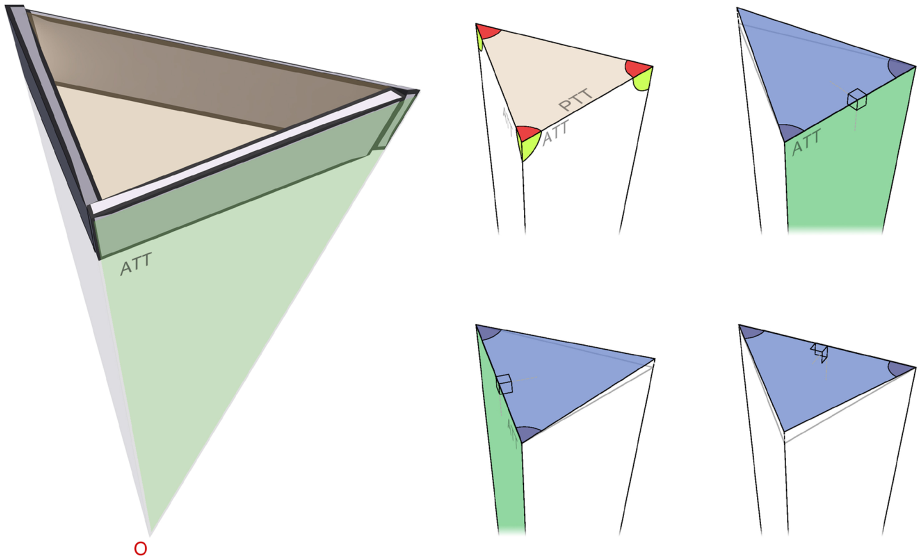

Fig. 2 - Internal Angles (red), Axial Angles (green) and Principal OrthoDihedral Angles (blue).

Fig. 3 illustrates the definition of the Tetrahedral Dihedral Angle [TDA] as the angle between a PTT and each one of its three ATT.

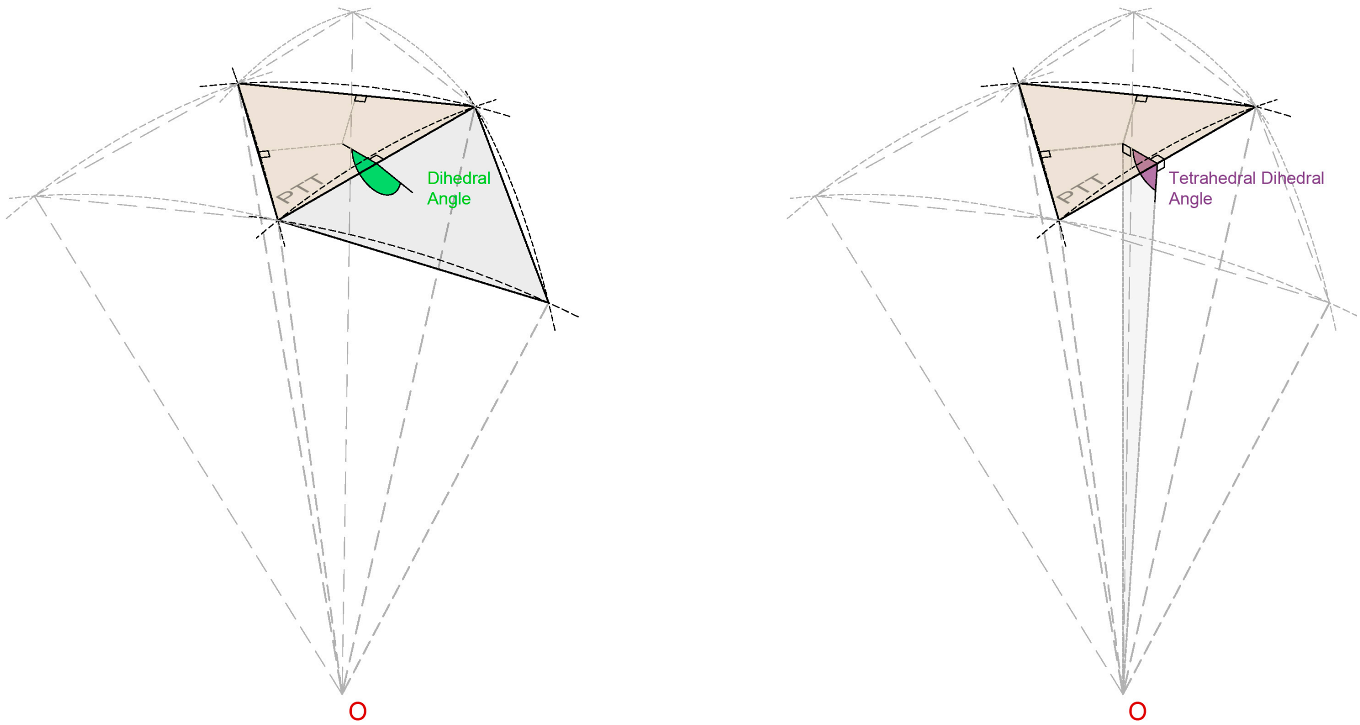

Fig. 3 - Dihedral Angle (green) and Tetrahedral Dihedral Angle (violet).

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A widespread error is to work on a spherical system as if it were a planar system, applying the properties of Euclidean geometry to it. Unlike a triangle designed on a Euclidean plane, the sum of the three POA of a spherical triangle should not only not be, but also cannot be 180º - in fact, it has to be greater than this.
For the calculation of the POA in the three vertexes and, consequently, for the design of the three-dimensional modules of the *GoodKarma* method, one doesn't work in the same plane, but in three different planes, because, in spite of representing a geodesic system as a series of plane facets, one continues to work on a spherical surface in which the axioms of Euclidean geometry are not valid.
As summarized in Table 1, comparing the data obtained by applying the concepts and operators introduced by the author's researches, with the ones proposed in the informal manuals of this construction method, we can see how 80% of the cuts would be made using different angles.

Table 1 - Comparison between Principal OrthoDihedral Angles and Manual's Angles
for the five PTTs of an "Icosahedron, v4, equal chords subdivision method" geodesic mesh.

Chord Factors					
A	B	C	D	E	F
0,25318	0,29524	0,29453	0,31287	0,32492	0,29859

v4 Comparison between Principal OrthoDiehedral Angles and Manual's Angles

A	B	A	PTT 1				
0,25318	0,29524	0,25318				Chord Factors	PTT's Total
72		54		54		Manual's Angles	180
71,8645	71,8645	54,6644	54,5849	54,5849	54,6644	POA	181,1138
CA	AB	AB	BC	BC	CA		
-0,1355	-0,1355	0,6644	0,5849	0,5849	0,6644	Deviation (Degrees)	
0	0	1	1	1	1	Deviation (Cut Angles)	

C	C	B	PTT 3				
0,29453	0,29453	0,29524				Chord Factors	PTT's Total
60		60		60		Manual's Angles	180
60,3810	60,3824	60,6248	60,6248	60,3824	60,3810	POA	181,3882
CA	AB	AB	BC	BC	CA		
0,3810	0,3824	0,6248	0,6248	0,3824	0,3810	Deviation (Degrees)	
0	0	1	1	0	0	Deviation (Cut Angles)	

F	D	C	PTT 5				
0,29859	0,31287	0,29453				Chord Factors	PTT's Total
63		57		60		Manual's Angles	180
64,2206	64,2137	57,9925	57,9637	59,2430	59,2789	POA	181,4562
CA	AB	AB	BC	BC	CA		
1,2206	1,2137	0,9925	0,9637	-0,7570	-0,7211	Deviation (Degrees)	
1	1	1	1	-1	-1	Deviation (Cut Angles)	

D	E	D	PTT 2				
0,31287	0,32492	0,31287				Chord Factors	PTT's Total
64		58		58		Manual's Angles	180
63,1498	63,1498	59,2427	59,2179	59,2179	59,2427	POA	181,6104
CA	AB	AB	BC	BC	CA		
-0,8502	-0,8502	1,2427	1,2179	1,2179	1,2427	Deviation (Degrees)	
-1	-1	1	1	1	1	Deviation (Cut Angles)	

E	E	E	PTT 4				
0,32492	0,32492	0,32492				Chord Factors	PTT's Total
60		60		60		Manual's Angles	180
60,5660	60,5660	60,5660	60,5660	60,5660	60,5660	POA	181,6979
CA	AB	AB	BC	BC	CA		
0,5660	0,5660	0,5660	0,5660	0,5660	0,5660	Deviation (Degrees)	
1	1	1	1	1	1	Deviation (Cut Angles)	

In addition, comparing the manuals data in Table 2 with the IA of the corresponding PTT, the most striking result is that, not only the values do not agree but, in an unexpected way, the values of four of the cutting angles reported in the manuals turn out to be smaller with respect to their respective AI. Being that the PTT is oriented towards the surface of the sphere, and being that surface convex by definition, this turns out to be simply impossible.
Comparing the POA with the manuals, 80% of the cuts would be made with a different angle. Comparing the POA with the IA, this deviation would fall to 53%, demonstrating that modifying the general geometry following the rules of planar geometry can lead to the creation of even more geometrical incongruities.

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Table 2 - Comparison between Internal Angles and Manual's Angles
for the five PTTs of an "Icosahedron, ν4, equal chords subdivision method" geodesic mesh.

Chord Factors					
A	B	C	D	E	F
0,25318	0,29524	0,29453	0,31287	0,32492	0,29859

ν4 Comparison between Internal Angles and Manual's Angles

A	B	A	PTT 1				
0,25318	0,29524	0,25318				Chord Factors	PTT's Total
71,3326		54,3337		54,3337		Internal Angles	180
72		54		54		Manual's Angles	180
CA	AB	AB	BC	BC	CA		
0,6674		-0,3337		-0,3337		Deviation (Degrees)	
1		0		0		Deviation (Cut Angles)	

C	C	B	PTT 3				
0,29453	0,29453	0,29524				Chord Factors	PTT's Total
59,9202		60,1595		59,9202		Internal Angles	180
60		60		60		Manual's Angles	180
CA	AB	AB	BC	BC	CA		
0,0798		-0,1595		0,0798		Deviation (Degrees)	
0		0		0		Deviation (Cut Angles)	

F	D	C	PTT 5				
0,29859	0,31287	0,29453				Chord Factors	PTT's Total
63,6689		57,5339		58,7972		Internal Angles	180
63		57		60		Manual's Angles	180
CA	AB	AB	BC	BC	CA		
-0,6689		-0,5339		1,2028		Deviation (Degrees)	
-1		-1		1		Deviation (Cut Angles)	

D	E	D	PTT 2				
0,31287	0,32492	0,31287				Chord Factors	PTT's Total
62,5649		58,7176		58,7176		Internal Angles	180
64		58		58		Manual's Angles	180
CA	AB	AB	BC	BC	CA		
1,4351		-0,7176		-0,7176		Deviation (Degrees)	
1		-1		-1		Deviation (Cut Angles)	

E	E	E	PTT 4				
0,32492	0,32492	0,32492				Chord Factors	PTT's Total
60,0000		60,0000		60,0000		Internal Angles	180
60		60		60		Manual's Angles	180
CA	AB	AB	BC	BC	CA		
0,0000		0,0000		0,0000		Deviation (Degrees)	
0		0		0		Deviation (Cut Angles)	

CONCLUSIONS

Just as there is no single geodesic geometry, there is no single construction method for its implementation. To understand these multiplicities, it is necessary to consider the geometric peculiarities of the "double curvature triangulated meshes" and configure specific tools to work on their definition.

At the same time, in order to define the variety of construction methods that exist, it is necessary to approach geodesic geometry with a new focus, and analyze the relationships that are established between a given geometric configuration and the specific geometric characteristics of each construction method applicable for real scale structures.

The existing methods of representation and definitions can easily lead to the error of working on geodesic structures as if they were a set of planar triangles. The new approach proposed in the author's research, with the definition of new operators and concepts such as POA and TDA, not only allows for a new reading of the existing methods but, as demonstrated by numerous experiences carried out in the field, this type of approximation allows the use of new construction methods, such as "*Brujodésico*" or "*Zdésico*", easily transferable to communities without previous knowledge of geodesic structures and implementable in low-tech environments.

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PAPERS' SESSION
PAPER 04

GEOMETRY AND EFFICIENCY IN THE JOINTS OF DEPLOYABLE STRUCTURES

Adolfo Pérez Egea[1], Pedro Miguel Jiménez Vicario[2], Pedro García Martínez[3], Manuel Alejandro Ródenas López[4] and Martino Peña Fernández-Serrano[5]

KEYWORDS: Deployable Structures, 3D Printing, Digital Manufacturing.

1. INTRODUCTION

The field of deployable structures underwent significant progress in the latter half of the 20th Century, thanks in part to aeronautical advances resulting from the two world wars (García et al., 2018).

Spain's Emilio Pérez Piñero is considered to be one of the pioneers in deployable structures. His work interested architects globally, such as Fuller and Candela, and in 1961, he patented his prototype in Spain (Piñero, 1961), as well as in America (Piñero, 1965). Fellow Spaniard Félix Escrig (1984) is regarded as the person who took on Piñero's research findings and developed them even further.

Such structural systems are essentially formed of rigid rods and articulated joints. Due to the great number of joints inherent in these structures, designing these elements has been a fundamental issue, so it is quite understandable that the invention process behind it is patented. From the last quarter of the 20th Century onwards, the application of deployable structures has increased significantly. Their ability for changing volume has allowed them[6] to be used in situations where load-bearing is not an important factor, such as in the construction of spatial modules (in the absence of gravity), furniture elements, or in the production of toys made popular by Charles Hoberman (1995).

Consequently, the materials used to make these joints have also changed. Piñero used, initially, aluminium to build the joints and rods. At that time in Spain, the only company able to build his designs was the aeronautical manufacturer *CASA*, whose technology was based on this material (Pérez and Pérez, 2016). With Hoberman's toys, a thermostable polymer was used. Nowadays, parametric design and digital manufacturing let us experiment with new methodologies and production processes that can be applied to these kinds of structures, where geometry performs an important role (Fig. 1). New collaborative methods and 'open source' processes amplify these phenomena.

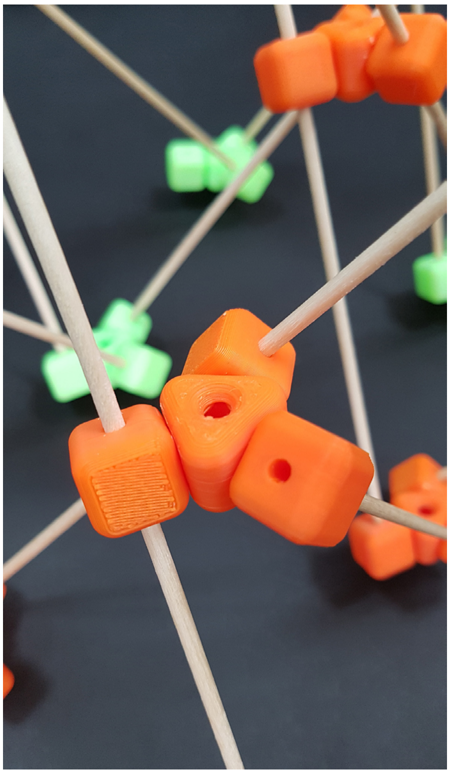
Fig. 1 - Joint created with parametric design software and 3D printing. Authors' image.

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[6] The contributions made by Charles Hoberman to the field of deployable structures are numerous and varied. His main patents, 'Reversibly Expandable Three-Dimensional Structure', 'Radial Expansion/Retraction Truss Structure', and 'Reversibly Expandable Doubly Curved Truss Structure' have had practical application in the development of toys, as well as in structural designs for architecture and engineering. See [01]).

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2. AIM OF THE PAPER

In this document, we focus on the geometrical factors prevalent in the design and manufacturing processes of basic deployable structures using digital tools, as well as some of the main variables to consider in such processes. As a result, we offer a concise classification of the structures' joints, specifically designed so they may be produced by 3D printing, particularly by fused deposition modelling (FDM). In addition, data is provided relating to the time and amount of material (ABS plastic) used in each case. Finally, we also describe criteria that can be considered when developing designs at a later stage, in accordance to the aforementioned.

3. DEFINITIONS

Studies, such as those by Hanaor and Levi (2001), lead us to remark that the descriptor 'deployable' encompasses very different kinds of deployable structures, including textile and inflatable ones. However, in this study, we will focus exclusively on those known as DLG (double layer grid), which combine rigid rods and articulated joints.

3.1 TYPES OF DEPLOYABLE STRUCTURES SYSTEMS STUDIED.

3.1.1 Tb systems.

The geometry of their basic module is similar to a tube bundle (Tb). Three or more rods gather in a central joint that connects them to each other. Piñero's structures (1965) are based on this scheme.

3.1.2 Ss systems.

The basic module of these systems is obtained when combining pairs of rods gathered in a central joint, forming a cross or a pair of scissors – hence the name Ss ('scissors systems'). Félix Escrig described them in his 1984 patent, generalising Piñero's earlier conclusions.

Depending on the system type and the number of rods used, deployable structures whose basic module adopts different geometrical shapes could be obtained. In this study, we will consider up to a maximum of six rods in Tb systems and eight rods for Ss systems, (Fig. 2).

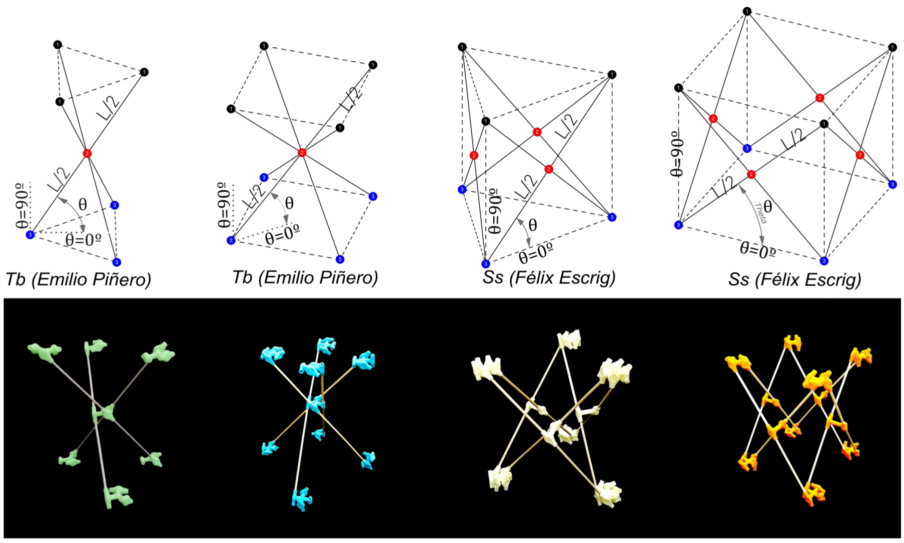
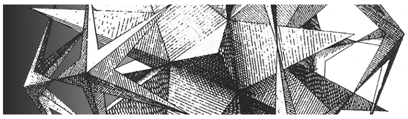


Fig. 2 - Classification of base modules according to the union of rods and joints, Tb (Emilio Piñero) and Ss (Félix Escrig). Authors' image.

3.2 TYPES OF JOINTS STUDIED AND NAMING CONVENTION.

Hereunder, we list the aspects we have studied and how these have been condensed into a code that allows us to classify and describe the joints in question:

- Number of rods gathering in the joint (1,2,3... etc).
- Type of system (Tb or Ss).
- Position of the joint in relation to the module (1: end, 2: centre, 3: end with connection, and 4: end or centre)
- Continuity of the rod in the joint:
  <: Discontinuous rod, an element of the rod appears either side of the joint.
  /: Continuous rod, joints do not interrupt the rod.
- Number of pieces integrated into the joint (1: joint made by one piece [continuous extrusion]; 2: joint made up of two pieces, etc.).
- Direction of rods' axes related to the geometrical centre of the knot (d: divergent; c: convergent).
- Family: Design of the general geometry of the joint (A - F)



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By way of example, a joint's naming convention could be codified in the following way:
3Tb4<1dD (Fig. 3)

Some joints may be used interchangeably across several systems. In such cases, the code is complemented with new items (separated by a hyphen).
Example: 6Tb4<1dB-Ss3<1dB

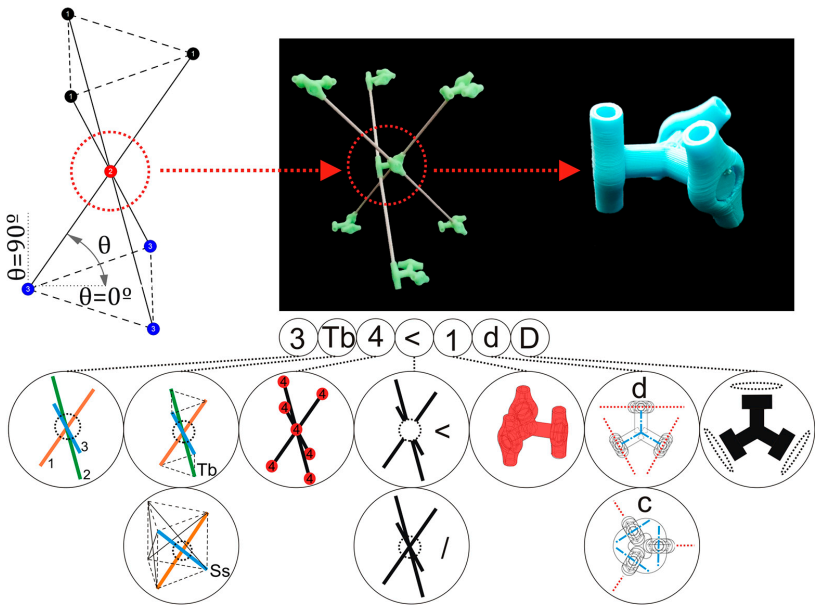

Fig. 3 - Codification for a joint based on the considered parameters. Authors' image.

4. METHODOLOGY

This study has been conducted using the joints described in the aforementioned Spanish architects' patents as a starting point, adapting them for manufacture via FDM printing. Three-dimensional modelling was carried out using *Rhinoceros* v5[7], supported by the *Grasshopper*[8] plug-in for parametric design. Next, the 3D models' meshes were verified, prior to the printing criteria being set up via *Z-Suite*[9], software owned by the manufacturer of the *Zortrax M200 3D printer* used for prototyping. This software enabled the generation of Z-code instructions files for the printer. The printing material was ABS plastic.
For the construction and testing of the described prototypes, the different joints were linked using two different systems: cylindrical wooden rods with 3mm diameter; rods generated by FDM 3D printing, with 3mm diameter and 3mm perforations.

5. RESULTS

In Figs. 4a and 4b, joints are represented graphically (to the same scale) as well as by their respective coding. Graphics 1 and 2 feature data pertaining to material usage (weight and length) and print time in each case.

6. RESULTS DISCUSSION

In Graphics 1 and 2, the proportional relationship between material usage and print time is apparent. Henceforth, we will consider there to be a greater efficiency in the design of the joints, the lower the value of both results.
With regard to the number of rods and making the remaining variables uniform, the design of the joint becomes clearly less efficient the greater the number of rods that converge into it.
Regarding the influence of the type of system (Tb or Ss) on the size of the joint, we have not found any evidence that this variable affects the design efficiency. Many of the joints are even compatible with both systems.
Considering the position of the joint in relation to the module, those joints in position 2 (centre), used in Ss systems, are the most efficient. This result is determined by the variable 'number of rods', as only two bars are converging into

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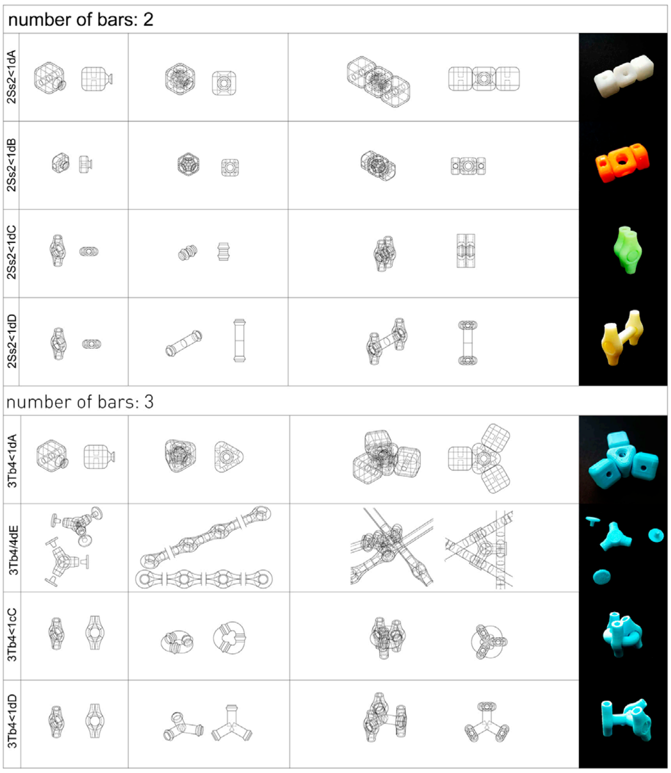

Fig. 4a - Classification of joints according to the number of rods that converge (I). Authors' image.

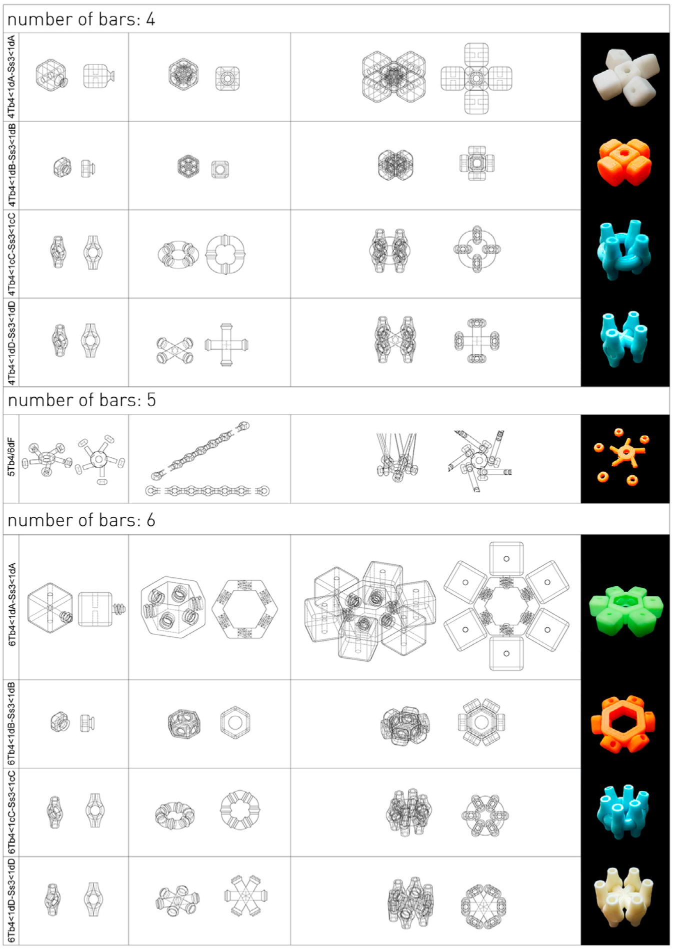

Fig. 4b - Classification of joints according to the number of rods that converge (II). Authors' image.

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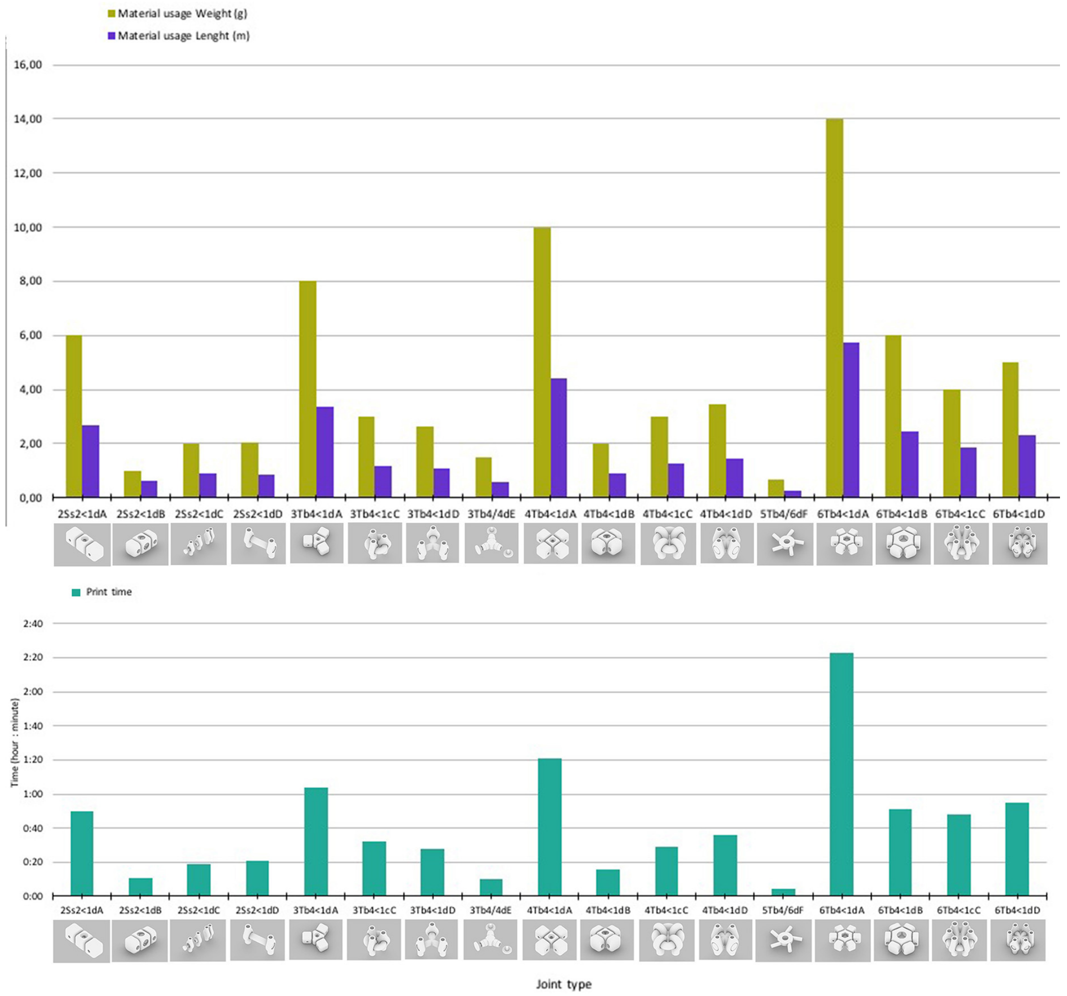

Graphics 1 and 2 - Relationship between material usage and print time used in the manufacture of each knot. Authors' image.

these knots. In the remaining positions, almost all joints end up being interchangeable (as seen from 3 to 6 rods), which renders it impossible to determine any significant effect this variable has on its own efficiency.

In relation to rod continuity, continuous rod designs are generally more efficient, although they require pre-machining on the rods, making a perforation in order to correctly enable the joining process. These designs are printed in several pieces and require a supplementary task of additional assembly.

Regarding the position of the bars relating to the geometrical centre of the joint, no conclusive data has been acquired determining greater efficiency of one compared to another.

When it comes to shape, the less efficient family is A, with B the most efficient, although a high ratio of defective prototypes was encountered. On the other hand, in absolute terms, family F ends up being the most efficient design, as the joint has been reduced to the minimum that is viable, providing only the necessary axes for the articulation of the rods.

7. CONCLUSIONS.

The joint design is a fundamental parameter in determining the efficiency (in terms of time and material) of its manufacture. Once the joint design has been optimised, the number of bars converging at the joint is the next key variable for determining its efficiency. Indeed, more rods converging at a joint result in greater material being used. It does not merely influence the increase in material usage corresponding to the number of rod-joint unions. In addition, the joint must grow to enable free movement, avoiding collisions with any adjoining bars, therefore increasing the singularity of the rod axis relative to the centre of the joint as the number of bars increases.

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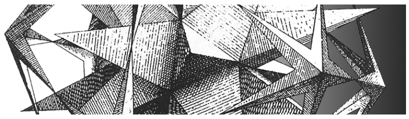

PLENARY SESSION
KEYNOTE SPEAKER

JAVIER BARRALLO

Javier Barrallo was born in Bilbao, where he studied and obtained a Ph.D. Degree in Computer Engineering at the University of Deusto.
Javier is Professor at the UPV-EHU (University of the Basque Country) where he teaches in the Department of Applied Mathematics at the School of Architecture in Donostia-San Sebastián.
Javier specialized in the relationship between Art & Science, dynamical systems including fractals and chaos, parametric programming of complex shapes and the scientific analysis for the structural maintenance and repair of heritage buildings.
Javier Barrallo has written over 100 papers and books, collaborated in 15 interventions on historical heritage buildings, organized 12 International Conferences and events and arranged 30 international exhibitions.

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PLENARY SESSION
KEYNOTE SPEAKER

POLYHEDRA: DIDACTIC EXPERIENCES

Javier Barrallo, Francisco González-Quintial, Antonio Sánchez-Parandiet[1]

We do not know the exact origin of polyhedra, but we know that there were examples of several regular polyhedra, if not of all, in the Neolithic era. This makes us think that the prehistoric society had a mathematical ability far superior to that recognized by archeologists, more in keeping with the artistic skills they developed in mural paintings such as those inside the amazing Altamira Cave. We must also imagine that the Neolithic expertise to carve stone polyhedra that have reached our days, could have been surpassed by wood carvings, probably of higher quality and lost by the passage of time.

The ancient Greek, Etruscan and Roman cultures greatly improved the manufacture of these models, and the mathematicians Pythagoras, Euclid and Archimedes described them accurately in their works. The philosopher Plato also granted them with an esoteric nature, by relating them to Fire, Earth, Air, Water and Ether. It will be several centuries more before scientists such as Della Francesca, Pacioli and Kepler return to show interest in polyhedra and that artists such as Da Vinci, Uccello and Dürer represent them masterfully.

Again, after a break of a few centuries, the emergence of artists such as Gaudi, Escher or Dalí will introduce polyhedra in the art of the 20th century. In the 21st century, George Hart and Rinus Roelofs, among others, will turn them into sophisticated and precious objects straddling Art and Mathematics.

This brief description through time shows us that polyhedra have evolved in parallel to human knowledge, and that their history is full of great names of Art and Science. This fact allow polyhedra to be used for an interdisciplinary education where History, Art, Geometry and Mathematics are intertwined in a fascinating way. The purpose of this article is not to teach and learn polyhedra as an end in itself, but a mean to introduce more advanced Geometry and Mathematics through them. And, furthermore, to leave an open window to explore other areas in a multidisciplinary way, such as Astronomy or Philosophy.

Polyhedra have exerted an enormous fascination not only in scientists, but among the population in general. Historically, they were the first three-dimensional objects to be classified, and even in our days, we are surprised by their properties, aesthetics, variety and applications. Any polyhedron is an educational element with enormous didactic possibilities and a great attraction for students who see them not as mere geometric objects, but as an entity that produces a great fascination. It is only necessary to observe a lesson of integration, equations, matrices, etc. and compare it with a class of polyhedra, to observe the enormous magic that they display to students, compared with the most academic subjects of Mathematics and Geometry.

Being that way, we ask ourselves why do polyhedra hardly have any theoretical importance in the academic curriculum of our students? It might seem that they are not very important in science courses and that they lack utility and applications beyond their beauty, but this is not the case. Polyhedra allow to introduce a large number of geometric concepts and develop many skills, both intellectual, to recognize and analyse them, and manual, for their physical construction.

The arrival of computers meant a drastic change in the way of designing and creating polyhedra. Most CAD programs include polyhedra in their graphic libraries and facilitate their design with multiple tools that avoid making cumbersome calculations of edges and angles. Computers also allow two objects to occupy the same physical space

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Fig. 1 - FabLab Donostia computer made polyhedra versus handcrafted polyhedra at the E.T.S. of Architecture in Donostia-San Sebastián.

(virtual in this case) so that intersections of elements can be made in a very intuitive way to generate new ones of greater complexity, which obviously cannot be executed in the real world.

To this computational facility to design new polyhedra, we must add the arrival of the new 3D printers, capable of producing complex models in a wide variety of materials and with a great level of detail. All these advances have caused a blooming of all types of polyhedra with magnificent finishes and have relegated traditional constructions of handmade polyhedra to a second plane.

In spite of having the modern and productive *FabLab Donostia* in our university, we consider that the technological advances in the design and production of polyhedra models should, in no case, be carried out at the expense of eliminating the hand work with polyhedra from our teaching programs. Starting from simple materials such as cardboard, scissors and glue, students get a much more enriching experience when building a polyhedron from scratch, than with a 3D printer. While both techniques provide educational values, they should not be exclusive. In addition, numerous interdisciplinary didactic projects can be carried out for students of Mathematics, Fine Arts, Engineering, Architecture ... and, in general, for any type of scientific discipline.

In this lecture, we present several didactic experiences carried out in the E.T.S. of Architecture in Donostia-San Sebastián at the University of the Basque Country (UPV/EHU). In these projects, we will travel through the polyhedral world, stopping in various areas of knowledge such as Fractals, Tensegrity, Fourth Dimension, Symmetry,

Fig. 2 - The Platonic Solids made with laminar reciprocal structures (Cube, Tetrahedron, Octahedron, Dodecahedron and Icosahedron).

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Topology, Geodesics... In our projects, we will use preferably simple and low-cost materials, but also easily accessible commercial products, such as the ones commercialized by the North American company *Zometool* and the South Korean company *4DFrame*, both with a significant presence on the Internet.
All these projects have been carried out satisfactorily with university students and, in many cases, before a public of all ages in Science Festivals and similarly popular events. The feedback of the students has always been very satisfactory, both for the work done and for the possibility of taking home one or more handcrafted polyhedra built by them.

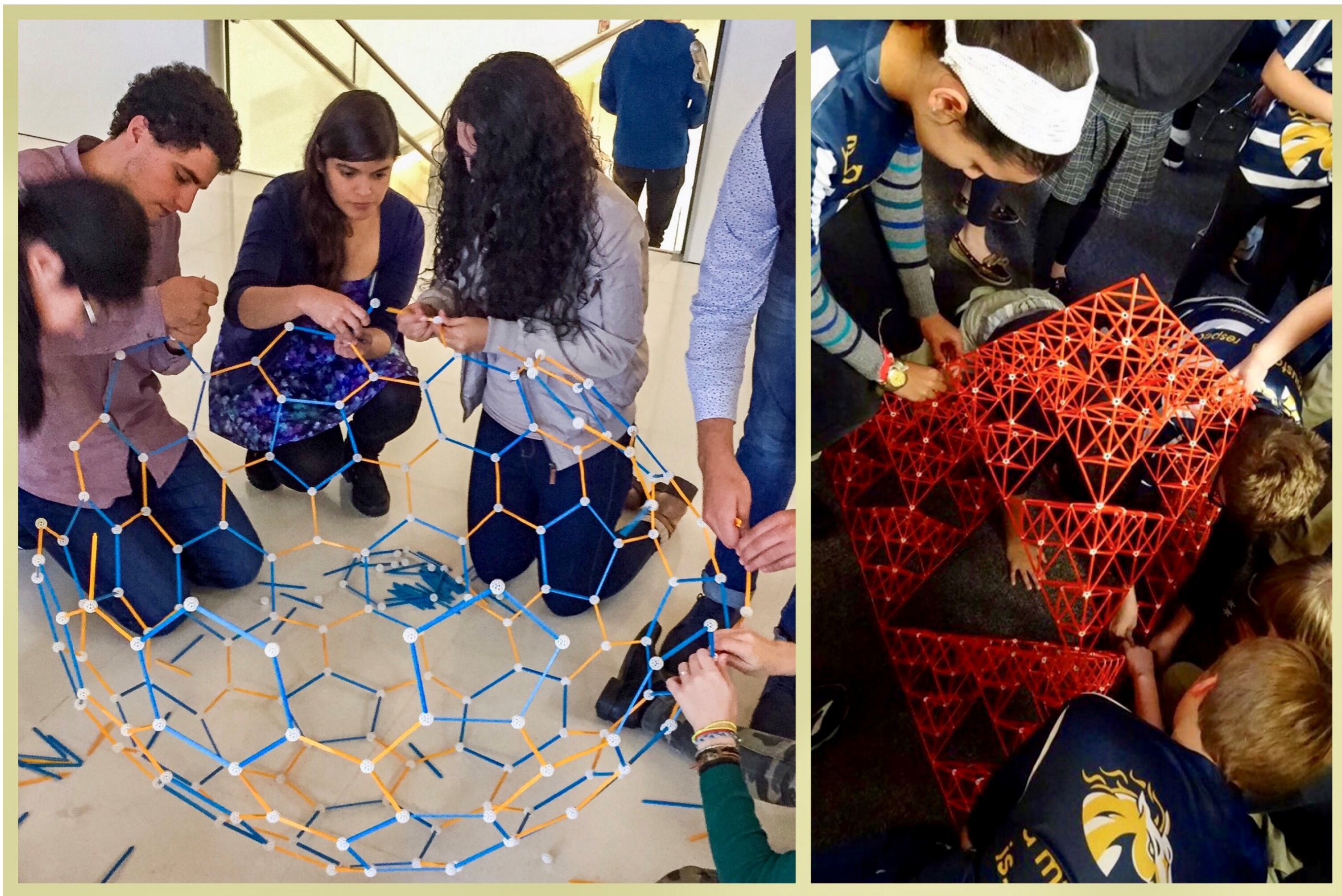

Fig. 3 - Collaborative works constructing a Fullerene C240 with *Zometool* at the University of the Basque Country and a Sierpinski Fractal Tetrahedron being assembled by young students at Breck Institute (Minnesota).

Before beginning to briefly describe these projects, which will be presented in greater detail during the conference, it is necessary to remember that it would be convenient for students to have an initial theoretical basis. In this way, although all our projects are eminently practical and designed to be developed manually, it would be necessary to begin by describing several basic concepts such as vertex, edge and face. Students should be able to count these elements in several polyhedra, and try to experimentally obtain Euler's Theorem for polyhedra (Faces + Vertices = Edges + 2).
This first project, eminently theoretical, can be complemented with the construction of the five Platonic Solids using a construction system as those described previously (*Zometool*, *4DFrame*). It will also be interesting to explain the concept of duality and concavity/convexity. This practice would be complemented with some concepts about symmetry, both in the plane and in space, and differentiating regular from semiregular and irregular polyhedra. Our second didactic project is the Stellation and the Truncation of polyhedra. The dodecahedron may be stellated easily, producing beautiful figures such as the one represented by Paolo Uccello at the Basilica of San Marcos in Venice. The icosahedron is a perfect polyhedron for truncation, resulting in the well-known geometry of the soccer ball.

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The third project has to do with folding and unfolding a polyhedron. As a theoretical starting point, we can start from the dominos, triminos, tetraminos (the pieces of the *Tetris Game*!), pentaminos and hexaminos. It is a good problem to find which of the hexaminos may be folded to compose a cube and which cannot. Students should then be able to create consistent unfolds of triangles for an octahedron, and pentagons for a dodecahedron.

The fourth project consists of generating virtual polyhedra from a flat figure, using several mirrors that act as a kaleidoscope. In a very simple way, we can generate a multitude of polyhedral images, both finite and infinite. As a parallel activity, it is also possible to create polyhedra with their outer faces made with any mirrored material and produce interesting reflections and lighting effects.

Our fifth project is the one for Polytopes, that is, the generalization to any dimension of a three-dimensional polyhedron. We will study how to generate a hypercube (fourth dimension) with soap bubbles inside a cube and the different ways of representing hypercubes in the fourth and fifth dimensions (and their projections in lower dimensions) through *Zometool* as constructing system.

The sixth project will introduce us to Topology, learning concepts such as the Hamiltonian Path and studying what type of polyhedra allow a sequential travel along its edges without repetitions. We will learn if it is possible to generate the edges of an icosahedron with a continuous thread starting from three golden rectangles.

Our seventh project will take a tour through reciprocal surfaces, starting with bars and experiencing specially with sheets, to generate an interesting collection of regular and semi-regular polyhedra in different sizes and materials.

The eighth project will present tensegrity. We will create different models with wooden bars and elastic bands, based on the experiments developed by Buckminster Fuller, from the icosahedron using six bars to more complex polyhedra.

The ninth project will introduce us to fractals. Triangles (Sierpinski) and squares (Menger) can be broken down into tiny copies of themselves. Extending this construction to the three-dimensional space, we can create incredible fractal structures, highlighting those of the tetrahedron and octahedron using *4DFrame*, whose large size from the fourth generation onwards makes it suitable for collaborative projects between several people.

In the tenth place, we will show how to build a Geodesic Dome and construct Fullerenes, studying their relationship with polyhedra. We can also build nanotube structures and, as it happened in the previous project, it is recommended to work in groups, given its complexity and the high number of pieces, as shown in Fig. 3.

The teaching project number eleven includes art and history as a curious challenge, in which students have to choose (or raffle) a polyhedron and a historical or artistic period. With the results, they must design their polyhedron decorated according to the period or associated artistic flow, as shown in Fig. 4.

Fig. 4 - Regular Polyhedra associated with artistic periods: Nazari (Alhambra) Tetrahedron, Vitruvian Man Cube, Manuelino Style Dodecahedron, Gothic Icosahedron, Pointillism Dodecahedron, Arts & Crafts Cube, Bauhaus (Mondrian) Cube, Abstract Expressionism Octahedron, International Style Icosahedron.

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For project twelve, we show how to make a bamboo construction using, as a frame for the assembly, a perforated polyhedron. There are multiple variants of this project, one of our students' favourites.

Project thirteen is based on *origami*, which allows us to create nested or intersected polyhedra by assembling their faces using cardboard or plastic trimmed flat pieces. There are numerous constructions of this type, being *origami* probably the most well-known source for the design and handcraft of polyhedra.

Project fourteen consists on building a lamp placing a light source inside a polyhedron. We remark that it is not enough to illuminate it; students should consider the material, the temperature, the perforations, the transparency, the shadows it produces, etc.

And our last project, fifteenth, has been intimately linked to art. The proposal consists on eliminating the maximum of elements of a polyhedron in an artistic way, but making it remain recognizable, as Sol LeWitt made, by leaving only some edges of a cube. Another example could be the work of the Basque sculptor Jorge Oteiza, who removed matter from the faces of a cube until making it extremely light, almost immaterial, but recognizable as a cube.

As a conclusion, it can be seen how the overall of these fifteen didactic experiences are not limited to Geometry or Mathematics, but that they cross over many areas of Art and Science in an interdisciplinary way, promoting learning and creativity. On the other hand, the response of the students is always enthusiastic, well beyond the usual scientific disciplines.

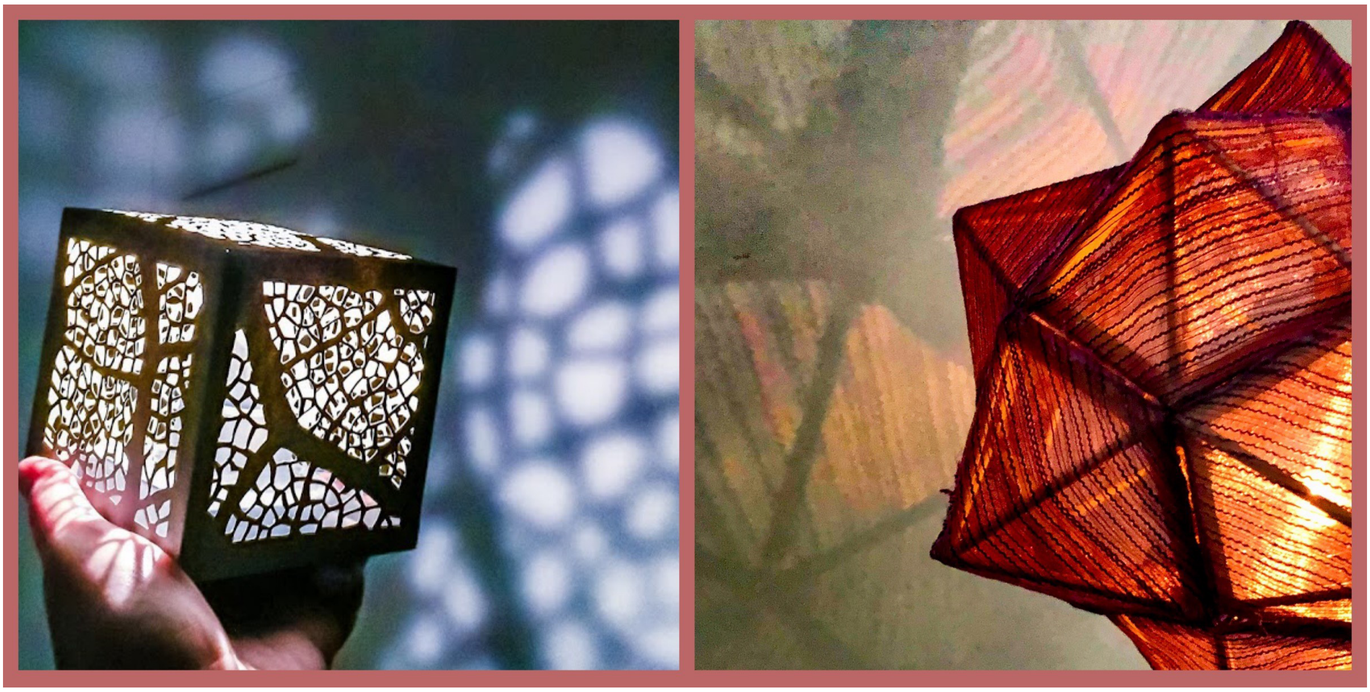

Fig. 5 - Polyhedral Lamps (Organic Cube and Stellated Dodecahedron).

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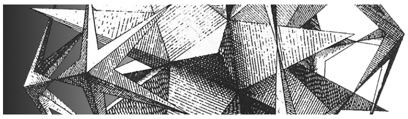

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POSTERS' SESSION
POSTER 01

GAUDI INVERTED: A LAYERED DESIGN PROTOCOL FOR INTEGRATING MATHEMATICAL INVERSION AND RULED SURFACES

Emmanouil Vermisso[1]

KEYWORDS: Design Computation, Ruled Surfaces, Inversion, Topology, Gaudí.

INTRODUCTION

This abstract discusses a generative design process for architecture, using mathematical rules and intersecting topology protocols, inspired by the use of geometry in the later stage (1915-1926) of the *Sagrada Familia*'s design by Antoni Gaudí. The rules for generating the cathedral windows are examined to propose new variations. An additional step is proposed, considering the inherent use of mathematical thought in the work of sculptor John Pickering. Both Gaudí's and Pickering's work regard mathematics as a generative platform for design, defining layered steps for the realization of *a-priori* unknown 2/3-dimensional outcomes. These studies originate in the work of Gaudi and subsequently, Mark Burry's contribution to formulating design models towards the completion of the S*agrada Familia* through parametric protocols, focusing on the use of ruled surfaces for architecture [01] (Fig. 1).

In our assessment of this particular building, we would like to propose the consideration of John Pickering's use of mathematical inversion to transform known geometrical surfaces, generating new topological conditions by numerical deformation (Fig. 2). This mathematical rationale can be integrated with Gaudí's approach, introducing a second stage of deformation in ruled surfaces, particularly looking at the Hyperboloid of Revolution, a geometry that is ubiquitous in the *Sagrada Familia* design.

RESEARCH

Gaudí had always regarded nature as a model for design, because he appreciated the ability in natural systems to successfully resolve the continuity between elements, avoiding the crude tectonics realized by human construction (i.e. the connection between the tree trunk and branching elements). Most of his work is abundant with fluid geometries

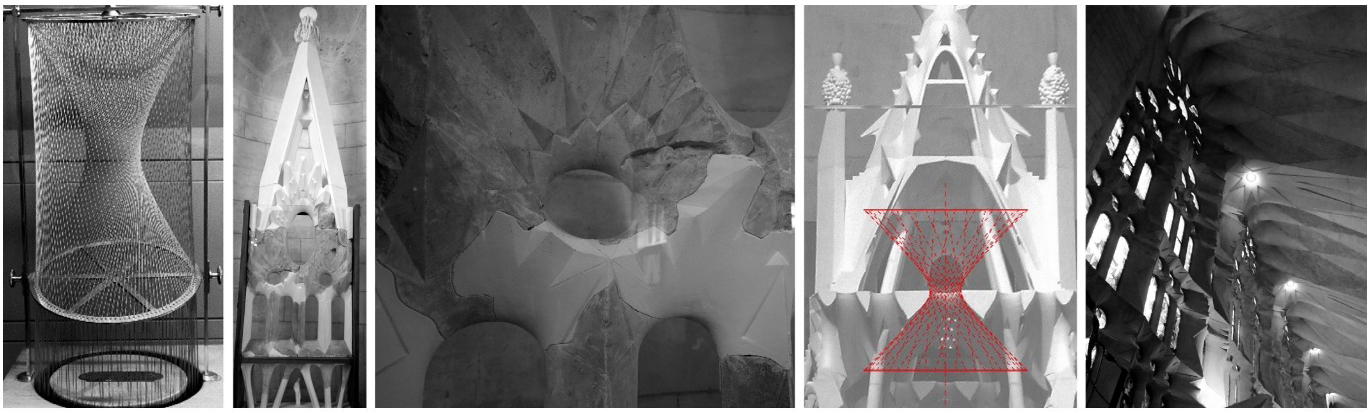

Fig. 1 - Hyperboloid of Revolution thread model, used for teaching descriptive geometry (19th c., Musée des Arts et Métiers, Paris);
Surviving 1:10 plaster model (reconstructed) for transept clerestory window of the *Sagrada Familia*;
Sectional Model from the "Gaudi Unseen" traveling exhibit with overlaid ruled surface to show opening in transept vault;
Interior of the lateral nave windows, showing light quality on the walls [01]. (image credits by author).

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and curvilinear shapes (i.e. *Casa Milà*, etc). During the later years of his life, nevertheless, he shifted towards a more austere visual expression for the design of the "inherited" commission for the *Sagrada Familia* cathedral, by adopting rigorously geometrical design rules. At first glance, this seems to move away from nature, yielding topologies which are more clearly defined. In fact, Gaudí embraced the natural example further, by understanding the mathematical rules inherent in natural systems. His process exploited in particular ruled surfaces like the Hyperbolic Paraboloid and the Hyperboloid of Revolution of one sheet (Fig. 1). Both of these geometries are defined by straight lines ("rulings"), giving an advantage in making the formwork for full-scale construction. Gaudí followed a profoundly "layered" process for the design of the wall, ceiling and roof openings in the *Sagrada Familia*. These are derived through a combination of Boolean operations. Specifically, the transept vaults and clerestory windows in the *Sagrada Familia* involved the subtraction of ruled surface volumes from a notional material solid [01], resulting in openings which are defined by doubly-curved hyperboloid surfaces (Figs. 1 and 2).

THE USE OF RULED SURFACES: THE HYPERBOLOID OF REVOLUTION

Ruled surfaces have played an important role in architecture of the early to mid-twentieth century, but it was Gaudí who introduced their use so extensively. The Hyperboloid of Revolution

> "*…can also be generated by rotating a straight line about a skew axis…The surfaces obtained by rotating conics are special types of a broader class of surfaces, which are called, for analytical reasons, "surfaces of second order"; these are the surfaces satisfying equations of the second degree in three-dimensional Cartesian coordinates*" [02]

In Fig. 2, we demonstrate the process by which several of the *Sagrada Familia* openings were generated, using Boolean operations of Hyperboloids of Revolution and elliptical hyperboloids (elsewhere in the building, Gaudí also employed hyperbolic paraboloids). We are interested in examining the resulting geometries, in reference to the rules for combining the ruled surfaces and their resultant light behaviour.

The use of the particular geometry is interesting, because this ruled surface preserves its overall topology after scaling. According to mathematicians David Hilbert and Stephan Cohn-Vossen,

> "*The two families of straight lines that lie on the hyperboloid of one sheet and the hyperbolic paraboloid have a surprising property. Let us imagine all these straight lines to be made from a rigid material and fastened together at all the intersections in a way that permits of rotation but not of sliding. It would seem reasonable to think that straight lines fastened in this way must form a rigid framework. But as a matter of fact, the framework is movable…*" [02]

Therefore, a physical model with fixed intersection of the rulings would be able to extend and shorten, changing the shape of the hyperbola it defines, while keeping the length of the rulings. We will keep this in mind during the parameter definition of the project; starting with one hyperboloid, several window variations may be created,

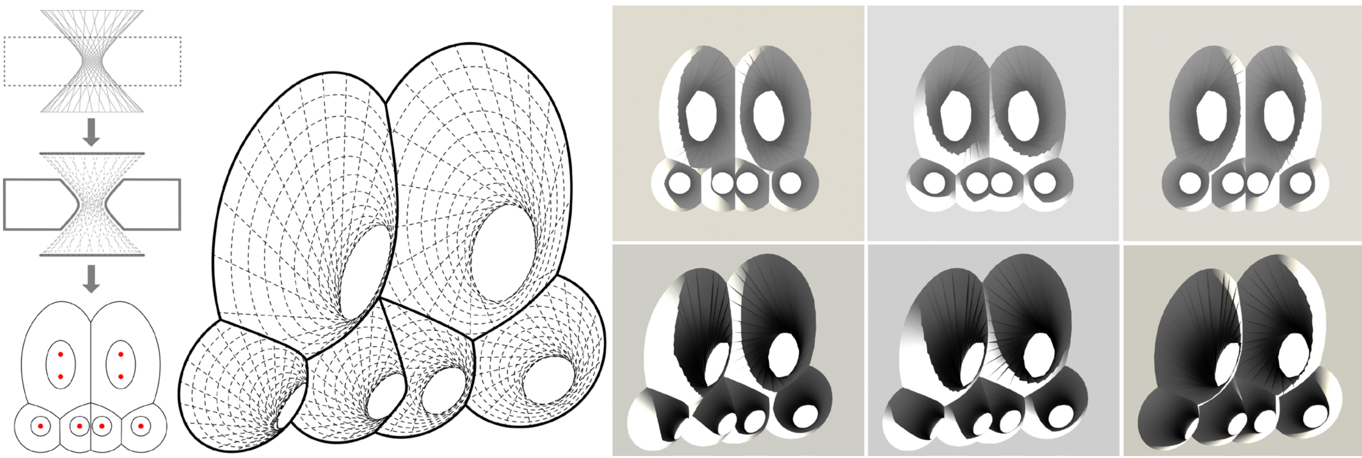
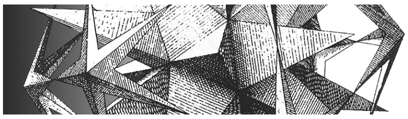


Fig. 2 - Opening generation using Boolean subtractions among 4 intersecting hyperboloids of revolution and 2 elliptical hyperboloids, after Gaudi's method; surface shadows at 3 times throughout the day (The topology is defined by rulings).



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modifying its length before the Boolean subtraction. The orientation of the subtracted hyperboloid in relation to the horizontal plane (*XY*), and the positioning of the rulings in relation to the axis of revolution is important, because it affects the way the light bounces off the wall or ceiling.

MATHEMATICAL INVERSION: JOHN PICKERING

Interestingly, artist John Pickering - in resemblance to Gaudí - experienced a shift in his work during the 1970s [03], when he "*became anti-nature…*", moving to strict numerical rules for his sculpture work[2]. His subsequent sculptures used a mathematical formula called "Inversion" ($R^2 = OA \times OI$), to introduce extensive controlled deformations in combinations of intersecting solids (Fig. 3).

The inversion process introduces curvature in all referenced geometries. Euclidean shapes are represented as two or three-dimensional curves (Fig. 3). Within the context of inversive geometry, a straight line corresponds to a curve. This type of transformation becomes more clear in relation to non-Euclidean geometries like hyperbolic geometry, where the equivalent of straight lines are represented as arcs of a circle within the Poincaré disc model[3].

Pickering's work entailed manual calculations of point coordinates in space to generate guidelines for the sculptures. In order to utilize this rule, an inversion script was developed in a parametric modelling environment and applied to transform a Hyperboloid of Revolution (Fig. 4). Examining various angles allows the investigation of both topological curvature variation, as well as shadow patterns on the resulting topologies. The script extracts the edges from a three-dimensional surface topology, divides these into sets of points (higher values increase output accuracy), and finds their inverse points in space, by calculating the distance of the original points from the centre of the "inversion sphere" with the inversion formula: $R^2 = OA \times OI$ (see Fig. 4). In the inversion of ruled surface topologies, moving the centre of the inversion sphere closer to the Hyperboloid of Revolution, causes greater deformation of the rulings.

The inversion algorithm was applied through a parametric 3D model to generate the inverted option of the window shown in Fig. 2. The original topology and its inverted result are shown in plan in reference to the sphere of inversion (Fig. 5). The results echo the original topology but are asymmetrically distorted, due to the location of the inversion circle. The new curvature creates interesting shadows thanks to the deformation. One thing to consider however, is the fact that the rulings are no longer straight lines after the process of inversion.

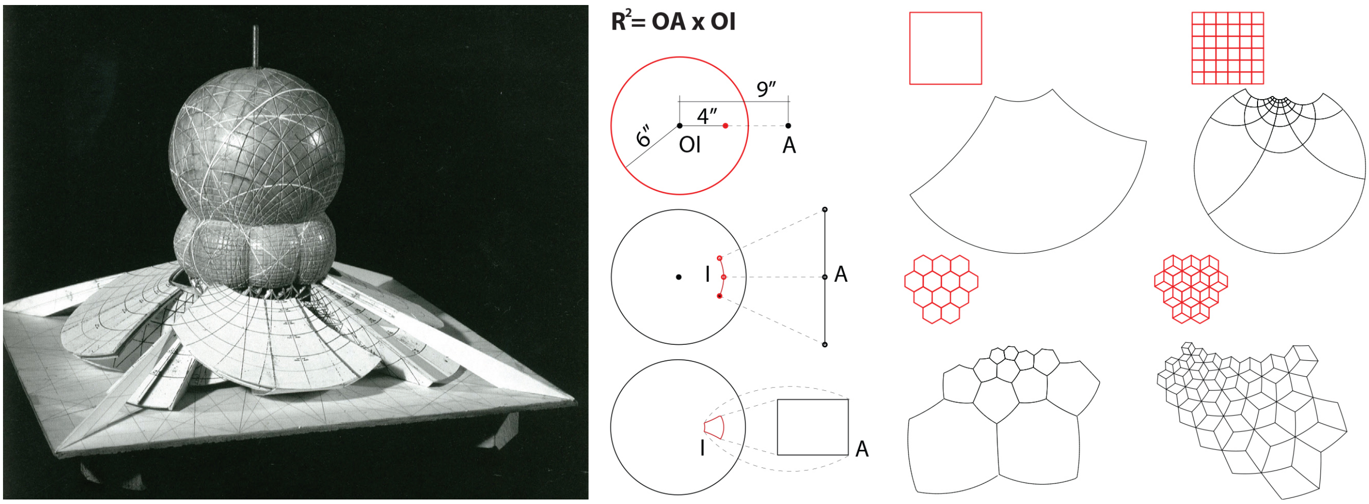

Fig. 3 - Sculpture, showing deformation of a 3-dimensional square grid, using the Inversion formula: $R^2 = OA \times OI$, where *O* is the centre of inversion, and *I* the Inverse point of *A*. The centre of inversion lies inside the grid (John Pickering, 1981-1983-1984); Diagrams: the inversion of a straight line results in a curve, while inversion of a rectangle yields a curved shape; Various grid shapes are shown in the original and inverted conditions.

[2] As in Gaudí's case, he actually got closer to nature; as Mohsen Mostafavi has observed, "Pickering's fascination with geometry, inversion and fractals seems part of a ploy to get closer to nature". [03]

[3] For further clarifications on matters of Hyperbolic and Inversive geometry, the author redirects the reader to available online resources like *Wolfram's MathWorld*.

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As straight lines are advantageous for fabrication purposes, the inverted surfaces could be reconstructed by a new set of curves which are geodesics (shortest path between two points on a series of subdivided inverted surfaces, Fig. 5).

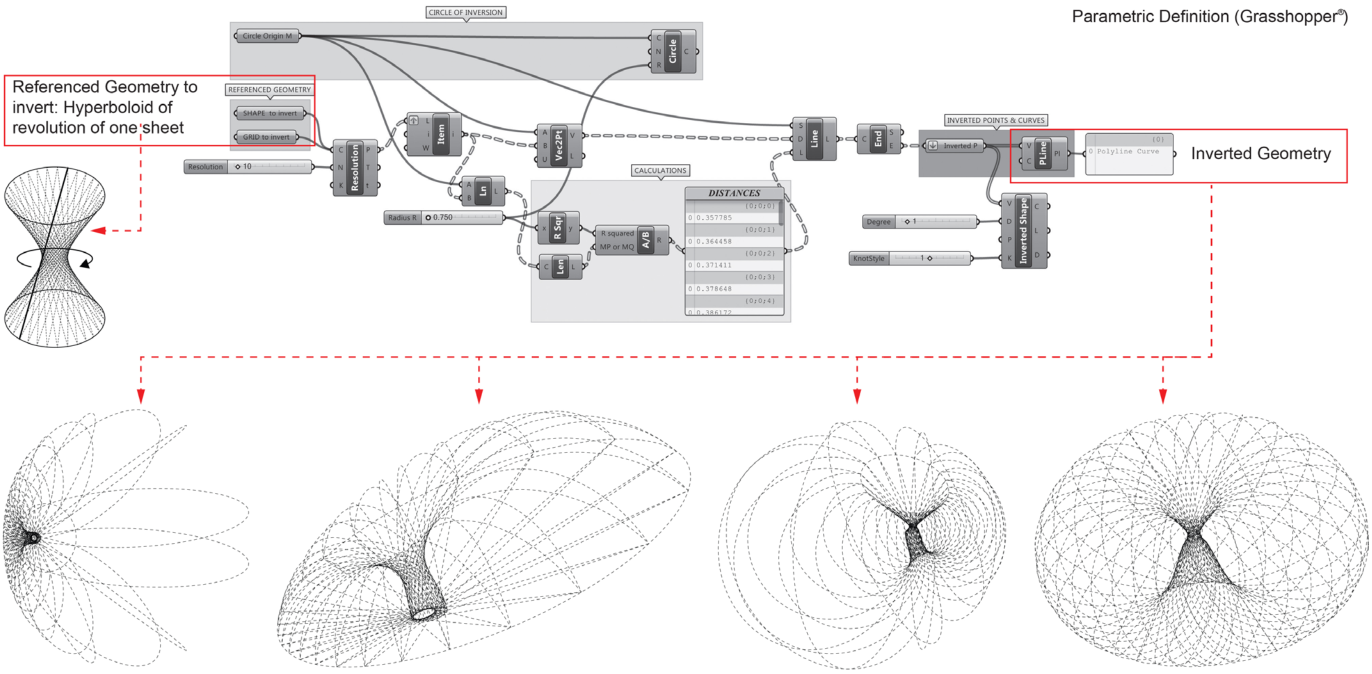

Fig. 4- Inversion algorithm in parametric modelling software (*Grasshopper*®). Four stages of deformation are shown.

CONCLUSIONS

We discussed the mathematical logic present in Gaudí's work *vis-à-vis* an equally rigorous analogue numerical methodology in Pickering's work. Echoes of underlying codification seem to exist in both: the significance of the invisible aspect which drives the final design.

In the *Sagrada Famili*a, ruled surfaces manifest in the building through their absence, what Mark Burry has called an architecture of "real absence and virtual presence" [04]. To appreciate the building complexity, one has to imagine the original topologies from which the built components emerge. In Pickering's work, the translation of certain "regular" topologies into non-Euclidean geometries using "mathematical inversion" is noteworthy, because it operates into the hyperbolic plane. His sculptural volumes warrant a similar consideration, asking the viewer to decipher the complex intersecting topologies by reference to their earlier, straightforward Euclidean counterparts. As a result, this notion of ambiguity in visual expression is present in the work of both designers, as one needs to decode their geometrical intent by tracing back the design transformation steps. If one were to speculate on a possible evolution of Gaudí's methodology by further implementing mathematical rules, it seems appropriate to integrate other layers of geometrical complexity like Inversion. This additional layer will promote even richer results, and consider, among other factors, wall surface illumination from a different point of view.

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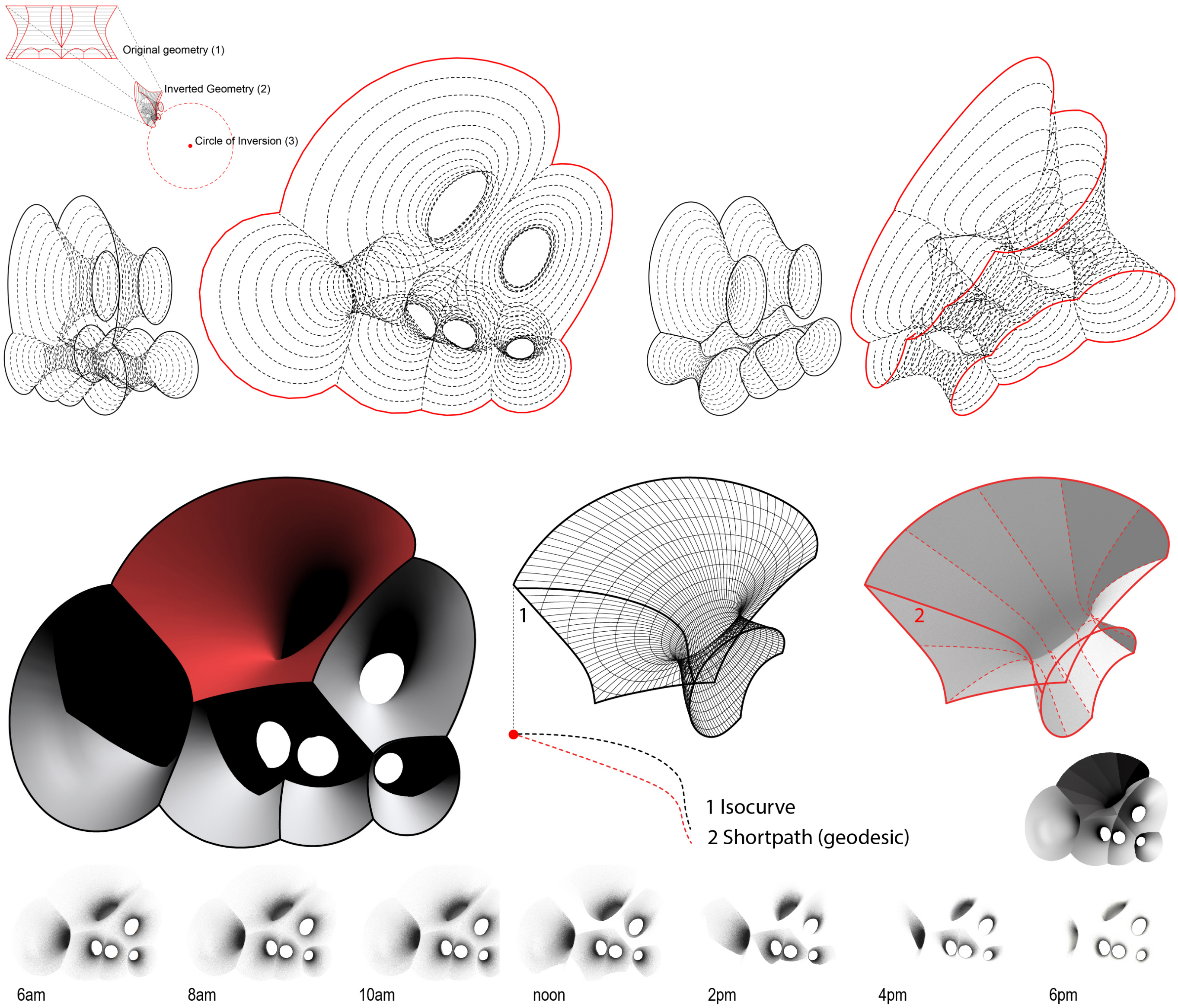

Fig. 5 - Comparison of the original and inverted topologies (front-back, inverted shape outlines are indicated in red); diagrams showing possible definition of conventional isocurve (1) surfaces through geodesics (2) in order to get developable curves which can be fabricated from straight lengths of material; Surface shadows on the inverted window topology at various times throughout the day (2 hour intervals from 6am to 6pm).

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POSTERS' SESSION
POSTER 02

STRAW MODELS REPRESENTING THE SKELETAL STRUCTURES OF *RADIOLARIAN PANTANELLIUM*

Takashi Yoshino[1] and Atsushi Matsuoka[2]

KEYWORDS: *Radiolarian* Skeleton, *Pantanellium*, Schlegel Diagram, Straws and Pipe Cleaners.

INTRODUCTION

Skeletal structures of radiolarians, a kind of marine plankton, have been studied by many palaeontologists because radiolarians are frequently used as indicators of age and environment in the Earth's history. Their structures have varied widely since the occurrence of radiolarians five hundred million years ago. Their variation is well-known through the publications of Haeckel's works [01]. The description of the shape of radiolarians has become more accurate with the progress of scanning electron microscopes (SEM) and micro X-ray computed tomography (CT) systems since Haeckel's era.

We focus on the genus *Pantanellium*, the representative radiolarian in the Mesozoic Era, in this study. The main features of *Pantanellium* are their thick pore frames in the cortical shell and two opposite spines (primary spines) (Fig. 1). The cortical shell consists of approximately thirty polygons: pentagons and hexagons. Matsuoka et al. [02] and Yoshino et al. [03] used Schlegel diagrams in order to represent the structures of the cortical shell. However, there is no suitable method to construct the skeletal structure models from the diagrams. In this study, we consider the model constructions using straws and pipe cleaners.

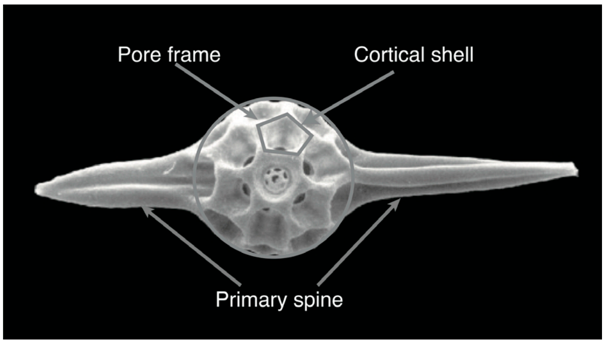

Fig. 1 - Representative SEM image of a fossil *Pantanellium* specimen.

RESEARCH

The representative SEM image and terminology of *Pantanelium* are shown in Fig. 1. The length is approximately 200 μm from tip to tip. The cortical shell consists of almost regular pentagons and hexagons. Primary spines are extended from the two vertices of the polyhedral frame in opposite directions. The cortical shell can be approximated to a frame of a polyhedron so that its structure can be geometrically represented by Schlegel diagram. Therefore, the structure may be reproduced according to the diagram.

The Schlegel diagrams of the skeletal structures of *Pantanellium* were obtained from the 3D data of real fossil specimens of the earliest Cretaceous age. We made eleven diagrams based on the different specimens presently. Since all of the diagrams were structuraly different, we concluded that the skeletal structure of *Pantanellium* was geometrically different within a single species. Fig. 2 shows the diagrams drawn by the authors, as examples. They have 27 or 28 pore frames (polygonal faces). Two open circles in each diagram represent the location of the primary spines. The central polygon corresponds to the front so that the outer frame corresponds to the frame on the back. Because the skeletal structures were not regular, the polyhedrons represented by the diagrams have no names. For this reason, each diagram has a number, and the number is merely the order in which the 3D image was taken.

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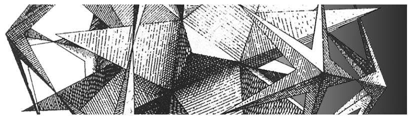

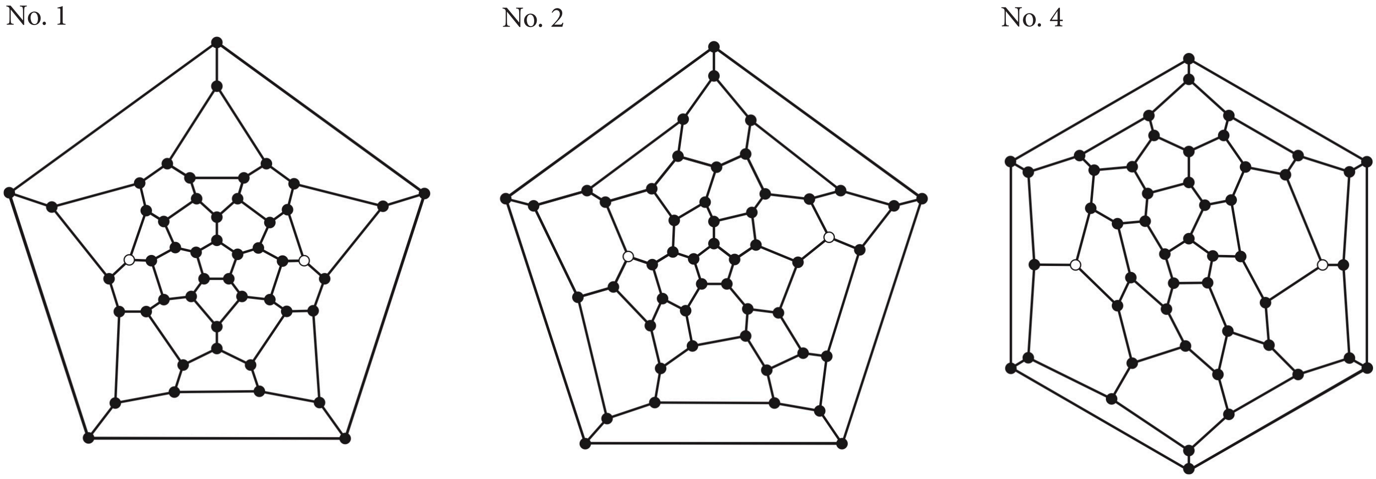

Fig. 2 - Examples of Schlegel diagrams representing the skeletal structures of *Pantanellium* specimens.

We used straws and pipe cleaners as materials of the skeletal model. The straws are used for the edges of the polyhedron, and the pipe cleaners are used for connecting the neighbouring straws (edges). We referred to the method developed by the *Mathematics Certification Institute of Japan* [04], summarized as follows:

Firstly, given numbers of straws and pipe cleaners are prepared. The lengths of the straws and pipe cleaners are approximately 20 mm and 30 mm, respectively. The lengths do not have to be exact.

Secondly, all of the pipe cleaners are folded at the centres. Fig. 3 shows the materials.

Finally, all of the straws are connected according to the Schlegel diagram. One end of the pipe cleaner is inserted into a straw, and the other end of the cleaner is inserted into another straw. Only the fold of the pipe cleaner is exposed.

Two pipe cleaners are inserted to one end of a straw in order to form a vertex. Therefore, n number of straws and pipe cleaners are necessary for a vertex of a degree of n. Fig. 4 shows a close shot of a vertex of a degree of three, for example.

The original dimensions of straws (*Shibase Industry Co., Ltd.*) and pipe cleaners (*Zonroic*) are approximately 180 mm in length and 4.5 mm in diameter, and approximately 300 mm in length and 6 mm in diameter, respectively. The diameter of the straws must be large enough for the insertion of two pipe cleaners, however, it must be small enough in order to construct a model. We failed to construct a model using another type of straw with a larger diameter (6.0 mm) and the same pipe cleaners.

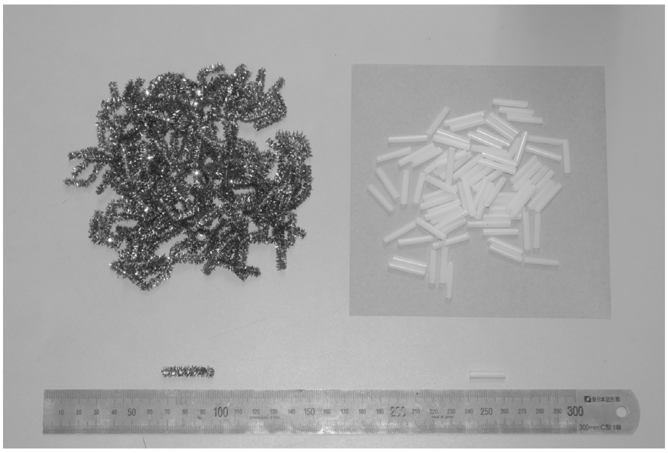

Fig. 3 - Materials used for model construction.
Left: pipe cleaners. Right: straws.

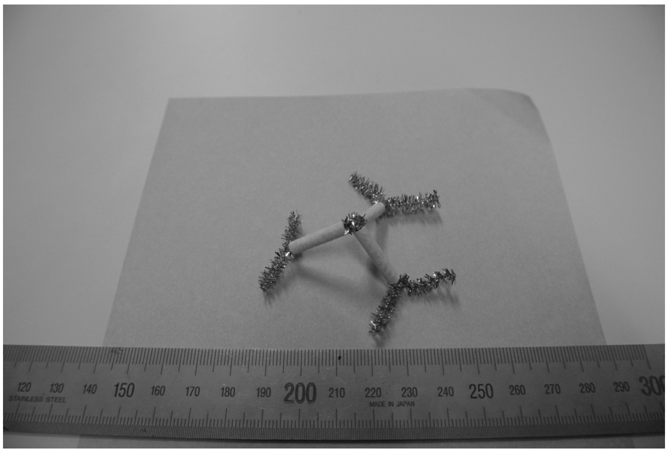

Fig. 4 - Close shot of a vertex
of a degree of three.

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The number of straws denoted by n_s is equal to the number of edges, and the number of pipe cleaners denoted by n_p is the summation of degrees of vertices for all the vertices. If all the vertices are degrees of three, the two numbers, n_s and n_p, can be described using the number of pore frames (faces), n_f, as:

$$n_s = 3\,(n_f - 2),\; n_p = 6\,(n_f - 2).$$

These formulae are useful for the preparation of the materials. It is notable that all the vertices are degrees of three for most of the cortical shells of the *Pantanellium*. Only one vertex is a degree of four among our 3D data of the eleven specimens.

We succeeded in constructing the cortical shell models of *Pantanellium*. The image of the resulting models of No. 2 and No. 4 in Fig. 2 are shown in Fig. 5. Both of the specimens have 28 pore frames (faces) so that the number of straws and pipe cleaners were $n_s = 78$ and $n_p = 156$, respectively. The time for construction, including the time for cutting the materials was approximately two hours in both cases. It was easy to connect the edges (straws), however, it was difficult to make the polygons according to the Schlegel diagram. The structures of the models were almost the same as the 3D models produced using the 3D printer. Therefore, we concluded that the straw model was an effective method to reproduce the shapes of the cortical shells of *Pantanellium*. As shown in Fig. 5, we added the primary spines to the model using the same type of straws as those for the edges although the length was different.

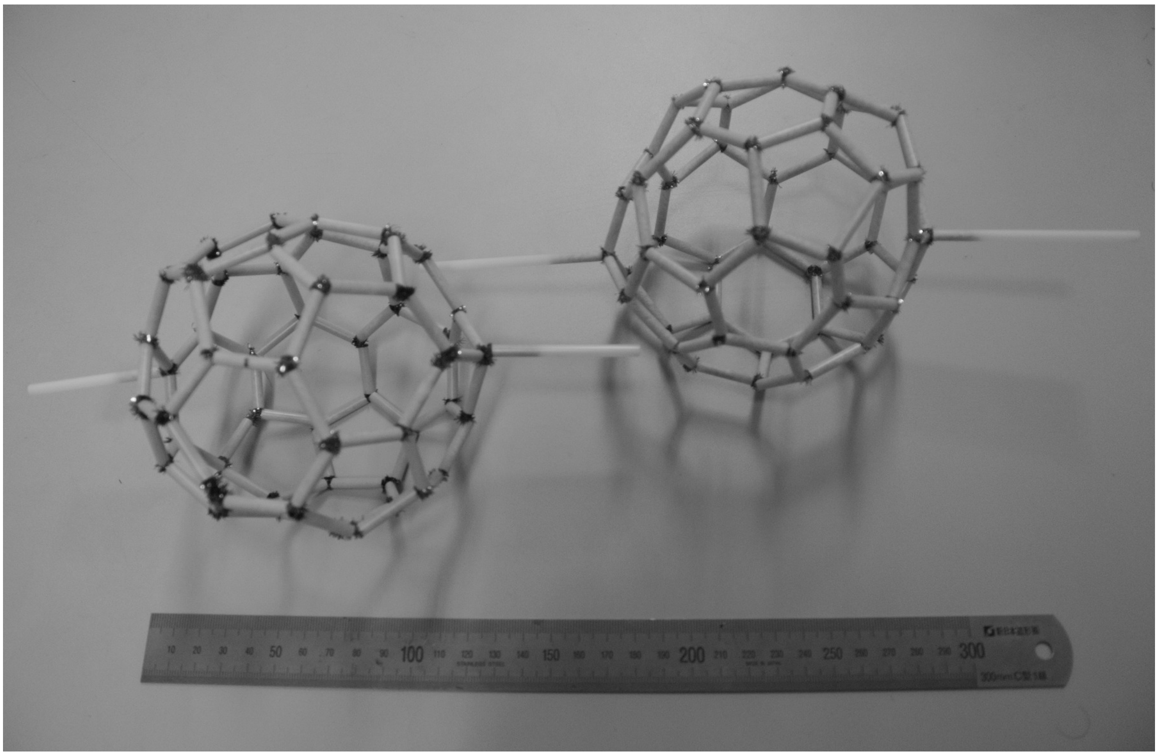

Fig. 5 - Resulting models of No.2 and No.4.

There are two advantages of the straw model over the unit *origami* model which we proposed formerly. The first feature is the time for preparation of the materials. The straw model takes approximately half an hour, on the other hand, the unit *origami* model takes more than one hour. The unit *origami* model needs more time for two reasons: the first reason is that the folding of one unit takes a few minutes; the second one is that the number of edges must be prepared in advance. The second feature is the representation of the primary spines. We must attach different shapes of *origami* to the unit *origami* model as the spines, however, we merely attach long straws to the straw model. The advantage of the unit *origami* model, on the other hand, is that the model uses only paper and glue.

Most of the commercial tools to construct polyhedrons assume that the polyhedrons are regular in shape. On the other hand, handmade parts, corresponding to edges in most cases, are suitable for non-regular polyhedrons. For this reason, we used straws and pipe cleaners or *origami* for the skeletal structure models. These materials are useful for

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such kinds of polyhedrons. However, we have to prepare the units ourselves. Therefore, the preparation must not be time-consuming; the number of units increases almost linearly with the number of faces (pore frames). The pore number of *Pantanellium* is the smallest among pore framebearing radiolarians, therefore, it is necessary to consider another approach to make a model which has a larger number of pore frames.
We will prepare some starter kits to take away for the participants who are interested in the model at the poster session. We will demonstrate how to construct the model using the same materials.

CONCLUSIONS

We constructed the cortical shell models of the radiolarian genus *Pantanellium*. The structures of the cortical shells were considered as polyhedrons, however, they were not always regular in size. For this reason, we did not use commercial tools but straws and pipe cleaners as materials. Straws represented the edges of the frame structures, and pipe cleaners were the connections. The models were made according to the Schlegel diagrams drawn, based on the 3D images of real specimens. We demonstrated the procedure of the construction and the resulting models. This method had advantages over the unit *origami* method: a simpler preparation of materials and representation of primary spines.

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POSTERS' SESSION
POSTER 03

POLYHEDRAL CHARACTERS AS A BASIS FOR TEACHING COMPUTER ANIMATION

Ratko Obradović[1], Miloš Vujanović[2], Igor Kekeljević[3], Ivana Vasiljević[4], Isidora Đurić[5], Lidija Krstanović[6] and Bojan Banjac[7]

KEYWORDS: Polyhedral Characters, 3D Animation, Platonic Bodies.

INTRODUCTION

Today's computer animation provides a wealth of possibilities for creating different types of animated films, designed for the entertainment industry, for the needs of various engineering disciplines or medicine, as well as for various aspects of education.

The main idea of this abstract is to present the process of creating simplified humanoid characters, obtained by the combination of Platonic bodies which holds great potentials for the education of students with a minimum knowledge in the field of computer 3D animation. We present a new and funny way of animating characters without requiring the standard skeleton and rigging in the creation a humanoid character.

RESEARCH

This research explores methods in education for teaching character 3D modelling and computer animation, focusing on the education of students with no prior knowledge of character 3D modelling. One of the possible approaches was presented in [01], proposing four related tasks for studying manlike 3D character animation. However, this method required that students possess a considerable knowledge of 3D modeling, texturing, animation of lights and cameras, as well as rigging. This abstract proposes a method for teaching students the basics of character 3D modelling and animation, which relies on the basic geometry of the Platonic bodies.

Polyhedra can easily be projected into a 2D plane, and can thus easily be painted, providing a sufficiently interesting basis for creating angular characters, like those (Figs. 1 and 2) presented in the *Alea Iacta Est* animated movie [02]. In [02], the cube is used as the main motive to represent everything that exists in nature. The main idea was to show that technology can simplify reality by bringing it down to basic geometric shapes. All objects such as plants, animals and the planet Earth, as well as humans, are cube-shaped, which facilitates their common coexistence.

This approach can be used to animate an interesting story whose quality depends on the imagination of the screen-writer, the director, and the art team. Namely, a good movie does not always require top technical conditions or good knowledge of hardware and software used for 3D animation, but, in fact, sometimes, a good or even an outstanding artistic film can be achieved with imagination and idea, or quality scenario. As early as 1978, Marr and Nishihara [03] offered a model in which all objects were presented with a cylinder.

In the animated film *The Dot and the Line* [04], a simple line (man) attempts to woo his true love, a dot (women), away from the unkempt squiggle she prefers. But he will have to learn to bend before she will notice him. Another representative examples are the humanoid polyhedral characters [05], illustrated in Fig. 3.

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Fig. 1 - Cube's characters, humans and car. From *Alea Iacta Est*,
by B. Knežević, R. Obradović, M. Vujanović, I. Nikolić and N. Kuzmanović, 2011, http://www.new-silhouette-studio.com/#aie [02].

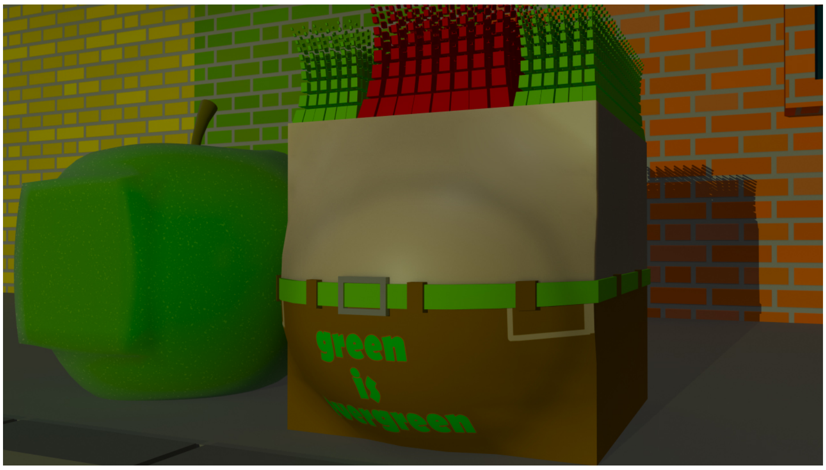


Fig. 2 - Cube's character with cubes hear and Apple as a Green Alien. From *Alea Iacta Est*,
by B. Knežević, R. Obradović, M. Vujanović, I. Nikolić and N. Kuzmanović, 2011, http://www.new-silhouette-studio.com/#aie [02].

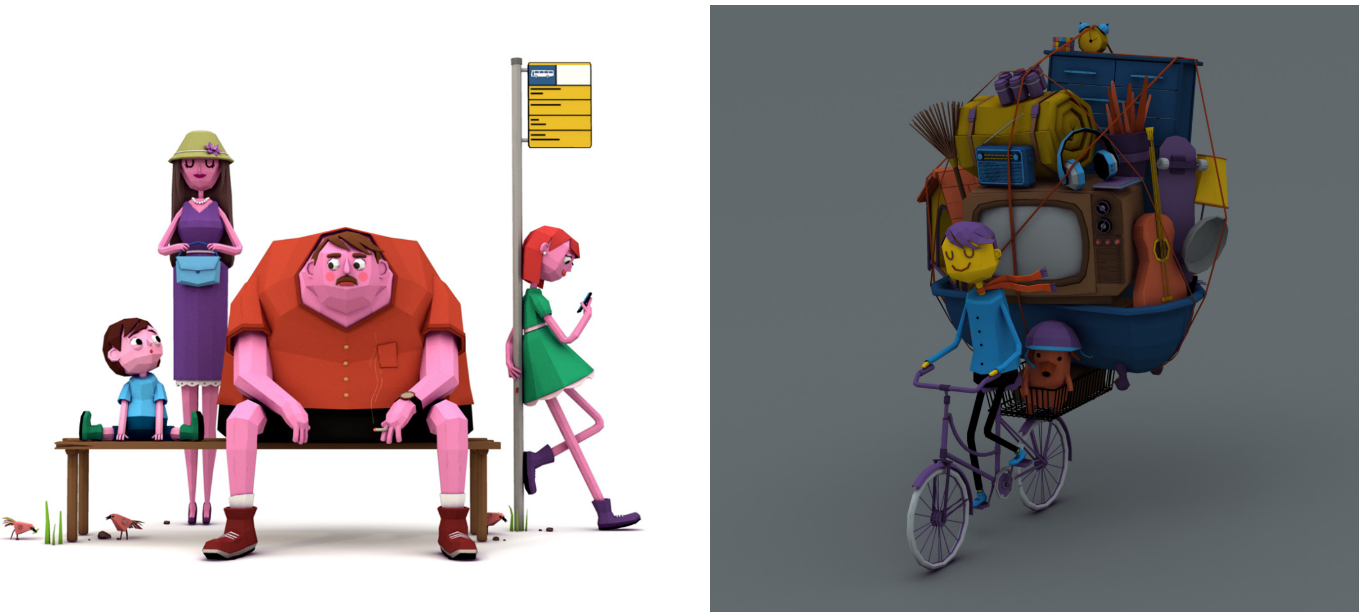
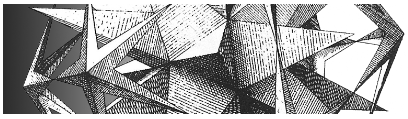


Fig. 3 - Polyhedral Characters by Sivan Baron, 2014.
From http://www.sivan3d.com/ [05].



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Four convex regular polyhedra were selected: tetrahedron, hexahedron, octahedron, and icosahedron [06], to present the idea of creating simplified humanoid characters. The concept of polyhedra was popularized by Plato. "Platonic bodies" might be a more popular expression instead of "regular solids", more commonly used in geometry. Plato describes them first in the *Timaeus*, by associating the pyramid (tetrahedron) with fire, the icosahedron with water, the cube with earth, the octahedron with air, and the fifth element with the 5-dimensonal space or universe. About the fifth element, Plato obscurely remarks: "...but since there is a 5th combination (of triangles) the God used it for arranging the constellations on the whole heaven." [07]. Plato connects them with an ancient model for understanding the world that is described by the four elements: fire, water, earth, and air, and the universe as the fifth element. Such a model with the mentioned four immutable elements, whose combination creates everything in nature, dominates the ancient philosophy and is mentioned at the earliest by Empedocles. He explains the connection in a picturesque way, through the experience that we would have when keeping these shapes in our hands. The tetrahedron represents fire, because its sharp edges are like flames of fire, the icosahedron rolls and it is elusive like water, the hexahedron stands firm and stable as the earth beneath our feet, while the octahedron is unstable like air [08].

Ancient Greeks believed that there was an increased presence of a particular element in body fluids in humans, which affected their behavior. They introduced an explanation for the four temperaments that were associated with a particular element and certain behavior. Thus, the choleric is a fire temperament, found with people who have individualistic traits that are aggressive, temperamental and ambitious. Phlegmatic temperament is related to water, with people who are sensitive, calm and withdrawn. Melancholy is a climactic temperament, and such individuals are rational, analytical and prone to perfectionism. Sanguine is an air temperament, with people who are intellectual, communicative and social.

Western astrology also relied on the idea of four elements, but also on the theory of four temperaments, in the ambition to understand human behavior and their fates. In astrology, there is a connection between people and certain celestial bodies, which influence a particular zodiac sign. Saturn affects the fire signs: Aries, Leo, and Sagittarius, who are passionate, energetic and ambitious in temperament. The Moon affects the watermarks: Cancer, Scorpio and Pisces, which are introvert, emotional and sensual. Mars affects the earthly signs: Taurus, Virgo and Capricorn who are stubborn, practical, materialistic in temperament. In the end, Jupiter influences air signs: Gemini, Libra, and Aquarius, which are abstract, intellectual and analytical.

The above theories were the starting point in the design of our characters, in the desire to establish a connection between geometric shapes, colors, and temperaments. Fig. 4 shows the concept of our four characters. Water is made up of icosahedrons and is painted violet, which is usually associated with emotions and feelings. Wassily Kandinsky says that violet is also the color of weakness and sorrow, because it is the chilled version of the red color. Fire is made of tetrahedrons and is painted red. Red is the color of life, passion, war, *Eros*. Earth is made up of hexahedrons and is painted in brown color, which is the color of reality, earthly, material. Air is made up of octahedrons and is painted blue, which is the color of the sky, immaterial [08].

In a semiotic domain of four elements, water is associated with a circle, a female principle, which denotes continuous movement, cycle, egg symbol, renewal of life. Fire is associated with a triangle, a male principle, an erection, a penetration, an elevation. The earth connects with the square, the female principle, because the earth represents fertility, birth, and motherhood. The air is associated with the rhombus, which is the male principle, the space between heavenly and earthly, air [8].

Fig. 4 - Concept of four Characters created from the Platonic bodies: Water, Fire, Earth and Air.

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Fig. 5 shows the character made of four hexahedrons, in which the details are shown through the texture. The whole process of the character creation consists of three main steps: 1) creating the 3D model based on the regular polyhedral geometry; 2) unwrapping 2D texture of the developed character 3D model; and 3) creating vector texture which is further projected back to the 3D model surface. The unwrapped texture (Fig. 5, down) illustrates the simplicity of the angular character creation only by using UV mapping technique. The texture is composed of vector segments that are separated in more layers and rigged in order to be able to be animated. Animation of the texture enables greater possibilities of poses and expressions, and even to make movements that are more attractive and more expressive than it would be possible with realistic characters, because it would be unnatural for realistic characters to develop in a plane or to become different geometric figures. According to that, the texture is visually simplified and reduced to geometric shapes in order to stylistically connect with the geometric design of the characters.

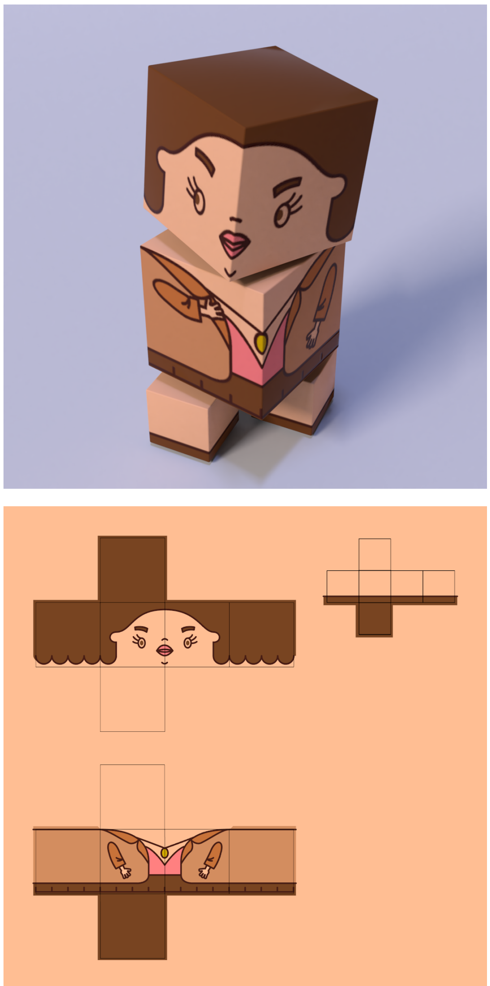

Fig. 5 - Character Earth and its unwrapped texture.

CONCLUSION

The analysis of this study shows that

> "*the effects of the observation of the object suggests that the representation is also defined by knowledge concerning the form, regardless of the angle of observation, distance, lighting and other current observing conditions.*" [09].

When observed, visual objects are represented in structural descriptions, such as that a man consists of one head, one body, two arms and two legs, and a bird of one head, one body, two legs and two wings. These structural descriptions may also be unrealistic. Contemporary practice in animation shows that such representation of objects can be carried out using polyhedra. In observation, visual objects are represented by a structural description and then a comparison is made of the currently observed image with characters stored in memory. So, in the first step, visual stimulation needs to be divided into constituent, primary, elements (polyhedra), and in the second, compare the structural descriptions and what we see with what we know about the character.

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POSTERS' SESSION
POSTER 04

ARCHITECTURAL APPLICATIONS OF GEOMETRIC STRUCTURES. CASE STUDIES OF SURVEY IN SOUTHERN ALBANIA.

Luigi Corniello[1], Enrico Mirra[2] and Lorenzo Giordano[3]

KEYWORDS: Architecture, Geometry, Survey, Fortresses, Albania.

INTRODUCTION

This research itinerary focuses on the study of defensive structures in southern Albania, with research operations of a cognitive nature which expose the geometric and structural beauty of the fortified architecture. Through the activity of reading, we could find identity issues of the design discipline and the relationships between representation and measure; the graphic aspects and the structural values of the fortifications, understood as traces of the past to be compared with the contingencies of the present. The representation activity of the castles of Porto Palermo, Saranda and Butrint was set up, providing, at an early stage, the execution of a survey that extended to the architectural bodies and surrounding territories, in order to define a primitive geometrical model. Subsequently, and in a second survey and restitution campaign, measurements of architectural details, structures and geo-referencing of the digital model were carried out. Appropriate photographic documentation was also produced in addition to bibliographic, archivistic, iconographic surveys.

RESEARCH

In order to outline a reading of the castles of southern Albania, we considered useful to deepen the issues related to treatises intended for defence and its mechanical and engineering implications. In this historical setting, the viewpoints characterizing the design of the fortifications, depicting fortified castles equipped with static defence systems (such as ditches, triangular bastions, watch towers and battlements), and dynamic devices (such as drawbridges, mobile walkways, wooden platforms), that represent a clear evolution of the two-dimensional planimetric scheme designed by Filarete and Giuliano da Sangallo. Francesco di Giorgio Martini contributed to this critical debate on defence architectures with the drafting of his main work, namely *Civil and Military Architecture*, developed between 1489 and 1492, the main parts of which are kept in the *Codex Magliabechianus*. The theme of fortifications is tackled by Francesco di Giorgio Martini in the fifth of the seven books, in which the second edition of his treatise is articulated. The attention given by Francesco di Giorgio Martini to the identification of a continuity between antiquity and his own time, in terms of the revival of an aesthetic and constructive dignity to be applied to architectures of a warlike type (such as fortresses for defence) is put into crisis by the clearly functionalist attitude expressed by Leonardo da Vinci through a reflection, written in 1490, involving man and fortified architecture: "It cannot be beauty and utility, as it appears in fortresses and little men". As we know, Francesco di Giorgio Martini, who lived in the second half of the fifteenth century, was an illustrious expert of art and military architecture and, at the same time, on the construction of fortresses, and wrote the Treaty on fortified architecture. The content of his work ranges not only on the fortifications, but deals also with all the fundamental aspects that synthesize the fortified architecture: places, cities, bridges, works of plumbing, temples, theatres, columns, and still geometric rules, levers, war machines. The fortress is considered as the fulcrum of the city, considered the head and compared to the human

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intellect, introducing, for the first time, a vision not only of a defensive and strategic function, but also for its symbolic representation, therefore, with military and political values. Subsequently to this theory, fortress and war become an instrument to obtain victory on the battlefield and the treaty itself, an almost construction site in constant search for practical demonstration and verification of the theory. The elaborations and continuous research on the quality of buildings are continually verified in practice, that thus becomes a means of proof and confirmation of theories and thoughts. The drawings, an integral part of any treatise, are, in part, survey obtained from the fortifications realized, as a direct and truthful demonstration of the building systems, which, in part, are theorizations based on practice and represented with geometry. The analysis of the treaty of Francesco di Giorgio Martini can, therefore, stand as a valid support in the study of fortified architecture suitable for the knowledge, protection and its recovery, the understanding of certain choices, constructive techniques and meanings for the architectural elements.

The path of knowledge of the fortified architecture in southern Albania, a territory characterized by an orographic conformation composed of mountain peaks, with large valley floors that bevel into plains along the coasts, deepens, through the actions of drawing and survey of the artefacts in the manors' sites in the coastal towns of Porto Palermo, Saranda and Butrint (Fig. 1). The castle of Porto Palermo (Fig. 2) rises along the southern coast of the Albanian territory on a peninsula facing west within the homonymous bay. The construction dates back to 1804 and is part of a network of defensive structures designed to provide shelter and defence for a conflict with the Turkish people. The structure, triangular in shape with three hexagonal towers arranged in the corners, is dominated by large protective battlements of the stronghold and the ramparts themselves, equipped with loopholes and cannons. The main entrance is characterized by a portal placed to the east, surmounted by a balcony. The castle is two floors above ground: the ground floor is characterized by barrel vaults with numerous interior spaces such as rooms, storage rooms and a mosque; and the stronghold, above the first floor, which is accessed via steep stone stairs, was the element of deposit of the war elements and, at the same time, the space for attack.

The castle of Saranda (Fig. 3), however, was connected to the castles of the coast such as Porto Palermo to the north and Butrint to the south. It stands on a hill in the eastern part of the town of the same name, in front of the Greek island of Corfu. This position, with a view of the entire gulf and the hinterland, guaranteed the structure complete control of the territory. Composed of a quadrangular plan with four circular towers placed in the corners and a central building raised above the outer walls is, nowadays, in a strong state of decay. There are few traces of the external walls, such as the southern towers with loopholes and battlements and the portion of the eastern entrance consisting of a round arch in white stone.

The fortification system on the *Unesco* site of Butrint is different, and survey was performed on the Triangular Fortress, the quadrangular Tower, the Venetian Castle and on the Castle of Ali Pascià (Fig. 4). The ancient center of Butrint offers readings that date from the ancient world to the present day, with textures of images and drawings of precious references for the Mediterranean societies. In particular, the archaeological site was declared by *Unesco* a *World Heritage Site* in 1992 and registered in the list of *Danger Heritage* in 1997. The site, which extends on a hilly

Fig. 1 - The bay of Porto Palermo, view of the peninsula with the castle.

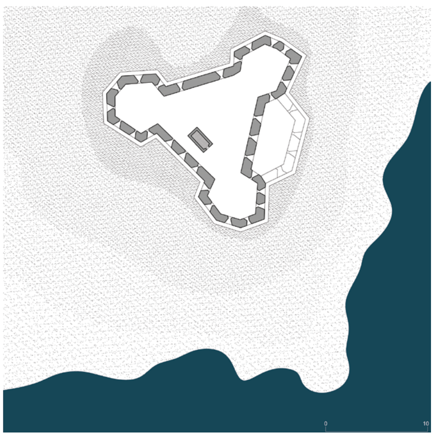

Fig. 2 - The castle of Porto Palermo, plan share + 7 m.

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peninsula in the south west of Albania on the border with the Greek state, includes some monuments that refer to more than two millennia, beginning with the Hellenistic temples of the fourth century. B.C. to the Ottoman defence systems of the early nineteenth century. The city was equipped with high defensive walls along the perimeter, terrestrial and marine, consisting of stone blocks of polygonal shape chiseled and laid dry. The entrance was guaranteed through five gates guarded by watchtowers, of which the quadrangular tower is still visible today. A three-stores structure overlapped of which a base part was used as deposits; the first floor covered by a barrel vault and accessible by an external staircase and connected to the tower by a wooden drawbridge; and an overhead guard terrace of size 6 m for 6 meters with loopholes and gunboats. Of great architectural and structural importance is the Skea gate, mentioned by Virgil on the occasion of the arrival of Aeneas, which is located in the eastern part of the wall. Five meters high and just over two meters wide, it is presumed to have had protective metal shutters. The Lion's Gate, on the other hand, takes its name from the base relief engraved on the weighty stone architrave depicting a lion that breaks an ox. At the top of the hill, rises the castle built in the Venetian era and now used as a museum. From the *Unesco* site guarding the Vivari Canal, stands the castle of Ali Pasha: the fortified quadrangular structure with four towers, two of which circular in the west, and two squares in the east; two entrances, one in the north, with access to a vaulted storage room and one in the east to the mainland. The aforementioned channel of the Vivari, which constituted the maritime access route to the city of Butrint, was also protected by the Triangular fortress. A triangular castle (Fig. 5) with three circular towers crenelated in the three vertices and three buildings inside: one circular. one square and the other. rectangular. Of the original structures there are traces of the two towers towards the east, more exposed to the attacks, and the walls that have undergone invasive restoration.

CONCLUSIONS

The reading and survey activities on the castles in southern Albania focused on the collection of general data, historical elements, many of which are traced from unpublished sources, historical and current photographic documents and manual and instrumental surveys on individual architectural artefacts.

Particular attention has been paid to the architectural structures and defence elements, such as the gunboats or helmsman, the openings in fortifications that are used for artillery sites in the most convenient places or in the bastions; many of them characterized by a neck and deep knee pads designed to defend the lower part of the human body. The stratifications of castles' structures have aroused considerable interest: this overlap due, as is well known, from the adaptation of the fortifications to the siege war, since the military convenience of neglecting the ancient and weak walls of linear form, was recognized as having led to the reinforcement and redesign of regular polygons with bastions at the top. The bastions or bulwarks replaced, therefore, the towers and the relative straight wall of the connections called curtains. This system of curtains and bastions, called the bastioned order, allowed the enemy who approached it to fall down on the side of the structure and is a characteristic element common among the castles of southern Albania.

Fig. 3 - The Venetian castle of Butrint seen from the west.

Fig. 4 - The triangular fortress, seen from the north.

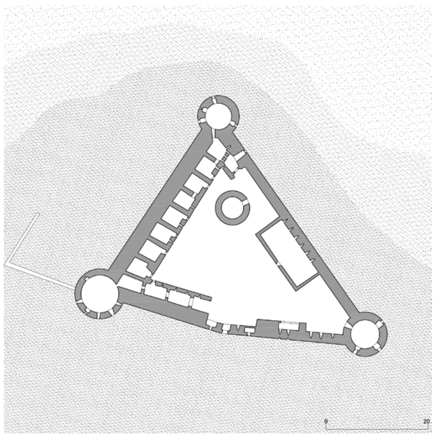

Fig. 5 - The triangular fortress, plan elevation + 1.20.

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POSTERS' SESSION
POSTER 05

AUTOMATED PREPROCESSING OF VIRTUAL 3D BUILDING MODELS FOR FLOW SIMULATIONS

Raul Piepereit[1] and Margitta Pries[2]

KEYWORDS: 3D City Models, *CITYGML*, *BRep*, Automated Preprocessing, Simulations.

INTRODUCTION

Over the past decades, the versatility of 3D city models has been increasing. Applications for such models range from simple graphic visualizations to complex numerical simulations [cf. 01]. For the latter, it has to be taken into consideration that not only a 3D city model is needed, but also a corresponding CFD (computational fluid dynamics) mesh. The quality of the mesh depends on the elements of the buildings' geometry which can have several negative impacts on the mesh creation and the simulation (e.g. high computation time; failure of mesh generation). Because of that, it is not uncommon that city models have to run through several preprocessing sequences before they can be used in simulations. These processes generally involve an extremely high manual and therefore uneconomical effort. Our goal is to reduce this manual effort, by implementing and combining different automated preprocessing strategies, such as Boolean operations, Minkowski sums, sweep-algorithms, as well as algorithms making use of freeform-surfaces.

RESEARCH

Advanced technologies such as aerial laser scanning and dense image matching have made it possible to create virtual 3D city models with a high degree of automation. Consequently, the availability of such models has been increasing rapidly in recent years [cf. 02]. The data format used frequently for storing and exchanging virtual city models is the well-established OGC standard *CityGML*. In this standard, the building models' geometry is defined by polyhedrons [03]. This data format is well suited for documentation, distribution and spatial analysis of geo data (cf. Fig. 1). It is not, however, automatically suited for engineering tasks such as CFD (computational fluid dynamics) simulations.

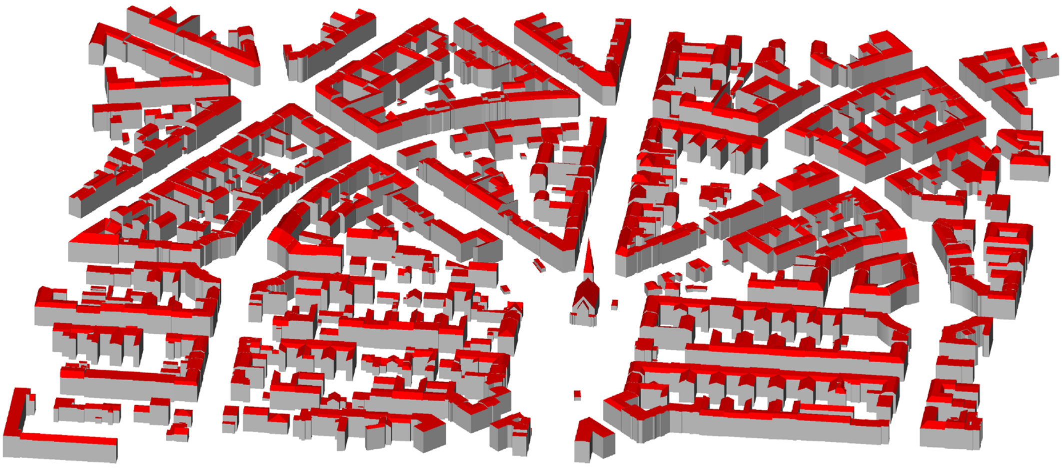

Fig. 1 - CityGML model of Friedenau, a district in Berlin (cf. https://daten.berlin.de/datensaetze).

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As was mentioned in the Introduction, a good CFD mesh is needed for simulation purposes [cf. 04], as it can increase the convergence, speed and accuracy of the simulation. A mesh of poor quality, on the other hand, can falsify the outcome of the simulation. These meshes of poor quality can occur, because automated meshing tools of common simulation software use the edges of the underlying polygons for the boundaries of their mesh cells. Polygons with short edges create small or narrow mesh elements which should be avoided, as too many or too small elements lead to a high computation time and the system of equations to be solved tends to be prone to failure. There are several reasons that cause the existence of unnecessary narrow polygons.

1. Building models with a high level of detail, for example, might contain elements, such as overhanging roofs, doors, windows, and chimneys etc., that increase the amount of small edges in the geometry and, therefore, must be removed.
2. Round surfaces are approximated by several narrow polygons, as the original geometry of a building model is saved in a polyhedral form without using freeform surfaces, as can be seen in Fig. 2.
3. Adjoining buildings (as depicted in Fig. 3) might create additional small elements (e.g. due to a small offset between adjacent surfaces).

Small polygons themselves are not the only reason that can create problems for meshing tools. Gaps between houses or polygons with sharp angles, for example, might cause the meshing tool or the simulation to fail and have to be addressed as well. Without preprocessing existing city models, it is practically not possible to obtain meshes suited for simulations. Our research goal is to create a set of algorithms which automatically preprocess city models for simulations, as this process generally involves an extremely high manual and therefore uneconomical effort. To obtain more control over the buildings' topology and geometry, we begin by converting *CityGML* models into a *CAD BRep* data structure.

In the following, we shortly introduce five algorithms that can be used for automated preprocessing of virtual building models. These algorithms were created and adjusted to our purposes as a preparation for this project.

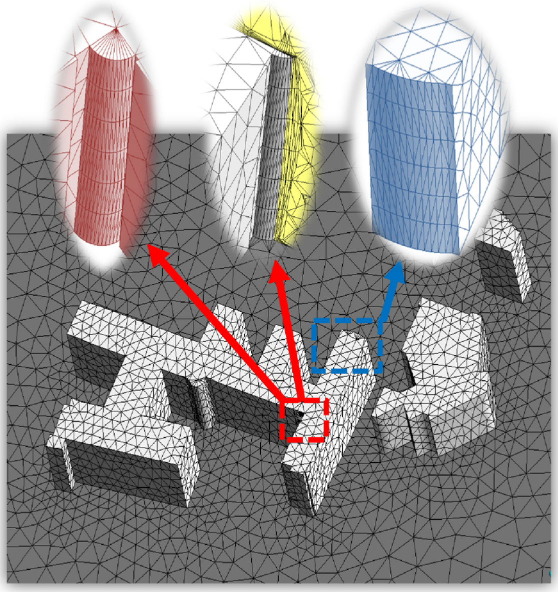
Fig. 2 - CFD mesh example of a virtual building model.

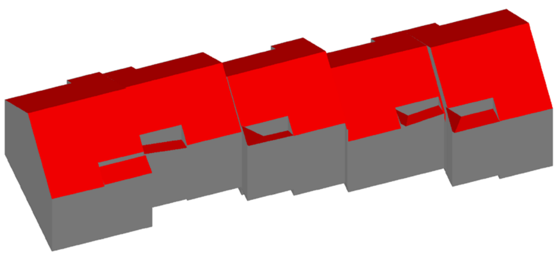
Fig. 3 - Several adjoining buildings.

To ensure a functional interaction between these algorithms during preprocessing, another fine tuning was and still is necessary. Once this is accomplished completely, a smooth implementation into the procedure of preprocessing 3D models is possible. These algorithms are no longer standalone solutions, they now interact with and depend on each other as they run through several iterations. The newly obtained set of preprocessing algorithms creates a system that increases the automated simplification drastically.[3]

1. *Merge Edges and Faces*: An easy but very important way to simplify building models, is to merge the edges and faces respectively, which are connected to each other and are, within a certain threshold, collinear or coplanar.
2. *Boolean Operations*: The basic operations of Boolean algebra are conjunction, disjunction and negation. For our purposes, we only need the conjunction. The union-operator of the *c++ library CGAL* can be used to merge buildings and building parts that are connected. As a result, other algorithms can be applied to more than just one single building at a time, but rather to several buildings of a whole city block at the same time, for example.

[3] For the automated preprocessing to work properly, we presume the building models (or their building parts) to be watertight and valid (e.g. the *CityGML* model has no degenerated polygons).

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3. *Minkowski Sum and Difference*: Using these operations, gaps and holes within a building can be closed. As a first step, the dilation of a building is calculated and created using the Minkowski sum. Employing the Minkowski difference, the erosion of the object is then calculated and created. Combining the Minkowski operations and the Boolean conjunction makes it possible to obtain the same results with building blocks. The Minkowski sums, like the Boolean operations, are implemented in *CGAL* and were documented in [05].
4. *Sweep-plane Algorithm*: The sweep-plane algorithm [cf. 06] iteratively eliminates edges with a length below a predetermined threshold, by moving corresponding faces along their normal direction into or out of the building. This algorithm allows for a specific elimination of small elements as they occur in offsets, bulges or corner offsets, for example. Building features, such as doors, windows or chimneys, can be erased in this manner as well.
5. *Coons Algorithm*: Round surfaces of buildings in *CityGML* are approximated by a plurality of narrow polygons. An approach to use the method of Coons in order to replace those polygons by freeform surfaces, such as B-Splines, is described in [07]. Using freeform surfaces enables the meshing tool to adjust the size of the mesh elements more freely. The Coons algorithm ensures that the models remain watertight.

After the above mentioned preprocessing, we export the model as a *STEP* file. These *STEP* models can then be loaded into simulation software (e.g. *ANSYS FLUENT*) and simulations, such as the wind simulation depicted in Fig. 4, can be run.

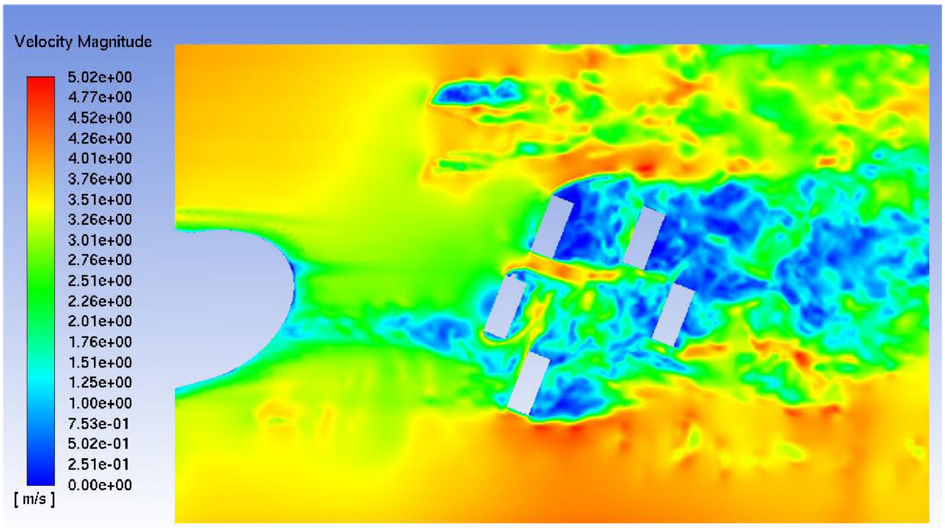

Fig. 4 - Example of a wind simulation.

CONCLUSIONS

By processing the geometry of virtual city models, the quality of the derived finite element mesh can be significantly improved. We concentrate on automatic preprocessing of city models with regard to simulations. Implementing the above mentioned approaches significantly reduces the manual effort required for preprocessing. By combining several of these approaches, a higher efficiency can be ensured. For certain complicated geometries, however, it must be noted that there is a need for further development of the above mentioned algorithms. For others, different approaches have to be designed. In addition, studies have to be carried out to determine the effects of the automated processing on the desired simulations.

ACKNOWLEDGMENTS

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DIGITAL FOLDING DESIGN AND DEFORMATION TEST OF ELEVEN *ORIGAMI* TESSELLATIONS

Samanta Aline Teixeira[1], Thaís Regina Ueno Yamada[2] and Galdenoro Botura Junior[3]

KEYWORDS: *Origami* Tessellation, Digital Folding Design, Deformation Test, Innovation Case Study, *Origami* Design.

INTRODUCTION

Researches on *origami* design presents several innovations for design and science, such as: automotive airbags [01], space telescopes [02], architectural structures [03], furniture [04] and self-folding robots [05, 06]. This article presents a deformation test of eleven *origami* tessellations, comparatively analysing their folds geometry and their interaction with the compression levels. The aim of the study is to contribute to the improvement of the *origami* design methodological basis, seeking, in practical and observational terms, planned geometric elements that, when folded, can be applied in products easier to compact and less complex to be manufactured in series.

RESEARCH

Origami design is based on planning the crease patterns. The folding map allows any three-dimensional structure to be sculpted from a flat material [06]. Starting from what is nowadays called active *origami*, i.e., the creation of functional structures through folding design, this abstract presents a deformation test of eleven *origami* tessellations geometry, electing the best and worst folding patterns based on two criteria: higher level of compression and less structural complexity. With this goal in mind, the methodology of data collection was performed by manual preparation of test specimens and measuring them when folded. The crease patterns design was built into a vector drawing software (Fig. 1). The deformation test of six *origami* tessellations was already performed in a previous article [07], and five new *origami* patterns are now added to these data and redesigned from important *origami* design publications [08, 09].
The grid base for the drawings was made from the repeating module of standards with the fixed height of 3 centimetres, and it can vary the width and fit of the modules within the total paper size of 12 centimetres. The finished *origamis* can be seen in Fig. 2.
After folding to its crease pattern, each *origami* was measured in its compact final forms under the x (length), y (height) and z (depth) axes. From these measurements, the total volume of each tessellation was calculated, where it was possible to list the more and less compact patterns, according to Table 1 and Fig. 3.
We analyse Table 1 and list the more compact patterns in four criteria: smaller compaction alone in the x (length), y (height) and z (depth) or total volume axes. Best illustrated in Fig. 3, the most compact patterns on the x-axis are A (blue), B (brown), F (orange), and H (red), tied with 0.5 cm. The E (light blue) pattern is the most compact on the y-axis, with 0.5 cm. The H pattern (red) is the most compact on the z-axis, with 0.4 cm. H pattern is also the most compact standard in total volume, at 1.54 cm. The patterns with the smallest measures on isolated axes are not necessarily the more compact ones in the total volume. This characteristic demonstrates that there is a more substantial complexity in the all sample than variables can reveal in isolation. Apart from this, compaction design in the independent axes can support several product designs, such as cardiac stents, where initial (compact) and final (expanded) occupancy require measurement accuracy for different application situations.

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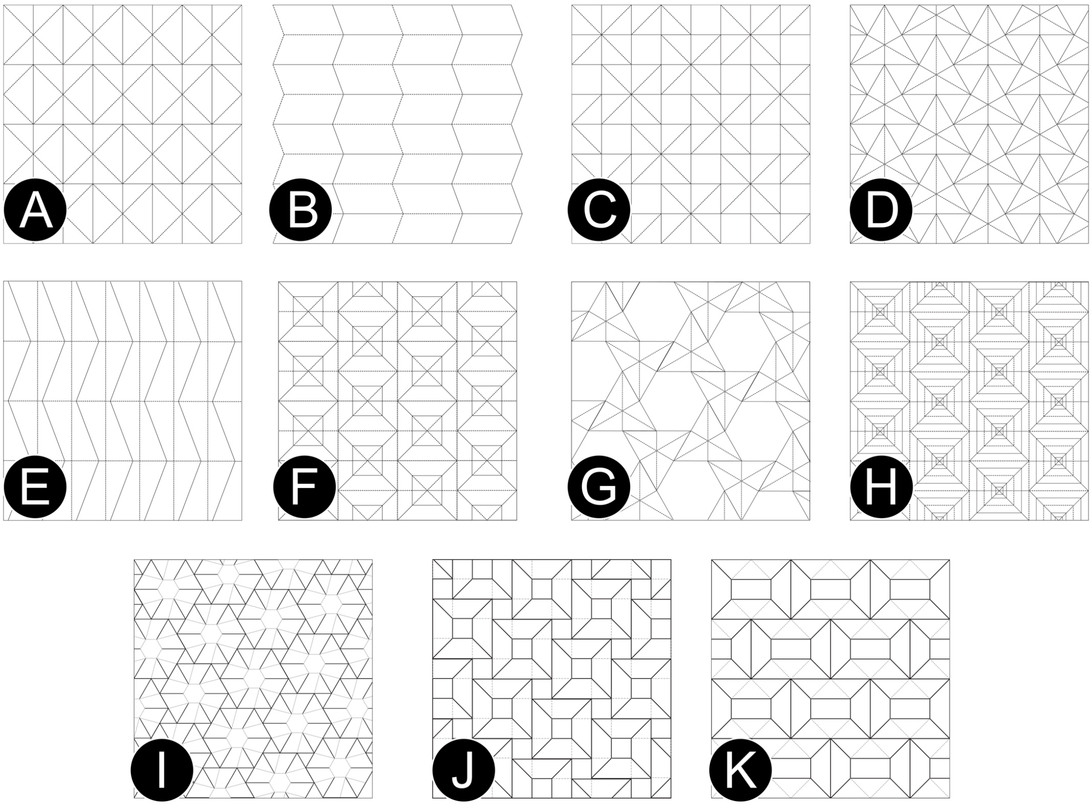

Fig. 1 - Vector drawing of crease patterns.

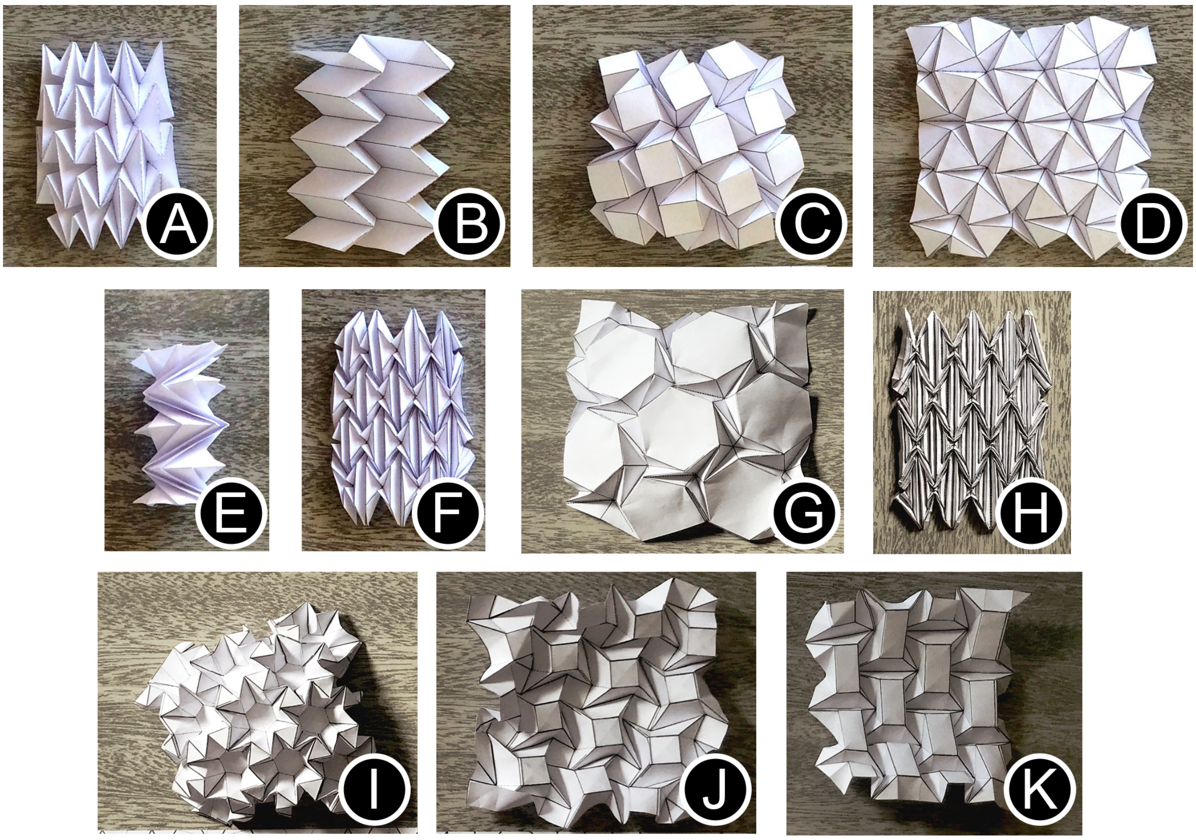

Fig. 2 - Folded Tessellations.

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Table 1 - Spatial measurements of tessellations.

Crease Pattern	Length (cm)	Height (cm)	Depth (cm)	Volume (cm^3)
A	0.5	1.5	7.2	5.40
B	0.5	2.2	5.5	6.05
C	6.0	1.5	7.2	64.80
D	7.5	1.0	6.8	51.00
E	3.5	0.5	3.7	6.47
F	0.5	0.7	7.4	2.59
G	9.3	0.9	8.0	66.96
H	0.5	7.7	0.4	1.54
I	6.0	6.5	1.0	39.00
J	6.5	5.5	1.0	35.75
K	6.5	6.5	1.0	42.25

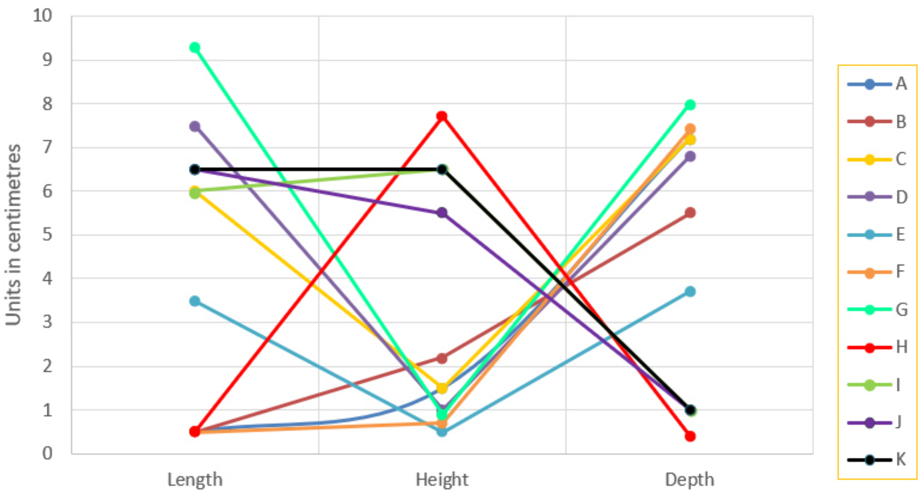

Fig. 3 - Measurements on the *x*, *y* and *z* axes.

The next step of the analysis is the number of folds in the tessellations. This step analyses the level of complexity required for the reproduction of each crease pattern and can be observed in Table 2.

Table 2 - Number of folds of tessellations.

Crease Pattern	Number of Folds (Valley and Mountain)
A	130
B	52
C	163
D	220
E	94
F	318
G	122
H	754
I	502
J	221
K	204

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According to the total count of folds (mountains and valleys folds), the sequence of the patterns with the highest to the lowest numbers of folds are these: H - I - F - J - D - K - C - A - G - E and B. Pattern H has the largest number of folds: 754, so it is the most complex to be reproduced. Pattern B is the least complex, because it has the smallest number of folds: 52. Of all patterns, nine were constructed from crease patterns already designed by *origami* artists and researchers such as Jun Mitani, Koryo Miura, Eric Gjerde, Ronald Resch, Yuri and Katrin Shumakov. The H pattern, created by the authors of this research, presented prominent measures in comparison to the others. The H pattern is the most compact in the total volume and the most complex, because it has the largest number of folds. However, the directly proportional relationship between complexity and compaction is not observed in all the analysed patterns. As in the previous study [07], the tessellation B, i.e., *Miura-Ori*, remains the simplest pattern and has medium compaction performance, between the eleven patterns. B is the fourth most compact in total volume. In addition to the data presented, other variables and tests may be performed in the future. One modification that can be explored is the variation of geometry size, increasing the height or width without changing the design base structure. To better understand the behaviour of the *origami* tessellation design, a stress test can be added to the deformation test performed here. The stress test is usually carried out by means of a dynamometer [10], bringing measures of compression and tensile forces.

CONCLUSIONS

This extended abstract presented a deformation test of 11 *origami* tessellations. As a result, the B pattern presented the simplest geometry and the H pattern presented the highest level of compaction in total volume. Future studies are being planned to better understand the relationships between crease complexity and structural compaction. Nowadays, the research on *origami* design is critical for engineering, design and science. We hope that the present study contributes to improve applications in technologies and product innovations.

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THE REPRESENTATION OF OBJECTS IN THE OBSERVATION DRAWING

Teresa Maria da Silva Antunes Pais[1]

KEYWORDS: Observation Drawing, Geometric Solid, Polyhedron, Sketch.

INTRODUCTION

In architecture training, Drawing is one of the curricular units taught in the first year of most courses.
According to the study carried out in 2007 [01], there are many affinities in the contents taught in this discipline in the various Faculties in Portugal, for example, the contact with the instruments and techniques, the objectives established or the approach to subjects such as objects, human figures or space. In terms of objectives, there are also similarities, notably, the development of the ability to observe, the dexterity, the knowledge of the very act of drawing or the sensitivity regarding aesthetic values.
As far as the exercises are concerned, the lecturers who participated in that study increase the difficulty of the topics throughout the school year. In general, assignments begin with simple tasks, such as the representation of small scale objects or spatial situations of level with a single vanishing point. Towards the end, the topics focus on the representation of the human figure or of space with a certain degree of complexity.
One of the subjects that is often dealt with, in the first classes of the different schools, is that of geometric solids. At the beginning of this school year, the first-year students of Drawing of the Integrated Master's Degree in Architecture of the University of Coimbra were invited to represent several solids: associated parallelepipeds, irregular polyhedra and regular polyhedra, in order to develop a process of comparative analysis.
The objective of the experiment was to compare the typical reactions to these families of objects as regards formal, procedural and plastic aspects, while understanding the advantages of each reaction in the development of skills related with the qualification of the drawings. This is particularly relevant in terms of the support occupation, perspective, proportions, or other aspects that are indispensable for a drawing to be made with intent and responsibility.

RESEARCH

In November 2018, it was suggested that 40 students draw some geometrical solids. For this purpose, a number of cardboard objects resulting from the association of parallelepipeds, whose proportions were carefully weighed, were used, so that the relationship between the measurements of the edges and the faces was easily perceived (Fig 1, left).
Students were at a level of development corresponding to the beginning of the second programme phase, and it was defined that students should make "sketches", a drawing process that is characterized by the paused graphic record, resulting from rhythmic and ambulatory gestures. The construction of the drawing is made from the general to the particular, implying a cognitive disposition of analysis and investigation of hypotheses, of perceptual adjustment to principles of morphological and/ or light similarity [02].
Each student made four sketches, each one representing a single different solid. It was established that each drawing should be carried out on an A4 format, with graphite, in a period of about 20 minutes.
As an element of research support, students were asked to capture their point of view for each of the solids represented by means of photography before starting their work, so that the photographs would serve as reference in the subsequent evaluation of the drawings.

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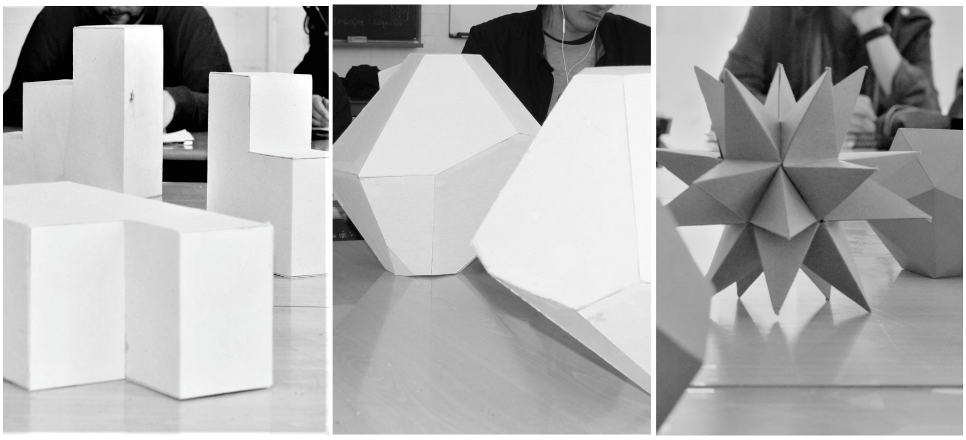

Fig. 1 - left: In the first group of exercises, students sketched cardboard objects resulting from the association of parallelepipeds, whose proportions were carefully conceived; middle: In the second group, the students represented irregular polyhedra, constructed by them within the scope of one of the practical exercises of Geometry; right: In the third group of exercises, the students represented regular polyhedra.

In the following class, the same students were suggested to make four more sketches.
The statement of the exercise remained, with the difference that the motives to be represented were some irregular polyhedra that had been constructed by them in one of the practical exercises developed in the course unit of Geometry (Fig. 1, middle).
The experiment ended in the following class with a third group of exercises, similar to those previously performed, except for the theme to be represented as regular polyhedra (Fig. 1, right).
After collecting and digitizing all the works (examples in Figs. 2 to 5), the next step was to generate instruments to analyze the drawings that fulfilled the established objectives and to select the parameters for their evaluation: the support occupation, the mastering of the mode, the ability to synthesize and the mastering of the perspective, form and proportions.
The process of comparative analysis of the drawings produced the results which are described below.

SUPPORT OCCUPATION

In the initial stages of learning, the support occupation is not, in general, an aspect to which the students attach great importance [03](Edwards, 1999).
In all the exercises proposed in this research, it was previously established that the scale of the representation would have to fit the size of the sheet of paper, meaning that students should reserve the distance of 2 cm between the limits of the image and the limits of the paper.
The analyzed drawings indicated that, in general, the students had some difficulty in carefully completing the support, both in the sketches made in experiment 1, 2 and 3 - with a tendency to draw smaller objects than was supposed. It was also observed that the students who were able to size the drawing to comply with the distance of 2 cm in experiment 1 were the same who could do it in experiments 2 and 3 (Fig. 2).

MASTERING THE MODE

In the first sketching exercises, mastering the sketch mode was no easy task. Students show a tendency to delimit shapes using the line, which tends to be clear and clean without hesitation, and to construct the drawing as a sequential sum of isolated elements, instead of evolving from the general to the particular [04]. In general, students do without auxiliary lines, and tend to simplify the process of assessing distances, angles and alignments, typical of this mode.
The analyzed data showed that the representation of the irregular polyhedra was the one in which the students had less tendency to make auxiliary lines, noting a greater propensity to use a process of image construction in which the evolution is made from particular to particular. On the contrary, the objects of experiments 1 and 3, especially in the first (Fig. 3), were the ones that motivated the construction of the drawing from the general to the particular, in

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a process of revision and gradual correction, overlapping and adding lines or patches to the model, and correcting information - indicating a mastering in the most effective and conscious way.

SYNTHETIC ABILITY

This concept involves the effectiveness that a given work has shown in terms of the choices made and the graphic options taken by the author, in the representation of certain forms or materials [05].
After the drawings were analyzed, it was found that there is a significant difference between the results obtained in the various experiments. The solids that motivated less advantageous results were the irregular polyhedra. On the contrary, the associated parallelepipeds and the regular polyhedra were those that provided for tendencies that showed more developed competences in the effectiveness of the representation of forms (Fig. 4).

MASTERING THE PERSPECTIVE

In order to evaluate the performance of the students regarding this aspect, it was assessed in which of the exercises they showed more difficulty in correctly evaluating the perceived angles.
The results between the sketches of the three experiments were very close. It was also observed that the students whose drawings presented a more discrepant angular evaluation between the angles perceived and those drawn, were the same in the various experiments.

MASTERING THE FORM

Form means the character assumed by the particular arrangement of the parts of a set that constitutes the image [05].
It was found that the polyhedra represented in experiment 2 were those in which the students showed greater difficulties in understanding the morphology, whereas the objects of experiment 1 and 3 seem to have motivated the students to understand the forms and, therefore, the structural rules and geometric properties inherent in the objects to be represented.
The collected data also point to the tendency of the students to differentiate between the faces of the solids with indication of light and shadow values, especially in the outlines of experiment 3, of greater formal complexity (Fig. 5).

MASTERING THE PROPORTION

Mastering the proportions means the ability to relate the measures of the various parts that constitute the object to be represented [06].
The drawings showed, unequivocally, that students are able to gauge more accurately the proportions of objects whose measurements are generated by clear geometric rules, which is the case of the objects of experiment 1 and the regular polyhedra of experiment 3, with emphasis on the former.

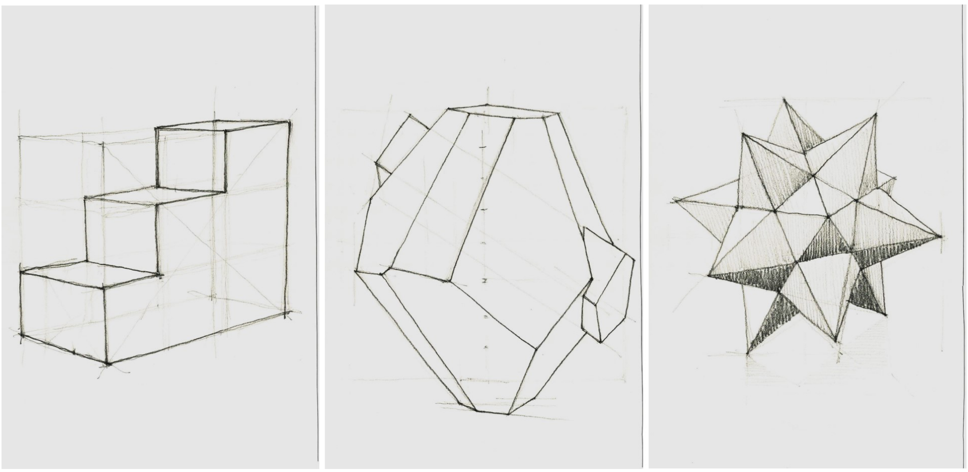

Fig. 2 - Exercises performed by one of the participating students, as an example of the response to experiments 1, 2 and 3.
In this case, the student was able to carefully occupy the sheet of paper in all experiments.

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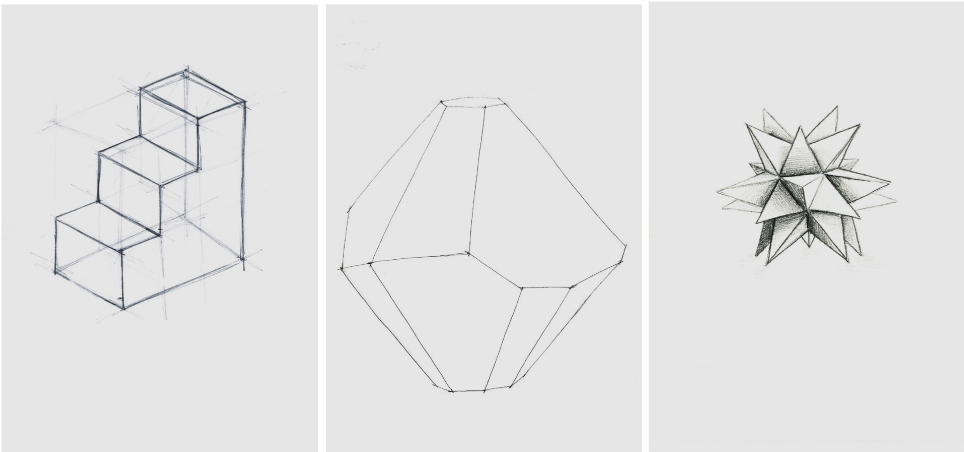

Fig. 3 - Exercises performed by one of the participating students, as an example of response to experiments 1, 2 and 3. The student used auxiliary lines to understand the structure of the object only in the drawing performed in experiment 1.

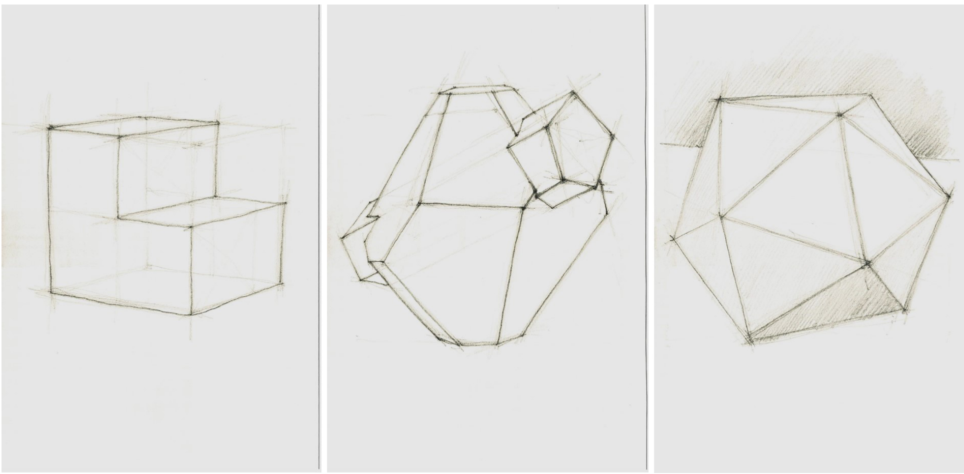

Fig. 4 - Exercises performed by one of the participating students, as an example of response to experiments 1, 2 and 3. In this case, the student was successful in representing all the forms requested.

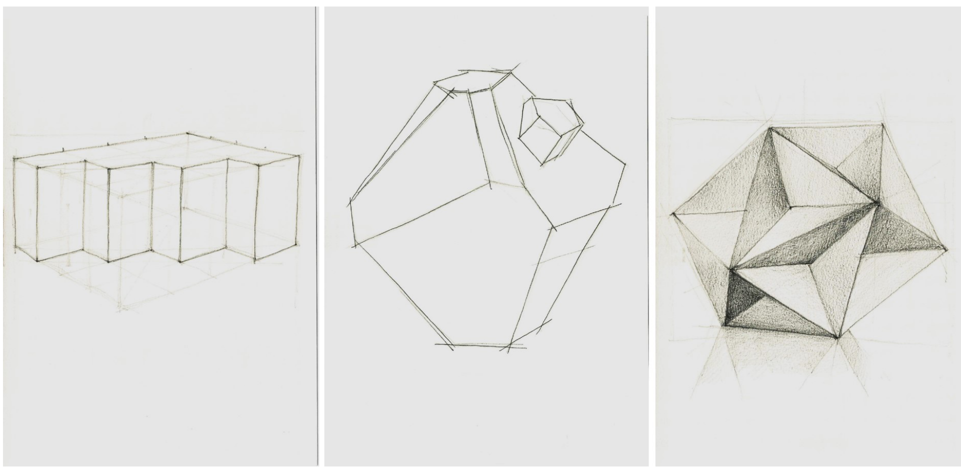

Fig. 5 - Exercises performed by one of the participating students, as an example of response to experiments 1, 2 and 3. The work of this student shows the tendency to value aspects related to light and shadow only in the drawing corresponding to the experiment 3.

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CONCLUSIONS

Concerning the occupation of the support and the mastering of the perspective, the results suggest that the formal characteristics of the represented objects do not influence the ability to fittingly occupy the support, or to control the perspective representation. It can be concluded that any type of solid used in the various experiments provides a good exercise, either for the adequacy of the scale of the drawing to the dimension of the support, or to reduce the tendency to emphasize the angles, regardless of the formal characteristics presented.

As for the ability of synthesis and the mastering of mode, the data collected indicate that irregular objects are those that, on the one hand, make it more difficult to lead an observer to the unequivocal identification of the objects represented and, on the other, provide the evolution of the image "from the general to the particular", according to hypothesis and correction tests. It follows that solids that do not have an intelligible relationship between the measurements of the sides or a clear or rational geometric structure inherent in their morphology, are less conducive to the development of these competences.

Also in the mastering of the form and proportions, it was observed that the difficulties are more accentuated when it comes to the representation of solids that present random measures, as in the case of those used in the experiment 2. The problems generally result from the sequential sum of parts and the difficulty of establishing relations to help control and understand the relationship between the whole and the parts.

Regular polyhedra, especially those with a high number of faces and greater formal complexity, gave rise to the differentiation of planes through light and dark notes, a feature that can be used as a motivation to view reality as variations of luminosity, providing an introduction to the theme of light and shadow.

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FCUP | PORTO | 05 . 06 . 07 SEPTEMBER 2019
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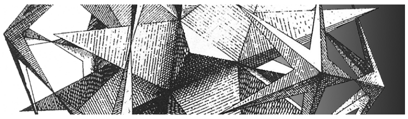

PLENARY SESSION
KEYNOTE SPEAKER

MICHAEL HANSMEYER

Michael Hansmeyer is an architect and programmer who explores the use of algorithms to generate and fabricate architectural form.

Recent work includes the design of two full-scale 3D printed sandstone grottos, the production of an iron and lace gazebo at the Gwangju Design Biennale, and the installation of a hall of columns at Grand Palais in Paris. He has exhibited at museums and venues including the Museum of Arts and Design New York, Palais de Tokyo, Martin Gropius Bau Berlin, and Design Miami / Basel. His work is part of the permanent collections of Centre Pompidou and FRAC Centre.

Recently, he taught architecture as visiting professor at the Academy of Fine Arts in Vienna and at Southeast University in Nanjing, and as a lecturer at the CAAD group of the Swiss Federal Institute of Technology (ETH) in Zurich.

He previously worked for Herzog & de Meuron architects, as well as in the consulting and financial industries at McKinsey and J.P. Morgan respectively. Michael holds a Master of Architecture degree from Columbia University and an MBA from INSEAD.

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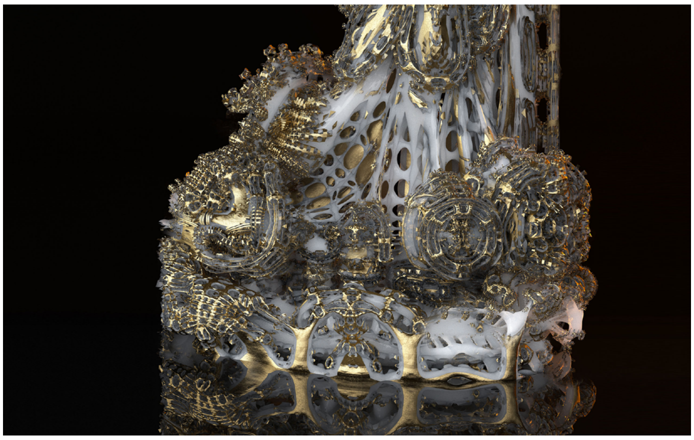
PLENARY SESSION
KEYNOTE SPEAKER

TOOLS OF IMAGINATION

Today, we can fabricate anything. Digital fabrication now functions at both the micro and macro scales, combining multiple materials, and using different materialization processes. Complexity and customization are no longer impediments in design.
While we can fabricate anything, design arguably appears confined by our instruments of design: we can only design what we can directly represent. If one looks at 3D-printed artifacts, there is oftentimes a discrepancy between the wonder of technology, and the conventionalism of design. It seems that our current parametric approaches operate within a too tightly prescribed scope. We appear unable to exploit the new freedom that digital fabrication offers us. In short: we can currently fabricate more than we can design.
What is needed is a new type of design instrument. We need tools for search and exploration, rather than simply control and execution. We need to redefine the relationship of the designer with the machine, thereby gaining entirely new spatial and haptic experiences and engaging design environments.

BEYOND THE PARAMETRIC

Traditionally, architects have designed primarily using plans and sections. The computer has allowed architects to transcend the plane and design directly in three dimensions. Recently, three-dimensional modeling software began to incorporate parametric design tools, and these are prevalent in many architecture schools and design practices today.
These parametric approaches enable an easier manipulation of free-form geometries, and facilitate working with repetitive yet variable geometries. They have brought us designs with complex curvatures and with highly varied façades.



Digital Grotesque II, 2017.

And yet, despite these formal freedoms, parametric modeling is essentially a constraint-based technique: every rule must be defined explicitly, and the model is established *a priori*. Parametric modeling works within a finite, prescribed, limited scope, rather constructing an open-ended solution space. As such, this approach implies thinking in terms of categories. Rather than allowing us to design any form that is conceivable - or even those that are still inconceivable - parametric modeling has led to designs that fall within fairly relatively range and are easily identifiable by their generative processes. These designs are often simply categorized as "parametricism".

TOWARDS AN AUGMENTED IMAGINATION

Yet the promise of today's digital technologies is that they allow us to escape existing categories. With these technologies, we no longer require descriptive words or labels, as we have the potential to create the previously unseen. Effectively, we can build tools to augment our imagination.
While largely amorphous, these new tools exhibit a primacy of the procedural over the parametric. For instance, we can conceive of simple procedures that continuously repeat a geometric operation to transform a basic volume into a highly articulated and elaborate form. By repeating this operation millions of times, the computer arrives at forms with minute details at the threshold of perception - algorithmic artifacts that would be almost impossible to draw by

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hand or traditional means. A simple and rational process can thus generate complex and seemingly irrational forms. This procedure can be rerun, using slightly different parameters, to produce countless permutations of a design. Each permutation has features in common with its siblings, while also having entirely unique traits. These permutations can be selected and crossed with each other to produce yet further variations, thereby successively developing the design.

While there is no randomness necessarily involved in these processes, there are so many individual steps in the procedure - so many possible interrelations - that the results are not entirely foreseeable. The computer obtains the power to surprise us. It generates forms within the space of what is possible that we would not likely have conceived of or found. Its output remains completely deterministic, while resisting both predictability and attempts at explanatory reductionism.

These tools redefine the process of design: the architect works in an iterative feedback loop with the machine, moderating processes, and incorporating feedback, surprises and proposals. Knowledge and experience are acquired through search, in a process that oftentimes resembles curation.

By designing in this manner, architects take the unusual step of ceding control over the exact nature of the forms they create. The computer, as a design partner, gains a degree of autonomy. The generated forms oftentimes manifest this tension of control and surprise. They are situated somewhere between chaos and order... both natural and artificial... neither foreign nor familiar.

Digital Grotesque II, 2017.

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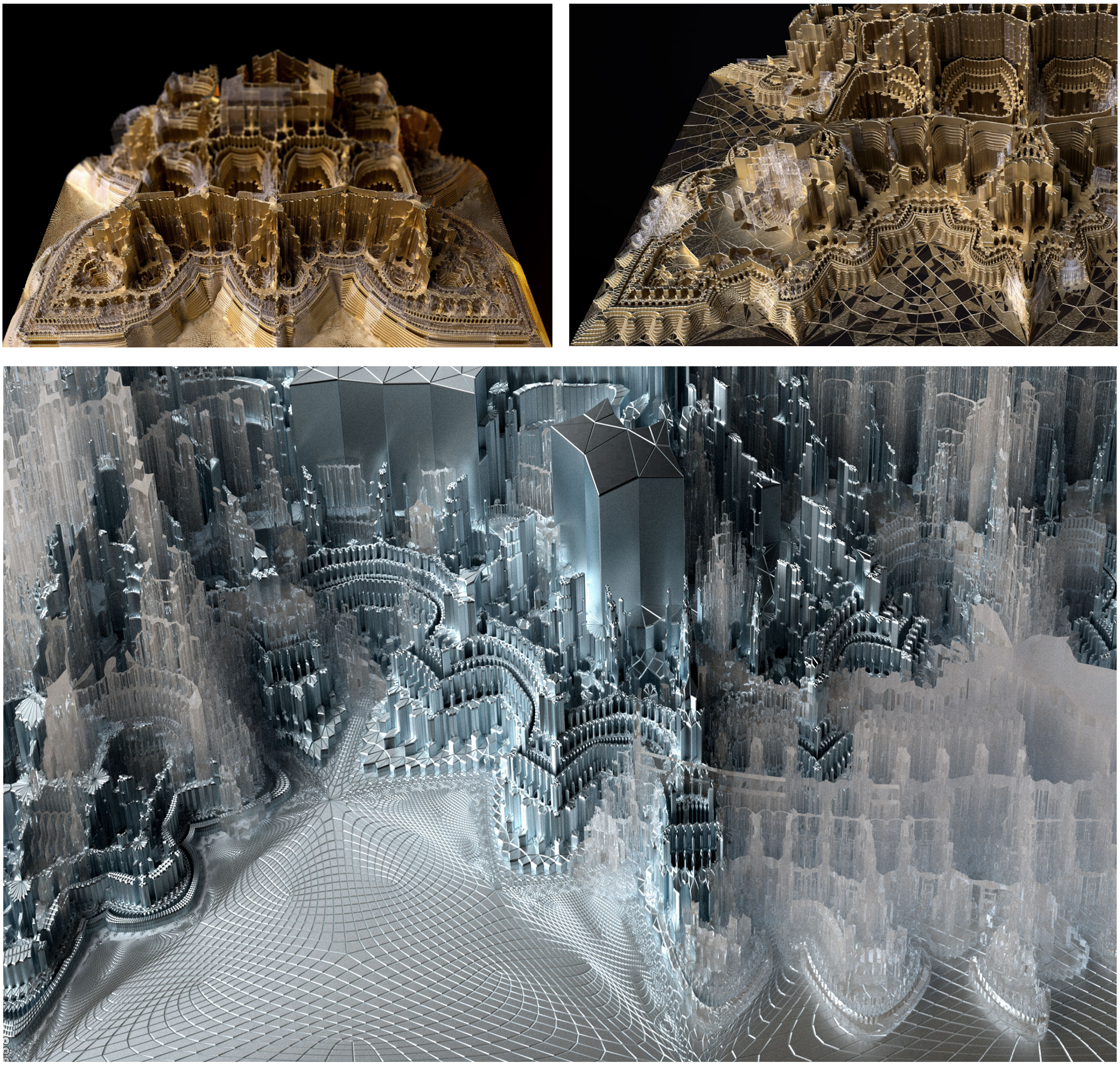
CELEBRATING THE UNFORESEEN

As of yet, we have countless tools to increase our efficiency and precision. Why not also create tools that serve as our muse, that inspire us and help us to be creative? Tools to draw the undrawable, and to imagine the unimaginable More than ever, seek an intuitive collaboration and productive dialog with the machine in the design process. We need to beyond a rational instruction and arrive at an intuitive interaction. It is Nicholas Negroponte, who in his book *The Architecture Machine* envisaged such a partnership:

> *"Let us build machines that can learn, can grope, and can fumble, machines that will be architectural partners, architecture machines."*

What we stand to gain are entirely new spatial and haptic experiences. A playful design that stimulates the senses, elicits curiosity, and invites interaction. A design environment that simultaneously allows control and surprise, and that embraces and celebrates the unforeseen.



Muqarna Studies, 2019.

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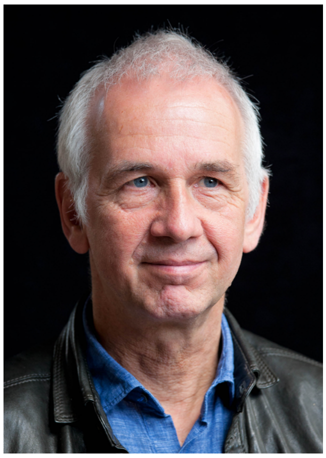
PLENARY SESSION
KEYNOTE SPEAKER

RINUS ROELOFS



Art about mathematics.

After studying mathematics for a couple of years (applied mathematics at the University of Twente, Enschede), I decided to switch to the school of arts. In 1983, I started my career as a sculptor. Inspired by the works of M.C. Escher and Leonardo da Vinci, my works became more and more some kind of expression of my mathematical ideas.

The main subject of my art is my fascination about mathematics - to be more precise: my fascination about mathematical structures. Mathematical structures can be found all around us. We can see them everywhere in our daily live. The use of these structures as visual decoration is so common, that we don't even see this as mathematics. But studying the properties of these structures and, especially, the relation between different structures can bring up questions. Questions that can be the start of interesting artistic explorations.

Artistic explorations of this kind mostly lead to intriguing designs of sculptural objects, which are then made in all kind of materials, like paper, wood, metal, acrylic, etc. It all starts with amazement, trying to understand what you see. Solving those questions often leads to new ideas, new designs.

Since I use the computer as my main sketchbook, these ideas come to reality, first, as a picture on the screen. From there, I can decide what the next step towards physical realization has to be. A rendered picture, an animation or a 3D physical model made by the use of CNC-milling, laser cutting or rapid prototyping. Many techniques can be used nowadays, as well as many different materials. But it is all based on my fascination about mathematical structures.

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PLENARY SESSION
KEYNOTE SPEAKER

HELICES IN UNIFORM POLYHEDRA

The way most people know about polyhedra is through physical experience (seeing, feeling) instead of theoretical understanding. To have a visual presentation of polyhedra seems to be necessary for the understanding of the objects in this field of mathematics. Maybe more important than the real three-dimensional models, are the two-dimensional pictures of these shapes. At some points in history, artists added new ideas for presenting polyhedra in such a way that certain properties could be explained. Leonardo da Vinci made clear drawings of the elevated polyhedra as described by Luca Pacioli; Albrecht Dürer explained in his drawings how polyhedra can be made from 2D plans; and M.C. Escher found nice ways to present the complex star-shaped polyhedra defined by Kepler. All these 2D representations were meant to show certain properties of the polyhedra.

In my presentation, I want to show how the helix can be used to explain the complex uniform polyhedra with, in most cases, intersecting faces. Starting with the Poinsot polyhedra, which I will show with the use of a method that I called "helixation". This is a new dynamic way to visualize the structure of a polyhedron, by showing how it can be generated. With this technique, we can also visualize things like duality.

In 2003, Branko Grünbaum described several ways to construct "new" uniform polyhedra. And again, there is a need for visualization of these new polyhedra. Also, in this case, the helix turned out to be a good help to come to a clear visualization. In some cases, I used the shape of a torus knot, which is, in fact, a helix around a circle, to make the visualization.

Maybe the most important part of the talk is the use of the helix to construct a complete new group of uniform polyhedra, which I called the helical star deltahedra.

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FILLING SPACE WITH GYROID SYMMETRY

Ulrich Reitebuch[1], Henriette-Sophie Lipschütz[2] and Konrad Polthier[3]

KEYWORDS: Space-Filling Solids, Minimal Surface, Symmetry.

INTRODUCTION

The gyroid is a triply periodic minimal surface, which belongs to the associate family of the Schwarz P and Schwarz D surface, and has several point reflection, rotational and translational symmetries. A discrete gyroid can be built from triangles - it is a discrete surface with the same symmetries as the smooth gyroid surface, and it is discrete minimal. Each, the gyroid and the discrete gyroid, split 3D space into two interlinked half spaces, which are symmetric to each other. We present a pair of solid building blocks, together filling space, each of them filling one of the half spaces created by the discrete gyroid.

MINIMAL SURFACES

In differential geometry, a surface that locally minimizes the surface area, is called a minimal surface. Many minimal surfaces are highly symmetric, having rotational, translational, mirror reflection, and point reflection symmetries.

DISCRETE MINIMAL SURFACE

A discrete surface consists of vertices, straight edges and planar polygonal faces. We call a discrete surface "minimal", if the surface area cannot be decreased by moving a single interior vertex, keeping all the other vertices fixed [01].

THE GYROID SURFACE

The gyroid is a triply periodic minimal surface, which belongs to the associate family of the Schwarz P and Schwarz D surface. It was found by Alan Schoen [02] and proved to be embedded in 3D Karsten Große-Brauckmann and Meinhard Wohlgemuth [03]. The gyroid has several point reflection, rotational and translational symmetries but, in contrast to the Schwarz P and Schwarz D surface, it does not have any mirror reflection symmetries.
The gyroid splits 3D space into two halves, which are mirror symmetric to each other.
A discrete gyorid surface can be constructed from triangles such that it is a discrete minimal surface and has exactly the same symmetries as the smooth gyroid surface [04]. Translational units of the smooth and discrete gyroid surface are shown in Fig. 1.

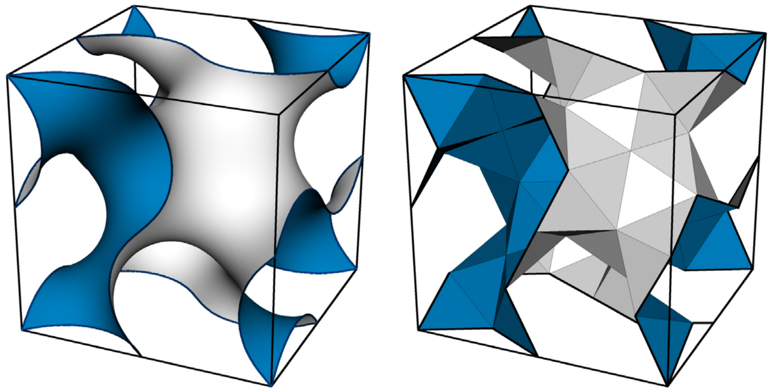

Fig. 1 - Translational unit of Smooth and discrete gyroid surface.

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SPACE-FILLING SOLIDS

We present a pair of solids, which together fill 3D space. We start with a truncated octahedron.We select a pair of opposite hexagons, and then select a set of short diagonals in the remaining six hexagons such, that none of these has a common vertex with the first two hexagons and such, that the six diagonals form a cycle. There are two possible choices of short diagonals fulfilling these conditions.

Now we connect each of these six diagonals to the centre of gravity of the truncated octahedron, building six triangles. The surface given by these six triangles cuts the truncated octahedron into two solids. Each of the two solids is bounded by the six cut triangles, one complete original hexagon and three complete squares from the truncated octahedron, and three big and three small parts of hexagons, cut by the selected short diagonals.

Neither of the two solids has any mirror symmetry, but they are symmetric to each other. The cut triangles are shown in Fig. 2 as white triangles, the remaining surface of the truncated octahedron is shown in blue.

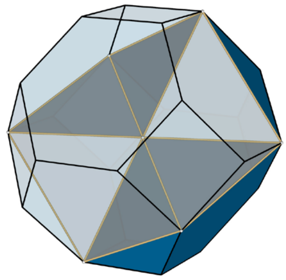

Fig. 2 - Cutting the truncated octahedron into two solids.

FILLING HALF SPACE

Using copies of just one of two solids, half of 3D space can be filled. Since all parts of the original faces of the truncated octahedron have a mirror symmetry, the solids can be attached to each other at these faces in different ways, as shown in Fig. 3: attaching small (triangular) parts of hexagons, attaching squares, attaching big (pentagonal) parts of hexagons or attaching complete hexagons. When attaching solids at square faces, there are four possibilities, for hexagons there are two possibilities up to symmetry. Here, we have to take care that all edges show combinations of faces, where a third copy of the solid can be attached to both solids. For the square, the vertex at the cut-triangle surface has to be aligned; for the hexagon, the squares of the two solids cannot share an edge. For the triangular and pentagonal faces at the original faces of the truncated octahedron, there is only one possible way of attaching a neighbouring solid face to face.

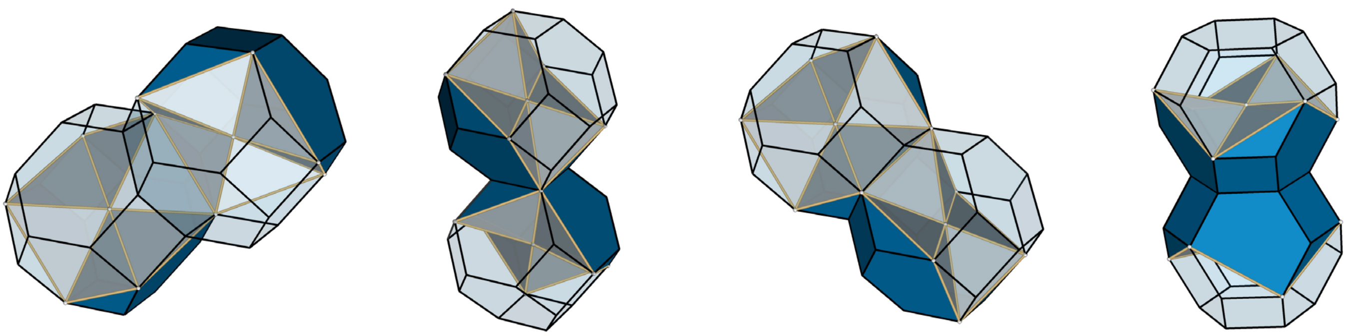

Fig. 3 - Four ways of attaching solids: At triangles, squares, pentagons and hexagons.

By these gluing rules, solids of the same type can be glued to all the faces that are not incident to the point at the centre of gravity of the complete octahedron, such that only these cut triangles remain unglued. In this way, the cut triangles form a discrete gyroid surface and one type of solid fills one of the two half-spaces bounded by the discrete gyroid; the other type fills the other half-space. A set of glued solids is shown in Fig. 4: the blue faces of the solids are glued and the white faces build the discrete gyroid surface.

CUTTING AT SMOOTH GYROID SURFACE

The same construction can be done with a curved cut surface through the truncated octahedron: if the truncated octahedron is placed with its centre of gravity in one of the points with point reflection symmetry of the smooth gyroid surface, the gyroid surface cuts the truncated octahedron into two parts that do not have any mirror symmetry, but are symmetric to each other. If the truncated octahedron has the correct scaling in relation to the gyroid surface,

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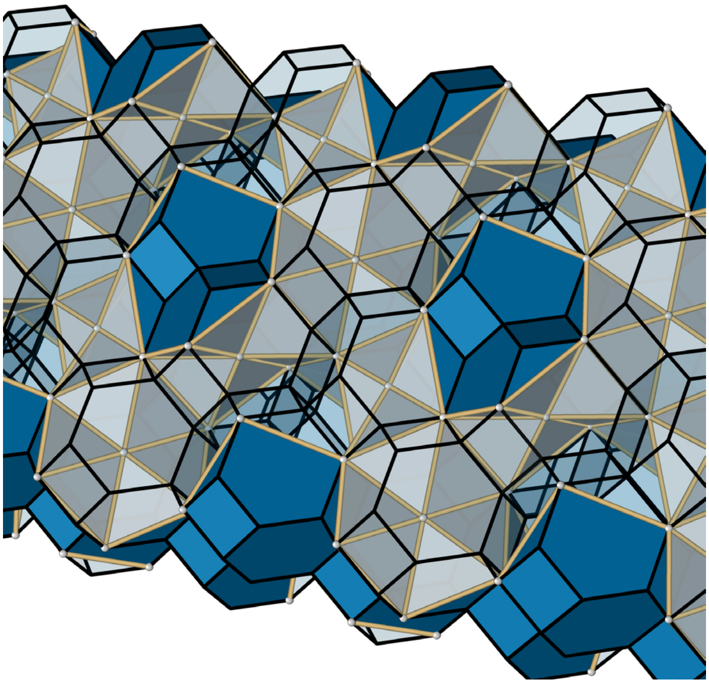
some of the gyroid's axes of 180° rotational symmetry coincide with face diagonals of the truncated octahedron, and the solids can be used in the same way to fill one half of 3D space, building a smooth gyroid surface at the volume boundary. The other solid type fills the remaining half-space. One solid with the smooth gyroid cut is shown in Fig. 5.



Fi. 4 - Gluing solids fills half the space;
the white surface becomes a discrete gyroid.

CONCLUSIONS

The truncated octahedron is a space-filling solid; if a tesselation of 3D space by truncated octahedra is scaled and positioned appropriately in relation to a discrete gyroid surface, the latter splits each of the truncated octahedra in the same way into two parts, which are symmetric to each other. Thus, each of the half-spaces bounded by the discrete gyroid surface is tesselated by one of the two half truncated octahedron solids.

If the same tesselation of 3D space by truncated octahedra is cut by the smooth gyroid surface, it also cuts each truncated octahedron in the same way into two solids, which are symmetric to each other. In this case, the cut surface is curved, and each of the two types of solids tesselates one of the half-space bounded by the smooth gyroid surface.

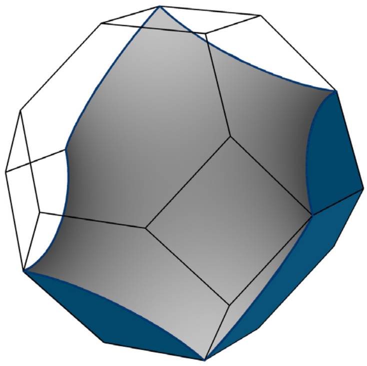


Fig. 5 - The smooth gyroid splits the truncated octahedron into two solids.

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PAPERS' SESSION
PAPER 06

CONCAVE DELTAHEDRAL RINGS BASED ON THE GEOMETRY OF THE CONCAVE ANTIPRISMS OF THE SECOND SORT

Marija Obradović[1] and Slobodan Mišić[2]

KEYWORDS: Antiprism, Deltahedron, Concave, Ring.

INTRODUCTION

Concave antiprisms of the second sort (*CA II*) are polyhedra which, similar to convex antiprisms, consist of two identical regular polygons connected by a deltahedral lateral surface. The lateral surface is created by 2π polar array of the spatial hexahedra composed of six equilateral triangles arranged around the common vertex. The term "second sort" originates from the fact that these solids have two rows of equilateral triangles in the lateral surface. There is an infinite number of *CA II-n* representatives, some of which (with base polygons of $n \in \{3, 4, 5...11\}$) are given in Fig. 1.

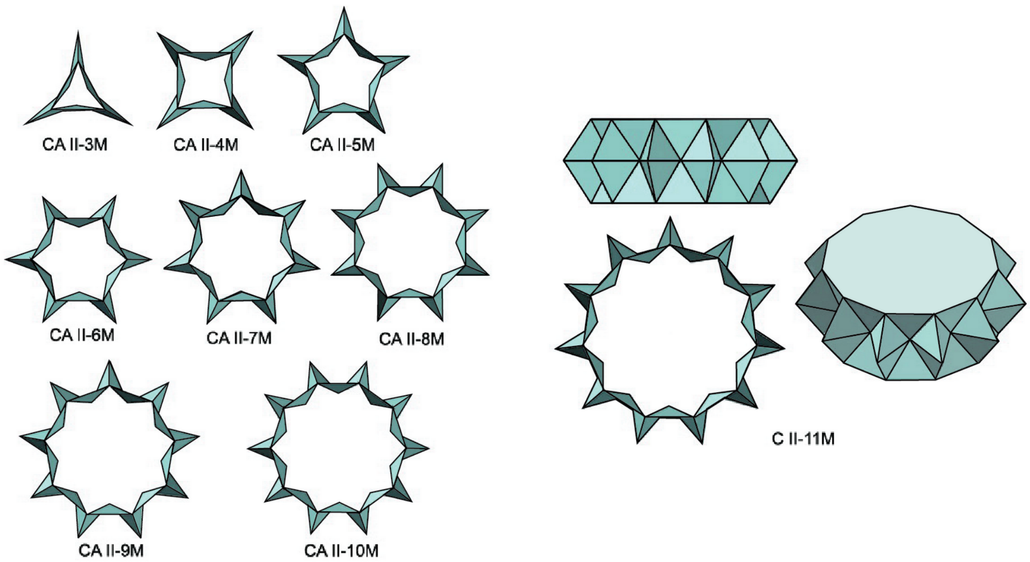

Fig. 1- Some representatives of CA II-*n*M.

A detailed description of the genesis of *CA II*, together with their geometrical properties is given in [01].
In this abstract, we search for deltahedral rings formed by a polar array of *CA II*'s fragments, thereby keeping the crucial linear and angular measurements of the initial solids. Our goal is to obtain forms of a high symmetry level, derived from a single element - an equilateral triangle.

INVESTIGATING THE POSSIBILITY OF DELTAEDAR RINGS' FORMATION

When forming the *CA II-n*, we start from the planar net of the spatial hexahedron - the regular hexagon subdivided into equilateral triangles. Depending on the way the net is folded, similarly to the other polyhedra of the second

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sort (*CC II*: [02]; *CP II*: [03, 04]), it is possible to form two types of the lateral surface. If the central vertex (*G*) of the spatial hexahedron is protruding, we obtain a type with a lower height of the lateral deltahedral surface, consequently termed minor, abbreviated as *CA II-nM*. If the central vertex (*G*) is indented, we obtain the type with a greater height, termed major, abbreviated as *CA II-nM*, which is what we focus on in this abstract (Fig. 1).

The key procedure for ring formation is to bring the middlemost pair of equilateral triangles from the spatial hexahedral cells of the two adjacent *CA II-nM* into the overlapping position (Fig. 2a). The planar symmetry of the spatial hexahedron itself makes this possible. In this way, we get a new cell, a spatial decahedron (Fig. 2b), which is a building block for a new deltahedral structure. Thereby, it takes over the measurements (points' heights, angles) from the initial *CA II-nM*. Then, we examine which multilateral reflection of this decahedral cell (Fig. 2c) produces a closed deltahedral ring - consequently concave and of the second sort (abbreviated: *CDR II*) - forming a full circle with no partial overlaps or gaps. The final result, therefore, comes down to the polar distribution of these cells, with *K* concave antiprisms in the array, where *K* is an integer.

The spatial decahedral cell can be positioned so that the space it surrounds is oriented towards the interior of CDR II, forming a flower-like ring, which we call the case *A*, or towards its exterior, forming a star-like ring, called the case *B*. In both cases, these decahedral cells can be combined with one or more hexahedral cells - the fragments of *CAII-nM* - between them. Given that the two different units are now involved in the ring's construction, such cases of CDR II are denoted by A_f and B_f.

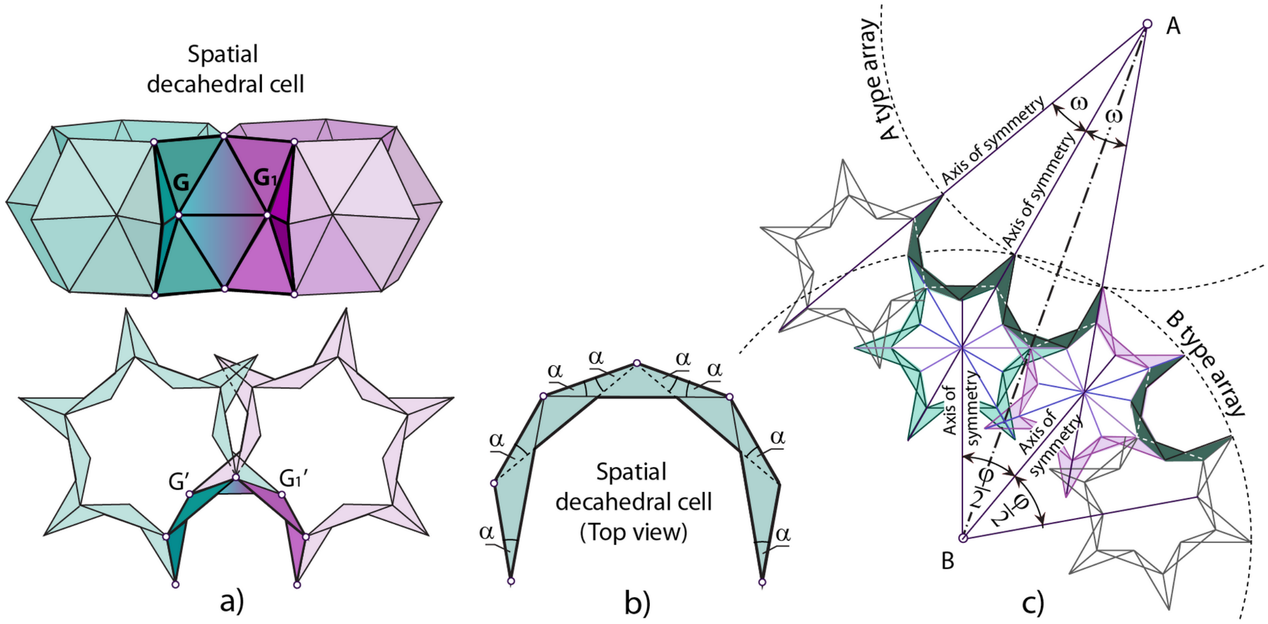

Fig. 2 - a, b) The formation of the spatial decahedral cell, c) Arraying the cells into a ring.

The *CA II-nM* themselves have a high level of symmetry. They are characterized by multilateral reflexive and rotational symmetry, while in the cases with even number of base sides ($n = 2t$) we also encounter point symmetry. Each base polygon of *CA II-n* has *n* axes of symmetry. To solve the task, we take two paired *CA II-nM* with a common decagonal cell and apply successive reflexive symmetries across the corresponding symmetry axes of their polygons (Fig. 2c) until we close the full circle. The solutions differ depending on the selected axes of reflection. As valid, we adopt only those with an integer number *K* of the cells in the circle.

Each chosen pair of the symmetry axes intersects in the same point (*A*, *B*,...) and defines the angle of the polar array between them. The angle determines the number of petals in the case A, or the number of star-points in the case B. Instead of testing each individual axis, we use a mathematical calculation based on the trigonometry of the paired *CA II-nM*'s orthogonal projections. We found formulae that define:

APROGED

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- base angles in the isosceles triangles - orthogonal projections of the *CA II-nM*'s faces, denoted by α;
- the angles between the paired reflection axes of the *CA II-nMs*, which provide the desired solutions, denoted by φ and ω.

$$\alpha = \pi \ \frac{n-2}{6n} \quad (1)$$

$$\varphi = \pi \ \frac{n-2}{n-2\alpha} \quad (2)$$

$$\omega = |10\alpha - \alpha| \quad (3)$$

Once the angles φ and ω have been calculated, we determine which number k, as their multiplier, produces a full circle, creating either case A or B. If k is not an integer, we search for the solution by additionally multiplying it by a minimal integer j that results in the integer K. The value of the variable j depends on the factual axes of symmetry that define the angles φ or ω. The cases with $j > 1$ will be those that we have denoted by A_f and B_f.
An overview of these values for the observed $n \in \{3, 4, 5 \ldots 11\}$ is given in the Table 1.

Table 1 - The number of petals / star-points in the CDR II depending on the *n* of *CA II-nM*.

n	α	φ	ω	$k_{A,B} = 2\pi/\varphi$		$. j =$	$K_{A,B}$	$k_{B,A} = 2\pi/\omega$		$. j =$	$K_{B,A}$
3	10°	40°	80°	$\mathbf{A_a}$	9	1	**9**	B	4.5	2	**9**
4	15°	60°	30°	$\mathbf{A_a}$	6	1	**6**	B	12	1	**12**
5	18°	72°	0°	$\mathbf{A_a}$	5	1	**5**	A = B	∞	1	∞
5	18°	72°	0°	B_f	5	1	**5**	A = B	∞	1	∞
6	20°	80°	20°	B	4.5	2	**9**	**A**	18	1	**18**
7	21.428°	85.715°	34.286°	B_f	4.2	5	**21 (3)**	$\mathbf{A_f}$	10.5	/	10.5
8	22.5°	90°	45°	$\mathbf{A_1}$	4	1	**4**	$\mathbf{A_2}$	8	1	**8**
8	22.5°	90°	45°	B_f	4	1	**4**	B_f	8	1	**8**
9	23.333°	93.333°	53.333°	$\mathbf{A_f}$	3.857	7	**27**	B_f	6.75	2	13.5
10	24°	96°	60°	B_f	3.75	8	**30 (3)**	**A**	6	1	**6**
11	24.545°	98.182°	65.454°	B_f	3.667	3	**11**	$\mathbf{A_f}$	5.5	2	**11**

To get the true deltahedral forms, we remove all the redundant faces, such as the ones that penetrate each other or do not participate in the formation of the ring.
To experimentally verify the research, we performed 3D models of the *CDR II* using the *AutoCAD* application.

FINDINGS OF THE RESEARCH

For the *CA II-nM* with $n \in \{3, 4, 5\}$, except for the case with a single congruent pair of triangular faces overlapping in the adjacent hexahedral cells (as in the formation of the decahedral cell, Fig. 1), there is also a complete overlap between two pairs of triangles, the plane symmetrical ones within the same spatial hexahedral cell (Fig. 3a).
Consequently, what results from such a match is the initial spatial hexahedral cell connected by a pair of equilateral triangles (Fig. 3b), instead of the decahedral one. In the polar arrangement, such cells produce regular double antiprisms: the case A_a (Table 1).
Still, by using the decahedral cells, it is also possible to form a star-like deltahedral ring as a case B (see Table 1 and Fig. 4).

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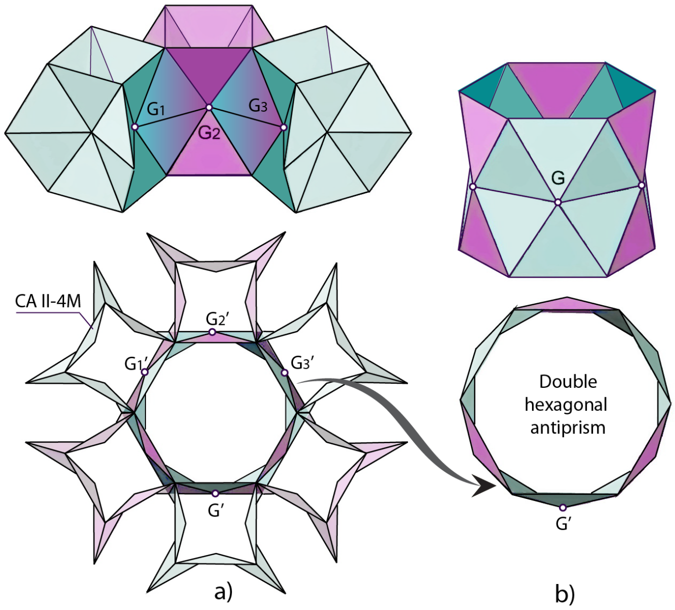

Fig. 3 - Formation of double antiprism (with *CA II-4M*) – case A_a.

When it comes to the *CA II-nMs* with bases of $n \geq 6$, the rule that a convex antiprism creates the common "core" will no longer be valid. Here, one can see a pattern of geometric limitation similar to the one observed in the formation of convex regular-faced pyramids or cupolae, i.e. Johnson solids J1 to J5 [05], where polygons with a number of sides $n \geq 6$ cannot participate in the formation of such solids' structure. However, although convex antiprisms are no longer commonly found as a case A_a of *CDR II-n*, the deltahedral rings, both of A and B variants still can be formed (Fig. 4).

Additionally, besides the results presented in Table 1, other interesting and highly symmetrical 3D forms can be obtained. It has been noticed that trilateral symmetry appears not only in the array of the *CA II-nM* fragments with n divisible by 3 ($n = 3t$), but also in the array of those with $n = 4$, $n = 7$, $n = 10$ ($n = 3t + 1$). For example, for such bases, we have discovered trefoil deltahedral rings (Fig. 4). On the other hand, for $n = 3t + 2$ bases, we encounter infinite linear series of *CA II-nMs*, both in x, y and z direction. This is closely related to Euclidean tilings with star-polygons [06].

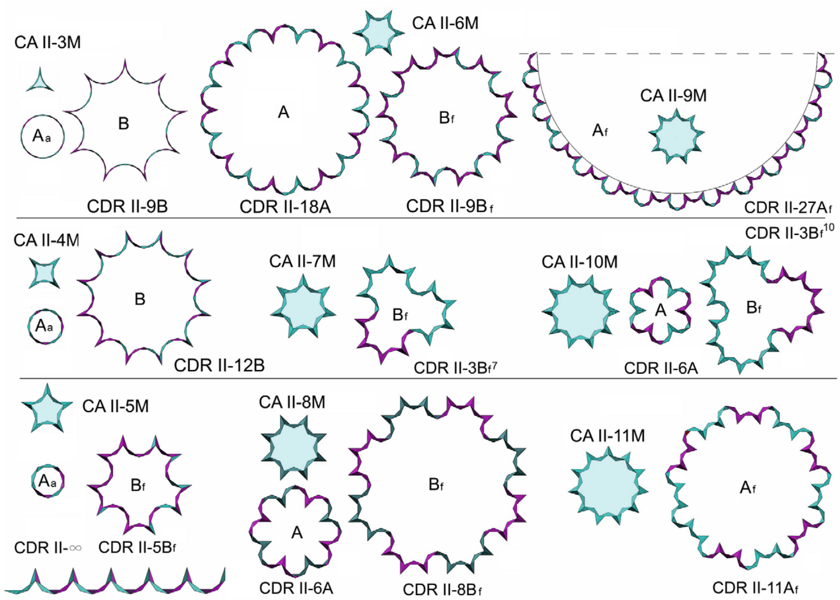

Fig. 4 - Representatives of *CDR II* obtained by fragments of *CA II-nM*, $n \in \{3, 4, 5 \ldots 11\}$.

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The most interesting results in this regard are obtained with *CA II-8M*. Here, the cases A and B are found within the same structure, for the same axes of symmetry and the same angle (φ or ω). Hence, two different solutions occur in both cases A and B: those with 4 and those with 8 *CA II-8Ms* in the ring. In this manner we obtain four different, concentric solutions: two flower-like and two star-like deltahedral rings.
In the case involving 8 *CA II-8M*s, another (double) *CA II-8M* can be inserted inside the *CDR II-8A*, playing the "core" role which convex antiprisms had in the examples of *CA II* with $n \in \{3, 4, 5\}$. Within this structure, we can identify 8 double square antiprisms surrounding the central double *CA II-8M*. The obtained composition can be arrayed in space infinitely, creating a 3D tessellation.

CONCLUSIONS

With this abstract, we have shown that:

- there is a link between the geometry of the *CA II-nM* with bases $n \in \{3, 4, 5\}$ and that of the convex antiprisms with the same bases;
- an integer number (*K*) of *CA II-nM*'s fragments, can be used to form a full multilaterally symmetrical ring of concave deltahedral surfaces, either flower-like (case A) or star-like (case B);
- the obtained rings can also be termed "of the second sort" (denoted by *CDR II-n*) as they inherit from the given *CA II-nM* the following: a) the linear and angular measurements needed for their graphic and mathematical elaboration, b) two rows of equilateral triangles in the lateral surface, and c) the high level of symmetry.

The possible formation of *CDR II-n* with the highest level of symmetry (i.e. excluding the cases A_f and B_f), and with the number of petals/star-points of any integer $K \geq 2$ can be a subject of further research.

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PAPERS' SESSION
PAPER 07

DISSECTION OF CUBES AND GOLDEN RHOMBIC SOLIDS

Izidor Hafner[1] and Mateja Budin[2]

KEYWORDS: Golden Section, Acute Golden Rhombohedron, Obtuse Golden Rhombohedron, Rhombic Triacontahedron, Dissection.

INTRODUCTION

Two combinations of polyhedra are equidecomposable if they have equal volumes and equal Dehn invariant. If Dehn invariant of a combination is 0, then the combination is equidecomposable with a cube of the same volume. Golden rhombic solids have Dehn invariant 0, so they are equidecomposable with cubes. A decomposition is simple if the number of its parts is small. There are only five convex golden polyhedra: the obtuse golden rhombohedron (*O*), the acute golden rhombohedron (*A*), the rhombic dodecahedron of the second kind (*B*, Bilinski's dodecahedron), the rhombic icosahedron (*I*), and the rhombic triacontahedron (*T*) [01:156, 02:339]. Investigation of golden rhombic solids has shown that, if a decomposition is simple, then the corresponding formula for the equality of volumes is simple as well. The following decompositions were known: $B = 2O + 2A$, $I = 5O + 5A$ [03, 04], $T = 10O + 10A$ [05, 06]. Recently G. Theobald has found a 29 piece dissection of *T* into a cube [07].

RESEARCH

Let a be a positive number. Then $(a + 1)^2 = a^2 + 2a + 1$, so a square of side $a + 1$ can be dissected into squares of sides a and 1 and two rectangles $1 \times a$, which can be dissected into triangles and reassembled into a rhombus. Fig. 1 shows a case when $a = \sigma$, where σ is the golden ratio, and we have a *golden* rhombus.

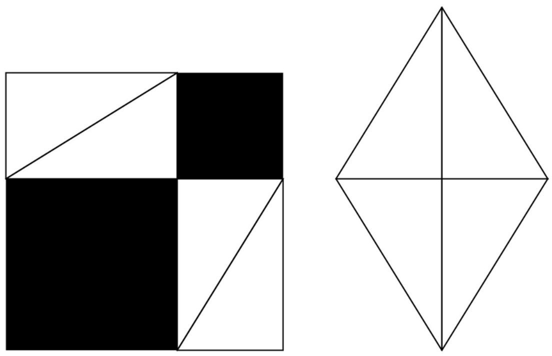

Fig. 1

The special Fibonacci sequence 1, σ, $1 + \sigma$, $1 + 2\sigma$, $2 + 3\sigma$, $3 + 5\sigma$, $5 + 8\sigma$, $8 + 13\sigma$, $13 + 21\sigma$, ... is also a geometric sequence 1, σ, σ^2, σ^3, σ^4, σ^5, σ^6, σ^7, σ^8, σ^9,...

Let the diagonals of faces of a golden solid be $d_1 = 2$, $d_2 = 2\sigma$.

Then the volume of the obtuse golden rhombohedron is $O = (d_1 d_2)/2\ (d_1/2) = 2\sigma$. The volume of the corresponding acute golden rhombohedron is $A = (d_1 d_2)/2\ (d_2/2) = 2\sigma^2 = 2\ (1 + \sigma)$. Since $B = 2O + 2A = 4\sigma + 4\sigma^2 = 4\sigma^3$, there is a simple dissection of a quarter of *B* into a cube of edge $d_2/2$. Fig. 2 shows this 4-piece dissection [08]. But a quarter of *B* can be dissected into $1/2O$ and $1/2A$ by a 2-piece dissection. Superposing these two dissections we get a 6-piece dissection of $1/2O$ and $1/2A$ into a cube of edge $d_2/2$. Fig. 3 shows this dissection, which is a geometric realization of the identity $\sigma + \sigma^2 = \sigma^3$. Note that the known geometric realization of $x^3 + y^3 = z^3$ (dissection of a cube into two cubes) needs at least 10 pieces [09: 239].

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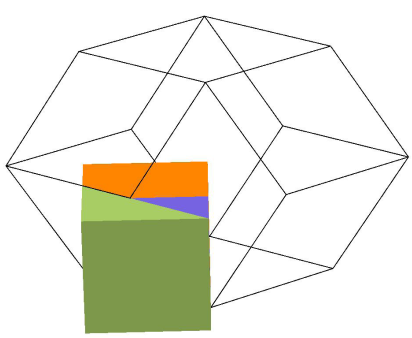

Fig. 2

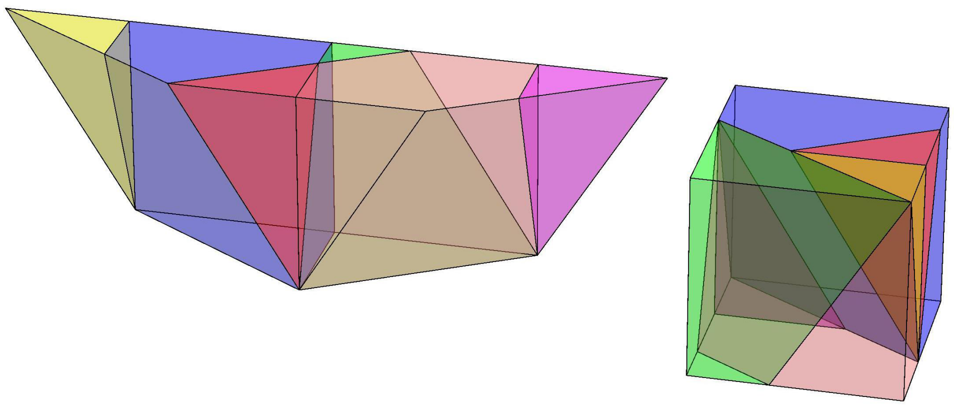

Fig. 3

The volume of $2B$ is $16\sigma + 8 = (2\sigma)^3$, and there is a simple dissection of $2B$ into a cube of the edge length d_2.
$(1 + \sigma)^3 = 8\sigma + 4 + 1$, and there is a dissection of a cube with edge length $(d_1 + d_2)/2$ into B and a small cube with edge $d_1/2 = 1$.
$(d_1 + d_2)^3 = 64\sigma + 40 = 6B + 8A$, and there is a dissection of a cube with edge lenght $d_1 + d_2$ into six B and eight A [02].
The volume of the rhombic triacontahedron is $T = 5B = 40\sigma + 20$.
The volume of the concave rhombic triacontahedron is $CT = 3B + 8 = 24\sigma + 20$.
So $T + CT = 64\sigma + 20 = (d_1+d_2)^3$, and there is a simple dissection of combination of triacontahedra together into a cube [02]. The following identies are also valid (Fig. 4):

$$(1 + \sigma)^3 = 8\sigma + 5 = 2\sigma + 2 + 3\sigma^3 = O + 1 + 1 + 3\,(d_2/2)^3,$$

$$(1 + \sigma)^3 = 8\sigma + 5 = 2\,(\sigma + 1) + 3\sigma^3 = A + 3\,(d_2/2)^3.$$

So there is a simple dissection of a cube of edge $(d_1+d_2)/2$ into A and three cubes of edge lenght $d_2/2$.

$$(1 + \sigma)^3 = \sigma^3 + 3\sigma^2 + 3\sigma + 1 = \sigma^3 + 3\sigma\,(\sigma + 1) + 1 = 4\sigma^3 + 1.$$

So there is a simple dissection of a cube of edge $1 + \sigma$ into four cubes of edge σ and one with edge 1.
The left part of Fig. 4 shows a (*Zometool*) dissection of a cube into an A and six eighths of B. The right part shows a dissection of a cube into an O, six eighths of B and two small cubes.
Numbers 1, σ, $1 + \sigma$ are algebraic integers in $Q\,(\sqrt{5})$, which is an extension of the field of rational numbers by adjoining $\sqrt{5}$. Norm of σ is $(1 + \sigma)\,(1 - \sigma)/4 = -1$. So σ is a unit in this field.
The equation $z^3 = 4y^3 + x^3$ has no solution in integers (if z, y and x are different from 0). But it has such solution in integers in $Q\,(\sqrt{5})$.
We know that a combination of polyhedra with Dehn invariant 0 is equidecomposable with a cube of equal volume. But is there a simple dissection of the combination into the cube?

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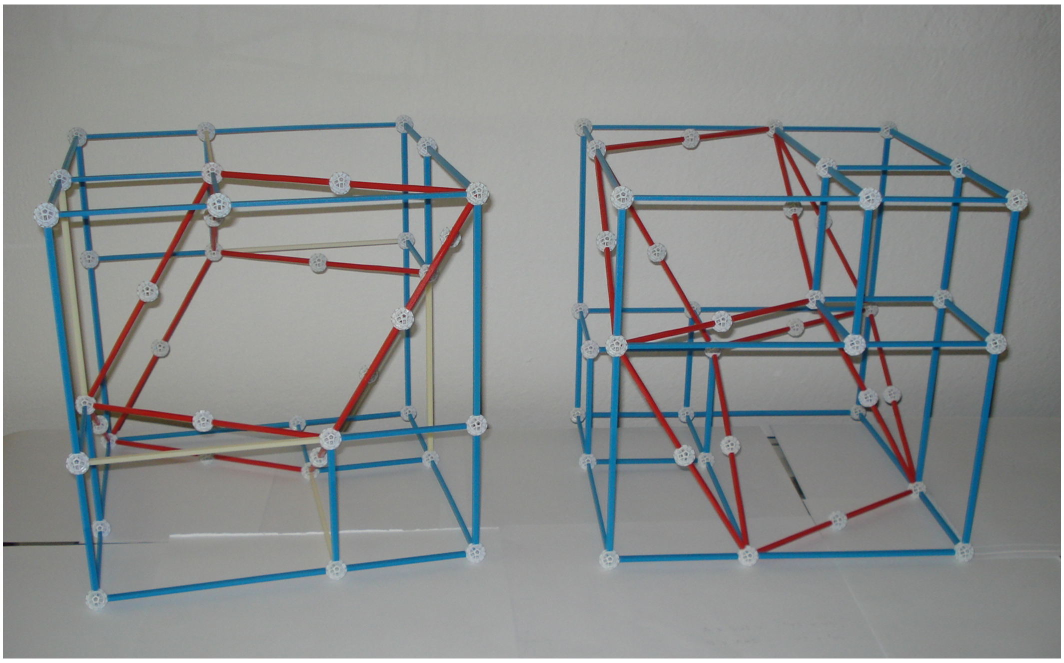

Fig. 4

CONCLUSIONS

If there is a simple dissection of two combinations of polyhedra, there is a simple formula for equality of their volumes. The opposite statement is doubtful. What about 4D? How does a dissection for formula $(\sigma + 1)^4 + 8 = (\sigma - 1)^4 + 8\sigma^4$ look like?

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PAPERS' SESSION
PAPER 08

EQUIFACED SIMPLICES AND POLYTOPES

Günter Weiss[1]

KEYWORDS: Simplex, Polytope, Cross-Polytope, Equifaced Polyhedron, Duality, Mongepoint, *Miura-Ori*.

INTRODUCTION

The concepts "simplex" resp. "polytope" generalize the concepts "triangle" and "tetrahedron" resp. "polygon" and "polyhedron" to higher dimensions. While for triangles, advanced elementary geometry provides a huge amount of research results (e.g. [01]), the research viewpoints for simplices and polytopes focus on the objects themselves, their classification and visualisation [02], and only a few references deal with what could be called "remarkable points and lines" of those objects in an n-dimensional (Euclidean-) space. Most references concerning "Solid Geometry" deal with objects in a three-dimensional Euclidean space, as in, for example, [03], [04] and [05].

In the following, we deal with some properties of low-dimensional cases of simplices and polytopes, which are generalisations of equilateral triangles and rhombuses resp. tetrahedrons and octahedrons. Some of these properties are already well-known. When they are presented here, too, it is, because we use synthetic proofs, often based on classical descriptive geometry, instead of analytical ones.

RESEARCH

Properties of polyhedrons or polytopes are either of projective geometric or of affine nature or their proper place of action is a metric space. An example for a rather projective geometric property is a pair of two perspective k-simplices, which reveal additional "remarkable points", c.f. [06]. Such perspective simplices can be interpreted as perspective projections of cross-polytopes in a higher-dimensional projective space. Thereby, the coordinate field can be chosen arbitrarily with the only restriction that its characteristic is unequal to 2 [06]. Examples of affine geometric properties are e.g. affine regularity and the centroid of a convex polytope (understood as a set of vertices or as a solid), while the centroids of lower dimensional "skins" of a polytope, e.g. its edges or 2-faces, are metric properties.

The concept "equifaced" is a matter of the considered metric. It might mean the equal content of d-faces of an n-polytope as well as congruent d-faced polytopes, ($1 \leq d < n$), whereby the congruence can be considered with respect to the Euclidean or to non-Euclidean geometries. Even the (Euclidean) similarity group might be allowed to generate polyhedrons, which are "equifaced" in the sense of having similar faces. There are examples of even closed and convex polyhedrons with similar triangles as faces, but it remains an open question, whether there exist polyhedrons with similar quadrangular faces or not.

Obviously, Euclidean regular polytopes (see e.g. [07]) are equifaced. Equifaced tetrahedrons are already mentioned in [08]. Starting with a (non-isosceles) triangle one can ask e.g., how many octahedrons can be built with copies of that triangle. One also might connect them with the space-filling set of problems.

Another (Euclidean or non-Euclidean) metric property of a polytope worthy to be mentioned are their different sets of altitudes: e.g. the set of "vertex altitudes" of a tetrahedron consists, in general, of four generators of a special regulus [09], while the set of "edge altidudes" of a tetrahedron consists of three concurrent segments [10]. It is also possible to generalize the concept "M-altitude" (midpoint altitude [08] for complete plane quadrangles to some special polyhedrons and polytopes. Here the concept "midpoint of a d-face" splits up to e.g. the centroid, the

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circumcentre or the incentre, if they exist, or even the Monge point. While for a quadrangle inscribed to a circle, the six M-altitudes are concurrent, they form again a complete quadrangle for a generic quadrangle. The circumcentres of the partial triangles of the original quadrangle form a quadrangle, which is in perspective position with the M-altitude quadrangle, thus defining a "quadrangle centre". Therewith, the question arises, if there exist analogues in higher dimensions, too, and what happens, if one considers iterations of the constructions.

With a general triangle, one connects the well-known Euler-line as a basic "remarkable object". For generic n-simplices, too, an Euler-line is defined containing the centroid, the centre of the circum-sphere, and the Monge point of the n-simplex [03] for the 3D-case. It suggests itself to consider the sets of Euler-lines of the $(n - d)$-faces of an n-simplex or of an n-polytope with $(n - 1)$-simplices as hyperfaces.

The concept "polytope" is neither restricted to the property of being closed nor of being convex. For example, one can generate equifaced polyhedrons starting with an arbitrary equilateral (planar) quadrangle receiving finite or infinite structures, which have, and already had, some applicability for architecture and civil engineering. Here, "foldable structures", i.e. extensions of the concept *origami* and especially of the *miura-ori* should be mentioned, as well as movable structures as those of the octahedrons of Bricard [11], [12] and [13] and the polyhedrons of Kokotsakis type [14].

Starting with a closed convex equifaced polyhedron, one can construct further polyhedrons, by intersecting the outer symmetry planes of adjacent face polygons. For example, in the case of a cube as start polyhedron, the first resulting polyhedron is the rhombic dodecahedron, which obviously has congruent faces. But this is not the only equifaced closed dodecahedron: the Bilinski dodecahedron is another one [15]. The construction of using the outer symmetry planes of two adjacent faces might be called "edge duality" as an analogue to the usual "duality", which we, in this connection, had to call "vertex duality", as it makes use of the symmetry planes of three adjacent faces. Obviously, this way of defining dualities can be extended to higher dimensions and, of course, also be iterated.

CONCLUSIONS

The aim is to show that there still is a wide range of open questions and viewpoints, in spite the many results presented e.g. in actual Wikipedia entries. Even so, the mentioned and partly new considerations are more or less restricted to equifaced polyhedrons and polytopes, so, this lecture will present not more than a review over the different topics.

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TOPOLOGICAL INTERLOCKING OF PLATONIC BODIES: GEOMETRY ENABLING THE DESIGN OF NEW MATERIALS AND STRUCTURES

A.J. Kanel-Belov[1], A.V. Dyskin[2], E. Pasternak[3] and Y. Estrin[4]

KEYWORDS: Topological Interlocking, Convex Polyhedra, Platonic Bodies, Materials Design, Engineering Structures.

INTRODUCTION

In this research, we look at an interesting application of convex polyhedra, specifically platonic bodies, in engineering of materials and structures with superior mechanical properties. The principle we used is referred to as topological interlocking [01-03]. The basic underlying idea is that it is beneficial to replace massive solids with an assembly of identical blocks whose geometrical shape and mutual arrangement provide interlocking of the blocks within the structure, so that no connectors or binder are required to hold them in place.

RESEARCH

Mathematically, this principle can be illustrated in the following way. Consider a set of convex figures in R^2. It can be proven that one of these figures can be moved out of the set by translation without disturbing the others. Therefore, any set of planar figures can be disassembled by moving all figures one by one. However, attempts to generalize this statement to R^3 have been unsuccessful and finally, quite unexpectedly, interlocking structures of convex bodies were found. These structures are important in engineering, as they provide a means to hold elements together without gluing. As a result, segmented solids can be constructed with enhanced damping, as separate elements can move relative to each other. In addition, a crack nucleated in an element will be arrested at its boundary. We have proved that such structures can be assembled with any type of platonic polyhedra (as well as their truncations), and they have a geometric beauty. It is interesting to mention that this principle was considered in a medieval work by Abeille [04], who suggested to construct vaults made from interlocked truncated tetrahedra.

In our publication [01], we demonstrated that identical tesserae in the shape of any of the five platonic bodies can be arranged in a monolayer, in which they are interlocked by virtue of their shape and mutual arrangement. These assemblies are sketched in Fig. 1. A recipe for generating such structures of topologically interlocked blocks by transformation of a tiling of their middle cross-sectional plane in the normal direction has been provided in [02].

In the years that followed the inception of this geometrical principle, we have demonstrated its numerous benefits for design of new materials and structures. These include the ease of assembly and disassembly (hence, full recyclability of the blocks), the possibility of combining dissimilar materials within a structure, which allows for a range of new functionalities, and responsiveness of the assemblies to external stimuli enabled by changes of the applied lateral forces in response to a stimulus [03].

Since the inception of the concept of topological interlocking of polyhedra, several research groups adopted it and a significant body of research emerged. In the realm of architectural design, the work by Fallacara [04], Tessmann and Becker [05], Weizmann [06], Miodragovic Vella and Kotnik [07], Brocato and Mondardini [08], Piekarski [09], Viana

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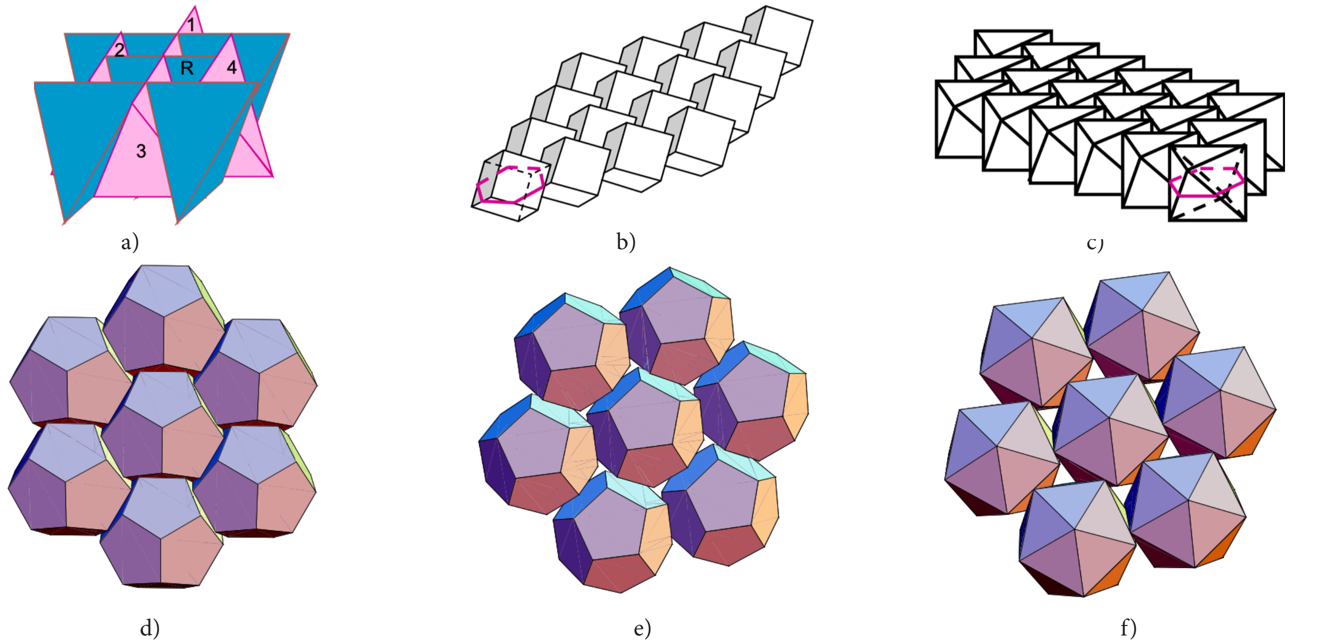

Fig. 1 - Interlocking arrangements of platonic bodies: a) tetrahedra; b) cubes; c) octahedra; d) and e) dodecahedra; f) icosahedra (after [01]).

[10], and others should be mentioned. In the context of materials design, research of the Siegmund lab at Purdue University and the Barthelat lab at McGill University has been particularly productive, as it provided insights in the manufacturing aspects of topological interlocking materials and outlined some important developments.

CONCLUSIONS

In this abstract, we give an overview of the geometrical principles underlying topological interlocking and its possible engineering applications, provide a critical assessment of the existing literature, and present some of our recent research into the engineering applications of this fascinating design principle.

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PAPERS' SESSION
PAPER 10

AN EULER-CAYLEY FORMULA FOR GENERAL KEPLER-POINSOT POLYHEDRA

Dirk Huylebrouck [1]

KEYWORDS: Euler Formula, Cayley formula, Kepler-Poinsot Polyhedra, Infinite Polyhedra.

INTRODUCTION

The generalized formula of Euler-Descartes for a polyhedron with V vertices, F faces and E edges that is the atom for a polyhedral structure with h loops is (see [01], [02], [03], [04])

$$V + F - E = 2\,(1 - h).$$

On the other hand, Cayley's formula for Kepler-Poinsot polyhedra with V vertices of densities b and F faces of density a, E edges and polyhedron density c is (see [05]):

$$bV + aF - E = 2c.$$

It was generalized to Archimedean polyhedra with V_j vertices and F_i faces of a given type and densities b_j and a_i respectively, E edges and polyhedron density c:

$$\Sigma b_j V_j + \Sigma a_i F_i - E = 2\mathrm{c}.$$

Here it is shown that both generalizations can be united in a single formula:

$$\Sigma b_j V_j + \Sigma a_i F_i - E = 2\,(c - a_1 h)$$

where V_j are the vertices, and F_i the faces of a given type and densities b_j and a_i respectively, E the edges and c the polyhedral density of the atom repeated in the structure with h loops connected on faces with face density a_1.
We illustrate the formula with several examples of generalized Kepler-Poinsot polyhedra.

RESEARCH

INTRODUCTION: THE WELL-KNOWN FORMULAS

The formula of Descartes-Euler states that for a convex polyhedron with V vertices, F faces and E edges:

$$V + F - E = 2.$$

Cayley generalized it to the Kepler-Poinsot polyhedra, using the face density a, the vertex figure density b, and the polyhedron density c:

$$bV + aF - E = 2c.$$

The face density a of a star polygon is 2 if one vertex is omitted to reach the next vertex of the subscribed polygon in a given star polygon (as in a star pentagon), 3 if two vertices are omitted (thus, for a star heptagon or octagon, a can be 2 or 3), and so on. It can also be seen as the number of times the star polygon goes around the center. Therefore, some call it the 'winding number', that is, the number of times a string surrounds a finger placed at the center. Yet another interpretation is that the central part of such a star polygon covers the surface as many times as given by a.
Similarly, the vertex figure density b tells us how many times a vertex should be counted when forming a polyhedron. It is the 'winding number of the vertex figure', the polygon that results when the vertex is slightly truncated. The polyhedron density c indicates by how many layers the polyhedron would cover a sphere if the polyhedron were blown up to a sphere. For regular convex polyhedra, all density values equal 1, and thus Cayley's formula is a generalization of Euler's formula.

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Cayley's generalization also works for star polyhedra 'of Archimedean type', that is, for star polyhedra with more than one type of regular polygons as faces. Suppose it has V_j vertices and F_i faces of a given type, with densities b_j and a_i respectively, then Cayley's formula generalizes to:

$$\Sigma b_j V_j + \Sigma a_i F_i - E = 2c.$$

Example 1:
Consider the truncated great icosahedron, formed by 12 pentagrams and 20 hexagons: $60 + (2 \cdot 12 + 20) - 90 = 2 \cdot 7$.
Example 2:
Kepler's star can be seen as a compound polyhedron formed by two joint tetrahedra: $8 + 8 - 12 = 2 \cdot 2$.
Example 3:
Consider an open pentagonal anti-prism (an anti-prism with triangular faces with pentagrams on the bottom and the top) on which stand star pyramids on the top and on the bottom: $(2 \cdot 2 + 10) + 20 - 30 = 2 \cdot 2$.

There is another generalization of the Descartes-Euler formula. Let h be the numbers of loops in a polyhedral structure. For an infinite polyhedron, this is defined by the number of loops in the repeating part. Others prefer to use the word genus in this context, but in the context of Kepler-Poinsot solids, with their many holes and cavities, this can be confusing (see [03], [05], [06]). Now the formula becomes:

$$V + F - E = 2\,(1 - h).$$

Example 4:
Consider a toroid of 8 open octahedra: $8 \cdot 3 + 8 \cdot 6 - 8 \cdot 9 = 2 \cdot (1 - 1) = 0$ (Fig. 1).
Example 5:
Consider Petrie-Coxeter's infinite polyhedron {6, 4} where four hexagons meet at a vertex and $V + F - E = -4$, since $V = 12$, $F = 8$, $E = 24$ and $h = 3$. Each 'atom' is formed by a truncated octahedron of which 3 times 4 vertices, 6 faces and 3 times 4 edges are removed. The removed parts fit on those of the atoms next to it (Fig. 1).

JOINING EULER'S GENERAL FORMULA AND CAYLEY'S FORMULA

The above formulas of Euler and Cayley can be united in a single formula. Suppose an Archimedean polyhedra with V vertices, F faces and densities b and a respectively, E edges and polyhedron density c is used to form an atom of a larger polyhedron. In that case, Cayley's formula says:

$$bV + aF - E = 2c.$$

It can be rewritten as:

$$b\,(V - n) + (b - 1)\,n + n + a\,(F - 2) + 2a - (E - n) - n = 2c,$$

or else:

$$b\,(V - n) + (b - 1)\,n + a\,(F - 2) - (E - n) = 2c - 2a.$$

When we connect those polyhedra in a series or loop, they are joined on a common face.
Thus, we should delete two faces, one to connect it to the 'previous' one, and one to connect it to the 'next' one. From one of them, we had to remove n vertices, but counted only once (that is, with density 1). The thus formed atom has $V - n$ vertices with density b and n vertices with densities $b - 1$. It has $F - 2$ faces with densities a and $E - n$ edges. Thus, the first expression, $b\,(V - n) + (b - 1)\,n$, is the number of vertices times their density of the newly formed polyhedron, while the second, $F - 2$, is its number of faces times its density, and the third, $E - n$, its number of edges. Or, using appropriate notations:

$$\Sigma b_j V'_j + aF'' - E' = 2\,(c - a \cdot 1).$$

Repeating this procedure 2h times yields:

$$\Sigma b_j V'_j + aF'' - E' = 2\,(c - a \cdot h).$$

It is straightforward to generalize this to the Archimedean case to get $\Sigma b_j V_j + \Sigma a_i F_i - E = 2\,(c - a_1 h)$ and it probably is possible to generalize this expression even further, so that not only the density a_1 is singled out, but other face densities too, in a general notation. In in view of the examples at the disposal, however, this seems unnecessary.

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In the case $a_1 = 1$ and $c = 1$, $\Sigma b_j V_j + \Sigma a_i F_i - E = 2\ (1 - h)$ and that is the general Euler-Descartes formula.
In the case $h = 0$, $\Sigma b_j V_j + \Sigma a_i F_i - E = 2c$, and that is the general Cayley formula.

EXAMPLES OF THE GENERALIZED EULER-CAYLEY FORMULA

Coxeter excluded shapes with adjacent faces in a common plane to be considered as polyhedra (see [02]). However, this rule hasn't been followed by all authors, such as J. R. Gott, who called some of the solids he discovered 'pseudo-polyhedra' (see [04]). Others did not make that distinction and simply called them 'polyhedra'. We will follow this approach, though it seems quite a topic of discussion and so we refer to several references, such as [01], [04] or [05].

Example 6:

Consider the closed toroid star polyhedron where 6 equilateral triangles meet in each vertex. All triangles are equilateral but some cut each other causing an apparent trapezoid intersection. Three triangles lay in one plane along an edge of a pentagram. Now $V = 20$, $F_1 = 0$ (the pentagonal star in the middle) and $F_2 = 40$, $E = 60$, $b_1 = 1$, $a_1 = 2$, $b_2 = 1$, $a_2 = 1$, $c = 2$ and $h = 1$: $20 + 40 - 60 = 2 \cdot (2 - 2 \cdot 1)$.

Example 7:

Consider rings of small ditrigonal icosidodecahedra. For one small ditrigonal icosidodecahedra, $V = 20$, $F_1 = 12$ and $F_2 = 20$, $E = 60$, $b_1 = 1$, $a_1 = 2$, $b_2 = 1$, $a_2 = 1$, and $c = 2$. To form one ring of 10 elements and thus an example with 1 loop, 2 pentagonal faces are removed to form the atoms, including, for one of the faces, the vertices and the edges, so that $V = 15$, $F_1 = 10$ and $F_2 = 20$, $E = 55$. There are 10 atoms in total:

$$10 \cdot 15 + (10 \cdot 2 \cdot 10 + 10 \cdot 20) - 10 \cdot 55 = 0 = 2 \cdot (2 - 2 \cdot 1).$$

The small ditrigonal icosidodecahedra can be placed in 2 rings, thus forming an example with 2 loops, but where 2 rings meet, 3 pentagonal faces now have to be removed from two of the atoms. There are 17 atoms:

$$17 \cdot 15 + (15 \cdot 2 \cdot 10 + 2 \cdot 2 \cdot 9 + 17 \cdot 20) - 17 \cdot 55 = -4 = 2 \cdot (2 - 2 \cdot 2).$$

They can be placed in 3 rings (or more), by removing 3 pentagonal faces from four of the atoms. There are 22 atoms:

$$22 \cdot 15 + (18 \cdot 2 \cdot 10 + 4 \cdot 2 \cdot 9 + 22 \cdot 20) - 22 \cdot 55 = -8 = 2 \cdot (2 - 2 \cdot 3).$$

Example 8:

We again consider open octagrammic star prisms, but connect them by open great rhombihexahedra. For a regular closed great rhombihexahedron, $V = 24$, $F_1 = 6$ and $F_2 = 12$, $E = 48$, while for an octagrammic prism $V' = 16$, $F'_1 = 2$ and $F'_2 = 8$, $E' = 24$. We remove all octagrammic stars, so that 3 open octagrammic star prisms and one open great rhombihexahedra together have 24 vertices, $36 = 12 + 8 \cdot 3$ faces and 72 edges: $24 + 36 - 72 = -12 = 2 \cdot (3 - 3 \cdot 3)$. Six squares meet at each vertex (Fig. 1).

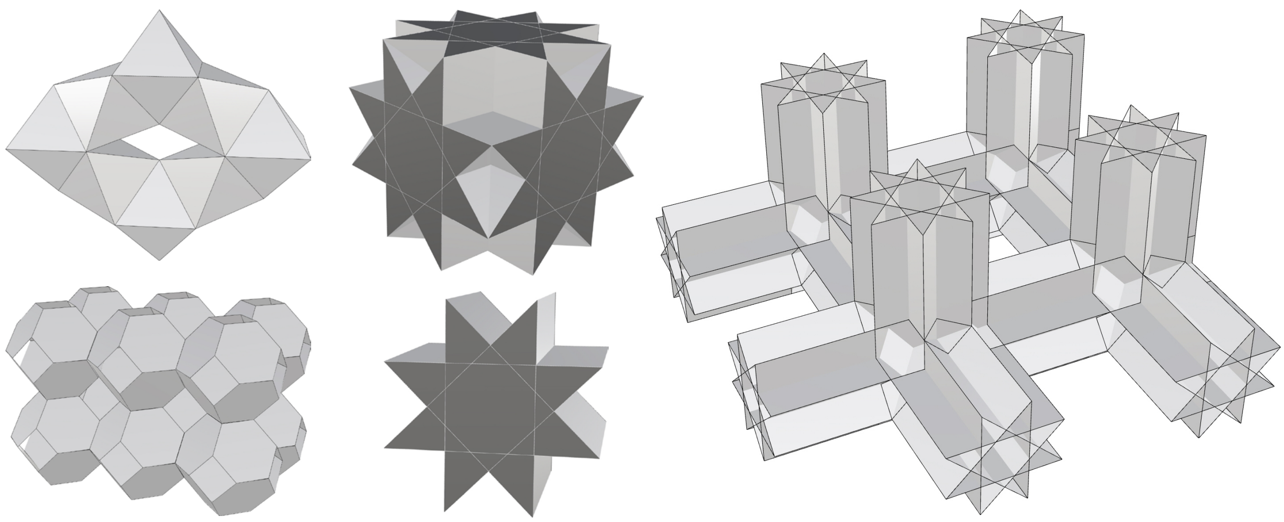

Fig. 1 - A toroid of open octahedra and Petrie-Coxeter's infinite polyhedron (left) and a polyhedron made from great rhombihexahedra and octagrammic prisms (middle, right).

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Example 9:

Consider a small cubicuboctahedron: $V = 24$, $F = 20$, $E = 48$. Now, $2 \cdot 24 + 20 - 48 = 2 \cdot 10$ and $c = 10$. We remove the 6 squares and their 24 vertices and edges to open the small cubicuboctahedron. Next, we remove 12 vertices and 12 edges to connect it to other adjacent small cubicuboctahedra: $12 + 14 - 12 = 12 = 2 \cdot (10 - 1 \cdot 3)$. Four octagons and two triangles meet at each vertex. Adding open cube tunnels between them turns it into an Archimedean example. An open cube has $V = 4$, $F = 4$ and $E = 8$ so that adding $V + F - E$ doesn't alter the Euler-Cayley-formula (Fig. 2).

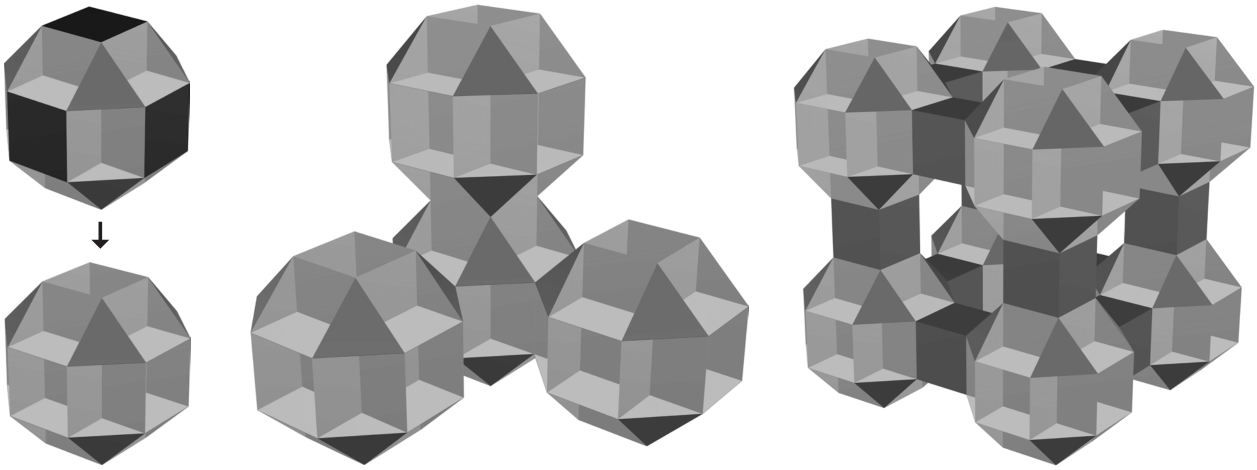

Fig. 2 - An infinite Kepler-Poinsot-polyhedron made from small cubicuboctahedra and an Archimedean case.

Example 10:

Consider a tetrahemihexahedron: $V = 6$, $F = 7$, $E = 12$. Now, $2 \cdot 6 + 7 - 12 = 2 \cdot 7/2$ and $c = 7/2$. We remove the 4 triangles to open the tetrahemihexahedron. From 2 of them, we remove their 6 vertices and edges: $1 \cdot 6 + 3 - 6 = 3 = 2 \cdot (7/2 - 1 \cdot 2)$. We add open anti-prisms tunnels between to get an Archimedean example, but that does change much as open anti-prisms tunnel has $V = 3$, $F = 6$ and $E = 9$ so that adding $V + F - E$ doesn't alter the Euler-Cayley-formula (Fig. 3).

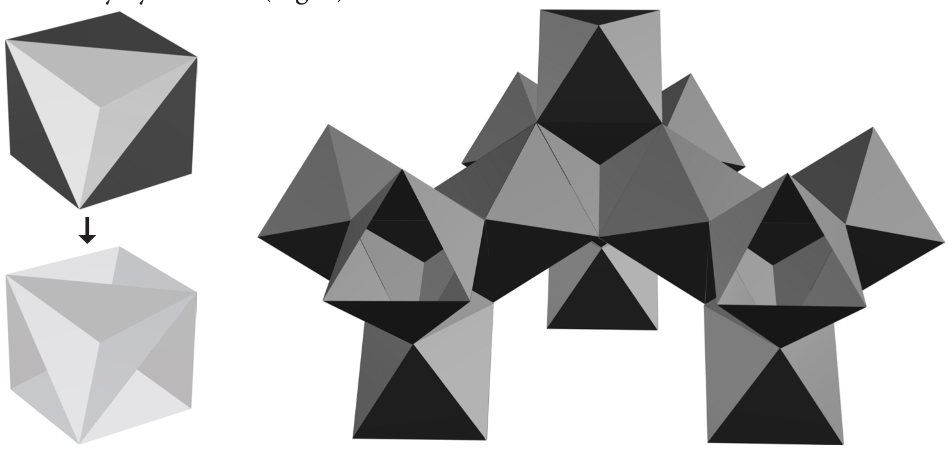

Fig. 3 - An infinite Kepler-Poinsot-polyhedron with tetrahemihexahedra and triangular anti-prisms.

Example 11:

Consider the well-known example of a ring of icosahedra connected by open triangular anti-prisms, but replace the icosahedra by great icosahedra. For a great icosahedron, $V = 12$, $F = 20$ and $E = 30$, so that $2 \cdot 12 + 20 - 30 = 2 \cdot 7$. We remove one triangle with its vertices and edges, plus one triangle (only the face): $2 \cdot 9 + 3 + 18 - 27 = 2 \cdot 7 - 2 = 2 \cdot (7 - 1 \cdot 1)$ and get one loop. Removing another triangle with its vertices and edges, plus one triangle (only the face): $2 \cdot 9 + 16 - 24 = 2 \cdot 7 - 4 = 2 \cdot (7 - 1 \cdot 2)$ and there are now two loops (Fig. 4).

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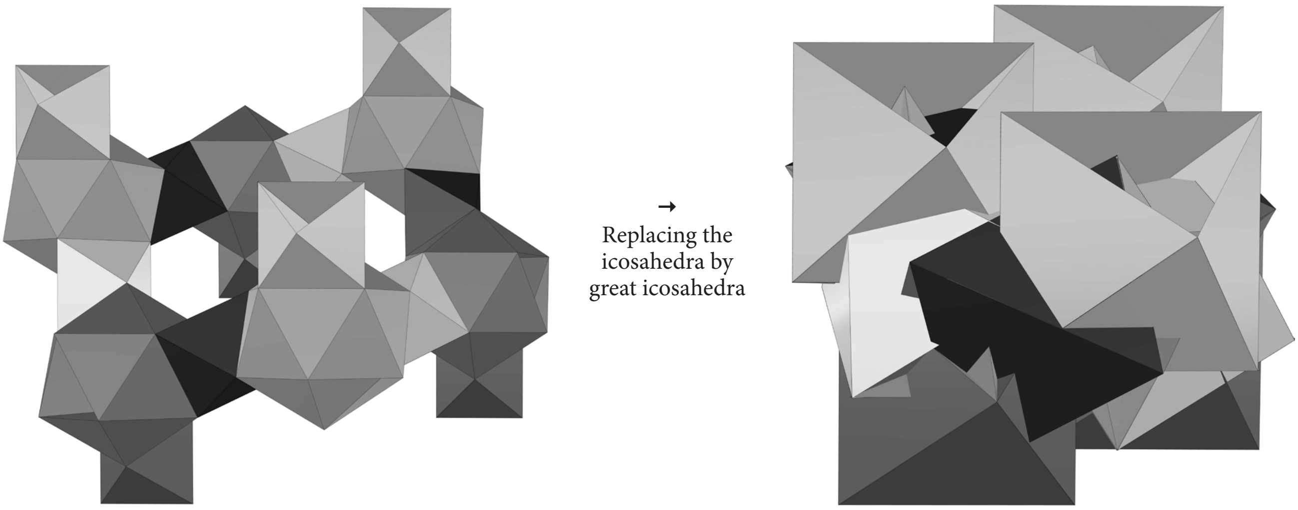

Fig. 4 - An infinite Kepler-Poinsot-polyhedron with great icosahedra and open triangular anti-prisms.

Example 12:

Consider a cubohemioctahedron: $V = 12$, $F = 10$ and $E = 24$ so that $2 \cdot 12 + 10 - 24 = 2 \cdot 5$ and $c = 5$. Without the squares faces and common vertices and edges: $V = 6$, $F = 4$ and $E = 12$ so that $12 + 4 - 12 = 4 = 2 \cdot (5 - 1 \cdot 3)$. Adding open cube tunnels ($V = 4$, $F = 4$ and $E = 8$; thus, $4 + 4 - 8 = 0$) to get an Archimedean derivative clearly confirms the number of loops is 3 (Fig. 5).

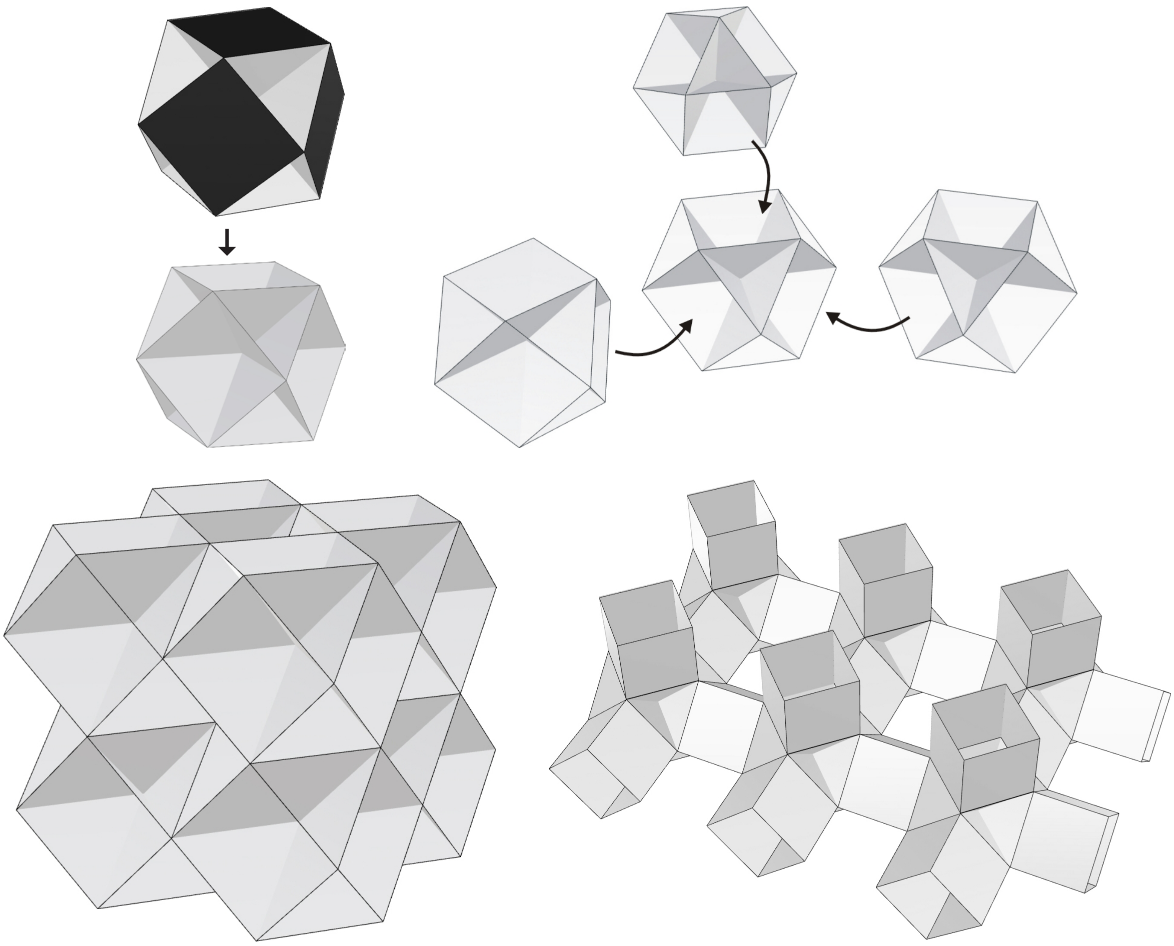

Fig. 5 - A regular polyhedron made from cubohemioctahedra and an Archimedean derivative.

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The infinite open cubohemioctahedron polyhedron is a new regular polyhedron (see [07]). In Schäfli notations, it is a regular {6, 8}. Two layers can be obtained from four layers of the regular Petrie-Coxeter {6, 4} and so, at first sight, it looks like a compound polyhedron just as Kepler's star is made from two tetrahedra. However, whereas Kepler's star still is a {3, 3}, just as its components, the infinite open cubohemioctahedron polyhedron is a {6, 8} while its components are {6, 4}.

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POLYHEDRAL TRANSFORMATION BASED ON NEW ROTATIONAL QUADRATIC SURFACE PROPERTIES

Andrés Martín-Pastor[1]

KEYWORDS: Quadratic Theorems, Graphic Thinking, Panelling, Discretization, Polyhedral Transformation.

INTRODUCTION

We want to evince the presence of polyhedra in the history of Descriptive Geometry as pieces that, beyond being merely symbolic, enable the most complex spatial transformations to be graphically understood. Following this inherited tradition, we have also relied on polyhedra to explore the properties of a transformation arising from the graphical conjecture presented in [01], in which a property of the rotational paraboloid, expounded by Archimedes (287–212 B.C.) in his work *On Conoids and Spheroids* [02], is generalized to include all rotational quadratic surfaces. The conjecture is summarized as follows:

> "*If two rotational quadratic surfaces share the position of one of their foci at the same point, then the intersection curves between the two surfaces are always planar*" (The oblate ellipsoid and one-sheeted hyperboloid are excluded) (Fig. 1).

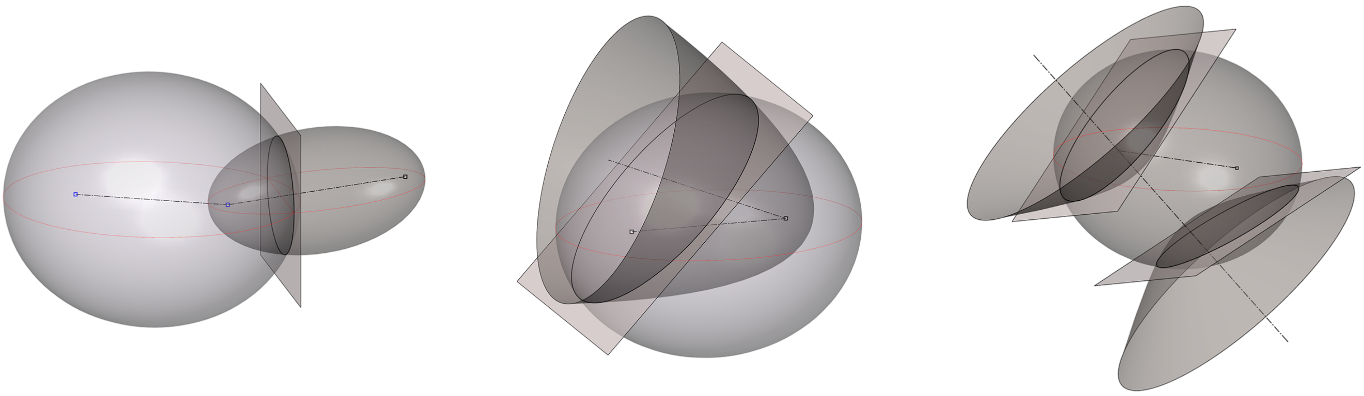

Fig. 1 - Graphic conjecture: Planar intersection of ellipsoids with paraboloids and hyperboloids.

Although the property is open to many other applications, in this abstract, we explore the capacity offered by this graphic conjecture to project a net of flat polygons, placed on a sphere, onto a rotational quadratic surface, while maintaining planarity after the transformation.

The problem of discretizing surfaces on planar surfaces constitutes a topic frequently tackled in the scientific literature, with numerous contributions in applied mathematics and in architectural geometry. The contribution of this abstract resides in the presentation of a straightforward reasoning that aids in the comprehension, from a graphical approach, of certain spatial properties that can be deduced regarding rotational quadratic surfaces, which, to date, have somehow been left relatively untouched by specialists. Based on these graphic speculations, and with the support of polyhedra, we present a new way to characterize 3D homology, by means of a sphere and an ellipsoid, and an innovative form to bevel the edges of a polyhedron by means of conical curves.

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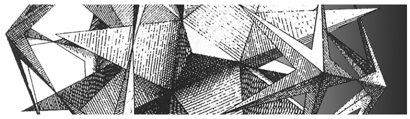

CONTEXTUALIZATION

By way of contextualization of this work, we want to show how polyhedra remain the most important pieces for an understanding spatial transformations in various graphic systems.

The relationship of polyhedra with perspective (the first codified system of representation) provides extraordinary examples in history: Daniele Barbaro, Vignola-Danti, Sirigati, Niceron, Dubreil, Albrecht Dürer and Wenzel Jamnitzer, among many other geometers. The formulae initiated by Luca Paccioli and Barbaro triggered an analysis of the internal geometry of the polyhedron, which substantially conditions the form of representation. The polyhedra must therefore be known *a priori* in order to be represented on the basis of their internal relations. The proposed strategy always links the polyhedron with the space to be represented and can be summarized as such: since the internal structure of the polyhedron is known, we can ascertain how space is transformed [03].

There are two examples of great interest, little known to specialists, that single out the use of polyhedra in the personal search for geometric relationships: The interpretation of the polyhedra of Wenzel Jamnitzer (1568) by the anonymous Spanish author of the manuscript *Artes Exçelençias dela Perspectiba* [04], dating from 1688 (Fig. 2, left); and the reasoning employed by certain geometers regarding the fourth dimension in the late nineteenth century (Fig. 2, centre and right).

The Spanish author carried out an analysis of the complex illustrations of Jamnitzer's perspective treatise, which lacked auxiliary lines and tracings. In the manuscript, the polyhedra were finally represented from another point of view and with all types of graphic and literary explanations. We are faced with a very interesting case, where a rigorous analysis of the polyhedra enables not only its true size to be deduced, but also the parameters that define the System of Representation [05].

The first geometers who strove to accurately graph the fourth dimension were faced with enormous problems: on one hand, they had to represent the invisible, while, on the other hand, they had to define a new form of representation. In these first attempts, the tetra-dimensional polyhedra, or polytopes, played a decisive role which still is in existence today [09].

We highlight the work of Van Oss [07], which included the first coding of a coherent graphic method to represent the tetra-dimensional space. This effort by Van Oss to plot the 4D space cannot be understood without the knowledge of polytopes, whose geometry was being deduced mathematically at that time. A few years later, this graphic method appeared in the work of Schoute [10] and Jouffret [08]. Since then, numerous attempts have been made to represent tetra-dimensional objects, always with the presence of polytopes.

Llorens-Herrero, in his Doctoral Thesis [11], finally managed to systematise the tetra-dimensional Euclidean space by means of a complete Graphic System.

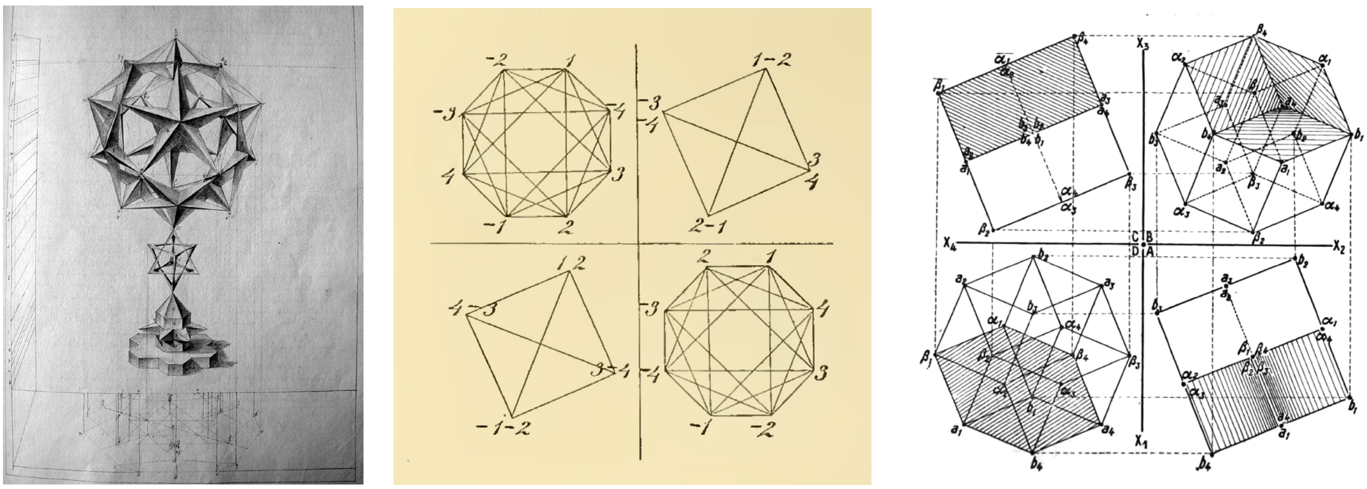

Fig. 2, left - The figure [04: Chapter 10, Fol. 72.] is an interpretation of *Perspetiva Corporum Regularium*, [06, table O, F, III,].
Fig. 2, centre: Representation of 16Cell, in the first 4D graphic system: 4-ortogonal axis projection system, [08: Tafel I].
Fig. 2, right: Representation of 8Cell in the 4-ortogonal axis projection system [07: 119].

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His method consists of extrapolating the reasoning of Monge's Dihedral system to an extra dimension: if a 3D object can be defined in an unequivocal way by means of its planar projections, then, a 4D object can also be defined by tri-dimensional projections. In his thesis, Llorens shows that all the lines of reasoning in Monge's *Descriptive Geometry* are equally valid for tetra-dimensional abstract elements.
This brief introductory reflection serves as a presentation of a study, which, as in the aforementioned examples, features polyhedra as the leading players in the role of geometric knowledge.

RESEARCH

AREA OF APPLICATION AND METHODOLOGY

The discretization of quadratic surfaces, especially ellipsoids, constitutes a wellknown topic in the field of surface discretization. Numerous methods of panelling are available for quadratic surfaces. We should bear in mind the Monge's ellipsoid, commented by Hachette [12], regarding the lines of principal curvature and its relation on current panelling methods. Pottmann are also studying this topic together with other more advanced topics [13]. There are many theses and research papers in the literature on surface panelling, where various panelling methods are proposed that are indirectly related to the present study [14, 15].
Our study is focused on the properties of rotational quadratic surfaces, in particular on a type of transformation, which projects planar polygons placed on a sphere, onto ellipsoids, paraboloids, and hyperboloids, while maintaining planarity after the transformation. The methodology employed for its demonstration is not based on any formal mathematical language, but on graphic thinking. Its validity has been fully tested through a heuristic method which involves checking the planarity on all possible combinations of quadric intersections in a necessary and sufficient number of cases in [01]. For this purpose, the power of CAD tools has been used as a true geometric research laboratory, where the validity of the theoretical approaches is subject to trial and error.

CASE 1: PROJECTING BY CONES

According to a specific case of the conjecture described above, a cone of revolution whose vertex is located in the focus of a rotational quadratic surface cuts this surface in accordance with a planar curve (Fig. 3a). Therefore, a family of revolution cones, whose vertex is located in the centre of a regular or semi-regular polyhedron, can project the points of the original polyhedron onto other rotational quadratic surfaces such as ellipsoids (Fig. 3b), paraboloids, and hyperboloids (Fig. 3c). The transformation maintains the flatness of the faces of the final projected polyhedron, which is inscribed in the second quadratic surface. In general, this property is fulfilled for irregular polyhedra whose points are all situated on the surface of a sphere, on the condition that the vertex of the projected cone is located in the centre of the sphere and in the focus of the rotational quadratic surface (Fig. 3d).

CASE 2. PROJECTING BY ELLIPSOIDS, PARABOLOIDS OR HYPERBOLOIDS

Instead of using conical projecting rays, we can also use ellipsoids, paraboloids or hyperboloids as projecting surfaces. These quadratics surfaces must share the position of one of their foci with the ellipsoid of reference and pass through the vertices of the polyhedron (Fig. 4, left). The points contained in the flat section of a sphere, face of polyhedron, transform into points contained in the flat section of another rotational quadric surface, just like case 1, but reducing the size of the transformed polygon (Fig.4, upper right). We verify that this transformation coincides with the operation of case 1, by projecting cones, applied to the same polyhedron with its edges bevelled. As a result, we have an operation that automatically bevels the edges of a polyhedron, inscribed in a sphere, according to different criteria (Fig. 4, bottom).

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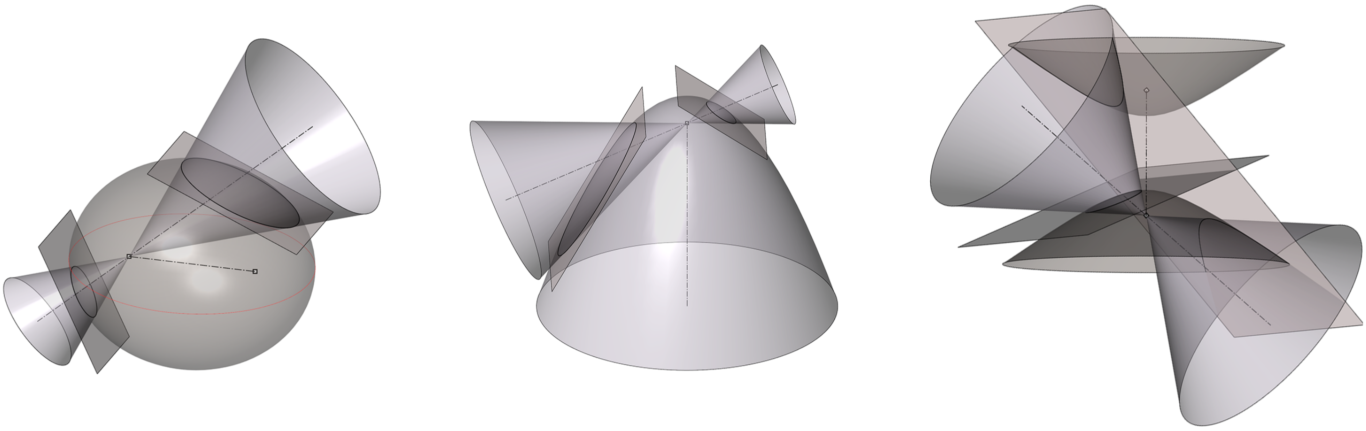

Fig. 3a - Projecting by cones from one focus.
Example of ellipsoid, paraboloid, and hyperboloid.

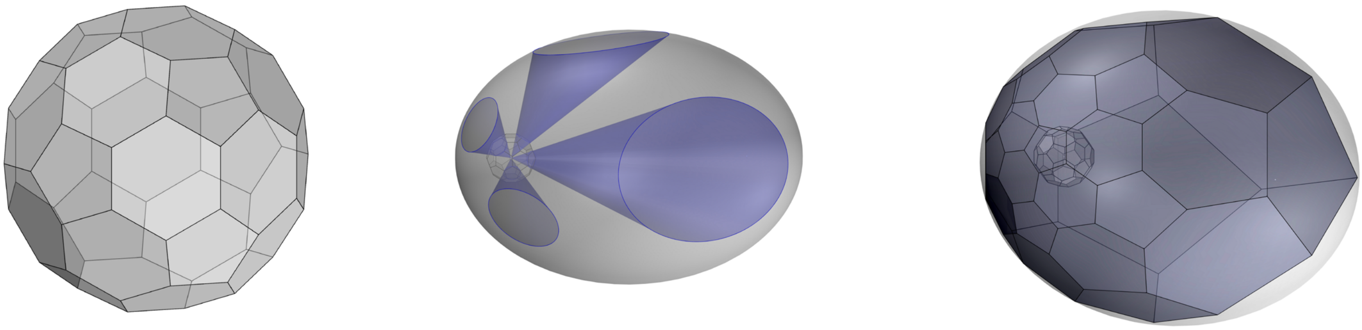

Fig. 3b - Polyhedral transformation Case 1: Projecting by cones from one focus.
Example of discretization of ellipsoid based in the truncated icosahedron.

Fig. 3c - Polyhedral transformation Case 1: Projecting by cones from one focus.
Example of discretization of paraboloids and hyperboloids based in the truncated icosahedron.

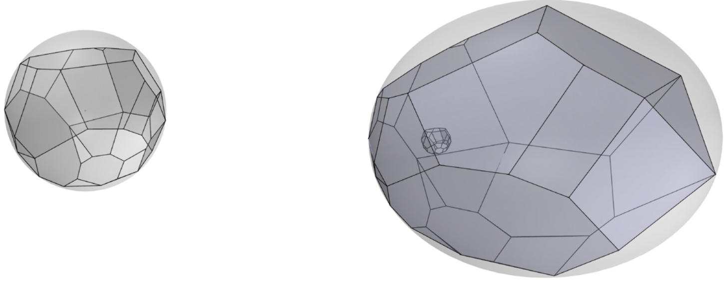

Fig. 3d - Polyhedral transformation of irregular polyhedra.

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GRAPHICAL CHARACTERIZATION OF 3D HOMOLOGY

The graphic study carried out in case 1, projecting cones, reveals that this transformation can be generalized for all points of space beyond the surface itself. If we can perform the transformation of a polyhedron whose vertices are contained in a sphere, we can also carry out the transformation of a polyhedron with vertices contained in various spheres. This is the case of the rhombic dodecahedron whose vertices are inscribed on two concentric spheres. (Fig. 5a) The imposition of flatness in the faces of the transformed polyhedra determines the major and minor axes of the second ellipsoid that shares the focus position with the first (Fig. 5b). It is deduced that the second ellipsoid is unique and it can be determined via the first sphere-ellipsoid set, which we call the "reference". Once this transformation is defined, the problem is generalized for any concentric sphere. Space continuum could therefore be understood as consisting of a set of spheres that are concentric to the reference sphere, where each one is transformed into a different rotational quadric surface (Fig. 5c). The transitions from ellipsoid to paraboloid and hyperboloid therefore carry a special importance, since these determine the singular points of such a transformation.

RESULTS

As a result of these graphical speculations, we deduce that the transformation carried out by this conjecture (case 1) is strictly a 3D homology (Fig. 5d). While this is a basic transformation of space studied in mathematics, it has not been as popular in the known courses in Descriptive Geometry. A rare example is described in Taibo [16: 273]. Through our graphic study, we present an innovative way to completely characterize this transformation. The graphical elements that define this homology (homology centre, homology plane, plane limit 1, and plane limit 2) remain implicit on defining the form and position of the reference sphere and the reference ellipsoid.

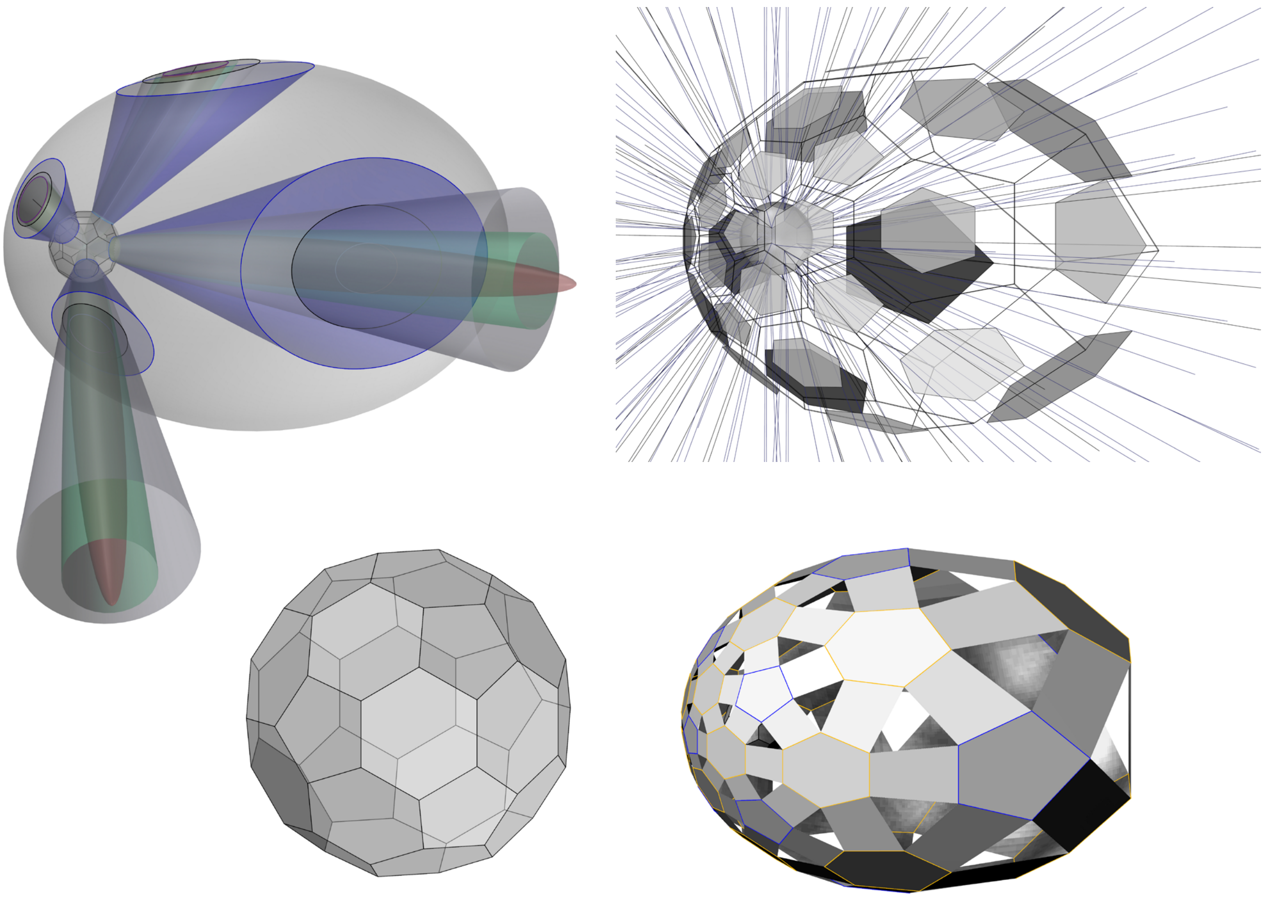

Fig. 4 - Polyhedral transformation Case 2: Projecting by ellipsoids, paraboloids or hyperboloids from one shared focus.
Example of discretization based in truncated icosahedron. The point are projected by hyperboloids.
The result is a truncated icosahedron with edges bevelled.

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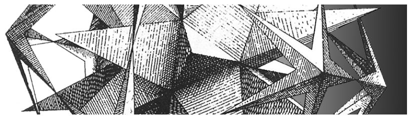

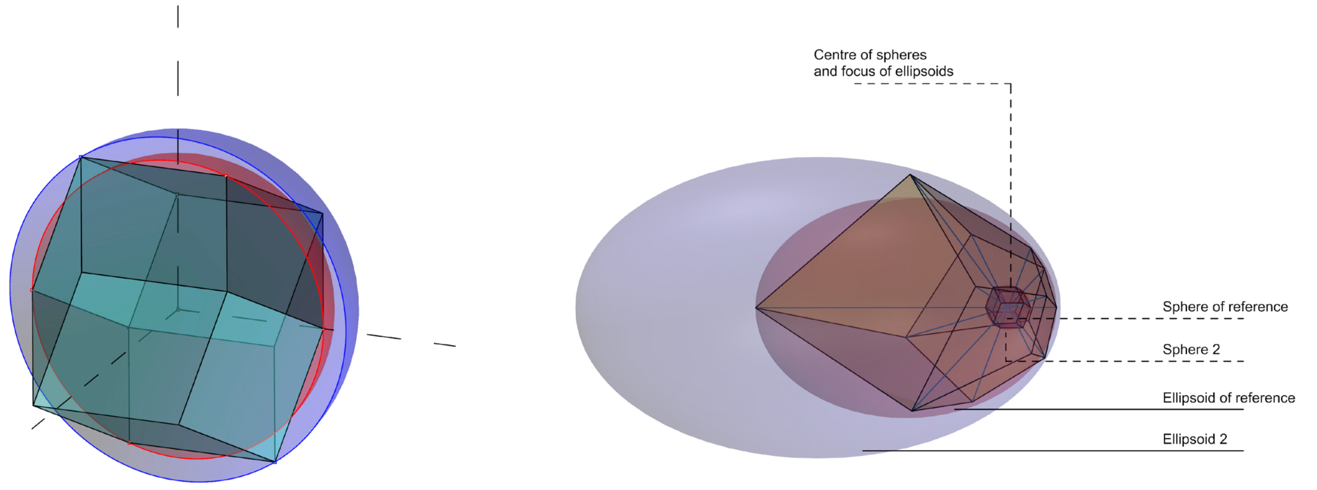

Fig. 5a and 5b - Transformation of the rhombic dodecahedron from two spheres (red and blue) to two ellipsoids.

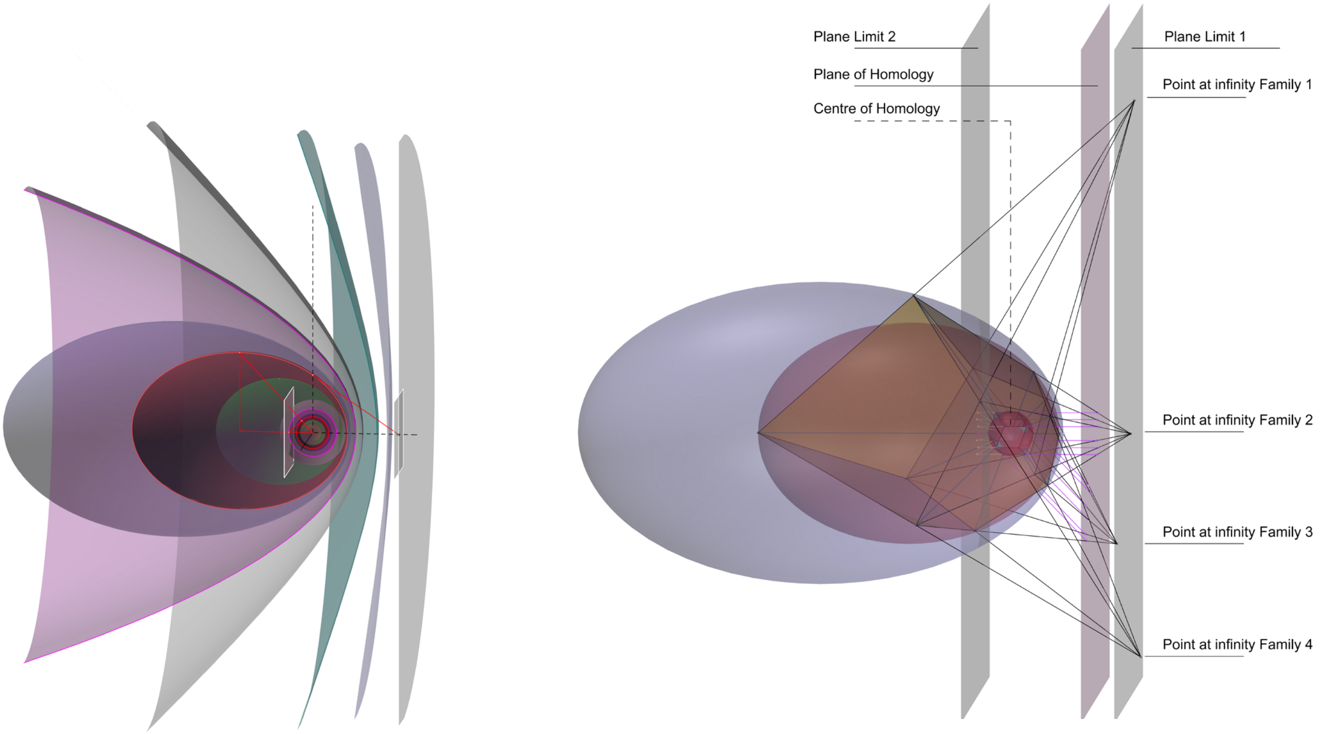

Fig. 5c - Generalization for entire space: each point of space corresponds to one sphere
and each sphere is transformed into a different rotational quadratic surface.
Fig. 5d - The transformated rhombic dodecahedron reveals that this transformation is strictly a 3D homology.

CONCLUSIONS

With respect to the applications of the conjecture to produce polyhedric transformations, we have differentiated between two types of transformation: the first by cones, and the second by ellipsoids, paraboloids, and hyperboloids. We have verified the correlation of the first transformation with a 3D homology. This leads to a new way to characterize 3D homology: by means of a sphere and an ellipsoid.

Newly discovered features of the second transformation include the fact that, instead of projecting by means of projecting rays, the projection occurs by means of conical curves with one of its foci located in the centre of the sphere. As a result, we have a new transformation that automatically generates bevelled polyhedral edges from other simpler polyhedra.

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Both procedures can be parametrized in *Grasshopper*, thereby providing a way of discretization for rotational quadratic surfaces. It should be emphasised that it is not only the new technologies and parametric tools that have made the definition of this transformation possible, but the capacity that new technologies offer in the production of augmented graphic thinking, which enables us to reinterpret inherited tradition.

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PAPERS' SESSION
PAPER 12

THE CONSTRUCTION OF REGULAR AND SEMIREGULAR POLYHEDRA THROUGH THE SYNTHETIC METHOD

Leonardo Baglioni[1] and Federico Fallavollita[2]

KEYWORDS: Descriptive Geometry, Mathematical Representation, Chiral Polyhedra, Snub Cube.

INTRODUCTION

In this study, we propose an explanation and application of the synthetic method to the representation of platonic and archimedean solids (PS and AS). The intention is to illustrate the potential of this method in a historical and theoretical context. The study of regular and semiregular polyhedra, in this sense, is an ideal theme to illustrate the heuristic potential of drawing. Therefore, some synthetic constructions of PS and AS are proposed, defining the constructive algorithms of these figures. The working environment used is the mathematical representation method; for some constructions, parametric and physical simulating tools were used. Particular attention is dedicated to two different synthetic methods; the first, is the construction of the snub cube through paper folding [01: 1-14] and the second, is a more general method that exploits a physical simulator engine.

RESEARCH

The synthetic method, as Gino Loria explains in the booklet *I metodi matematici* [02: 77-83], is part of the mathematical methods. In particular, we refer to what Loria defines as the *method of existential construction*. He inserts this method among those special to geometry. He explains that "Euclid […] never reasons on a figure whose construction he has not previously taught". As an example, he proposes the proof of the existence of the PS through the known Euler formula

$$F + V - E = 2$$

Then he says n is the number of sides of each face and m is the edges, and he declares this:

$$nF = 2E, mV = 2E$$

He deduces and concludes that the only possible regular solids are the five PS. "To prove their existence, not only *arithmetic* but also *geometric*, it is useful to start from their actual construction" [02: 77]

In these constructions, Loria explains how to build the five PS in space by giving instructions that are nothing more than algorithms that step by step allow us to build the solids. This existential method for Loria has a mathematical dignity equal to all the other methods illustrated in the essay. In particular, at the beginning of the essay, Loria distinguishes the analytical method from the synthetic one, trying to give a definition to both.

> "*Following the first (Analysis) we reduce the theorem to be demonstrated or the problem to be solved to another that is judged to be simpler, to the new one we apply the same procedure and so we continue until we get to a known or already treated proposition. Following the second one (Synthesis) a series of considerations is established which gradually leads to the desired purpose* ". [02: 03]

The synthetic method, therefore, is typical of descriptive geometry, that uses drawing as a research tool for the properties and relationships of figures in space.

Today, the synthetic method, thanks to the advent of the digital revolution, has acquired particular importance in the world of geometry. The digital drawing has a higher accuracy than the analogical one and, above all, allows us to

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draw directly in space. Nevertheless, there is a character of representation that more than any other has been enhanced by digital, and that is its constructive aspect. Geometry deals in abstract terms with procedures and methods that can be replicated in reality by means of physical instruments. Construction as a mental process at the base of geometric operations in the plane and in space, finds a fertile field of application in the virtual world, greatly enhancing the heuristic power of representation. This makes it possible to review old and new geometry problems that previously were impossible to solve synthetically [03: 7-28].

Taking inspiration from Loria's considerations, we propose the construction of some PS and AS through the synthetic method and attempting, in this way, to show the heuristic potential of drawing. In particular, we focus on the construction of the snub cube and we propose constructions that can be implemented in a parametric modeller (*Grasshopper*) and a physical engine (*Kangaroo*). For example, we can construct the dodecahedron starting from the basic pentagon face and from two adjacent faces, replicating the same operations that we could conduct in reality. In this way, by simulating the geometric movement of rotations in space of the two adjacent faces, it is possible to find the meeting point of the edges and stop the rotation at the exact point (Fig. 1). Furthermore, in the proposed algorithm, we have identified a solid section composed by a rectangular shape strip, forming a vacuous solid similar to the treaty by Luca Pacioli.

Synthetic constructions have also been studied and proposed for the AS that, as known, can be obtained from PS. These solids are convex polyhedra, composed of faces of regular polygons of different types but with edges of equal length; for this reason, they are called semiregular polyhedra. The last two solids enjoy the chiral property, for which the symmetry generates differences. All AS can derive from PS by means of three main types of operations followed by different variations (Table 1).

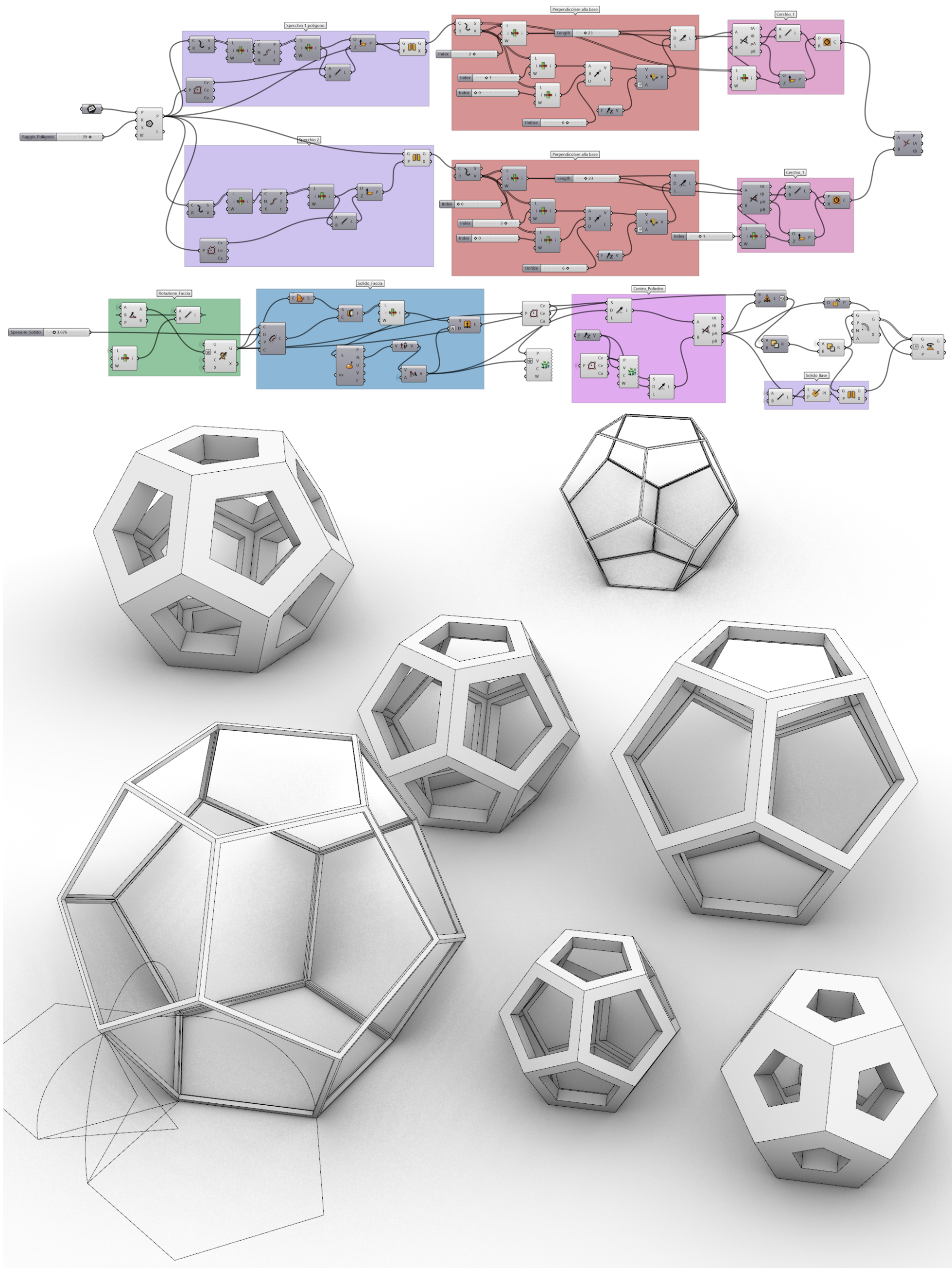

Fig. 1 - Construction process of thr dodecahedron through the synthetic method.

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Table 1 - Geometrical operations to construct the Archimedean Solids from the Platonic Solids.

Operation	Variation	Derivation
1. Planar section of the PS symmetric with respect to the vertices and...	1.1 ...passing through the edges centres	Cuboctahedron from the Cube and the Octahedron
		Icosidodecahedron from the Dodecahedron and the Icosahedron
	1.2. ...passing through the third point of the edges	Truncated Tetrahedron from the Tetrahedron
		Truncated Octahedron from the Octahedron and the Cube
		Truncated Icosahedron from the Icosahedron and the Dodecahedron
	1.3. ...so that the central segment of the edge connects twice the number of face sides	Truncated Cube from the Cube and the Octahedron
		Truncated Dodecahedron from the Dodecahedron and the Icosahedron
	1.4. ...beyond the midpoints of the edges and leading to reversely homotetical or inverted polygons inscribed within the faces	Truncated Tetrahedron from the Tetrahedron
		Truncated Octahedron from the Octahedron and the Cube
		Truncated Cube from the Cube and the Octahedron
		Truncated Icosahedron from the Icosahedron and the Dodecahedron
		Truncated Dodecahedron from the Dodecahedron and the Icosahedron
2. Planar Section of the PS symmetric with respect to the edges followed by symmetrical sections at their vertices	2.1 ...to obtain polygons homotetical with respect to the faces centers inscribed within the faces	Rhombicuboctahedron from the Cube and the Octahedron
		Rhombicosidodecahedron from the Dodecahedron and the Icosahedron
	2.2 ...to obtain polygons having twice the number of sides inscribed within the faces	Truncated Cuboctahedron from the Cube and the Octahedron
		Truncated Icosidodecahedron from the Dodecahedron and the Icosahedron
3. Inscribed within the faces of the PS, a polygon having the same number of sides, but rotated in the same direction through a certain angle		Snub Cube from the Cube and the Octahedron
		Snub Dodecahedron from the Dodecahedron and the Icosahedron

All the synthetic methods proposed allow us to obtain accurate constructions of the AS, since they are perfectly executable with a straight edge and a compass (SE&C) (Fig. 2).

The difficulties are found only for the last two AS, those called chiral. To construct the snub cube, it is possible to adopt the graphic method proposed by Dragomir and Gheroghiu [04: 214-215]. The method consists in dividing the edge of the PS starting from a ratio defined by the equation:

$$2\alpha^3 - 4\alpha^2 + 4\alpha - 1 = 0$$

From which the α value is equal to 0.352... The problem in the construction of the mathematical model consists in the approximation that the procedure involves, because the value of α is a relationship that we cannot express through a graphic construction with SE&C. This leads to an error in the construction of the solid, because the tolerance of the program, in most cases, does not recognize the vertices of the solid, as constructed, belonging to the circumscribed sphere and does not verify the solid's topology. The same problem arises for the construction of the last chiral solid, the snub dodecahedron.

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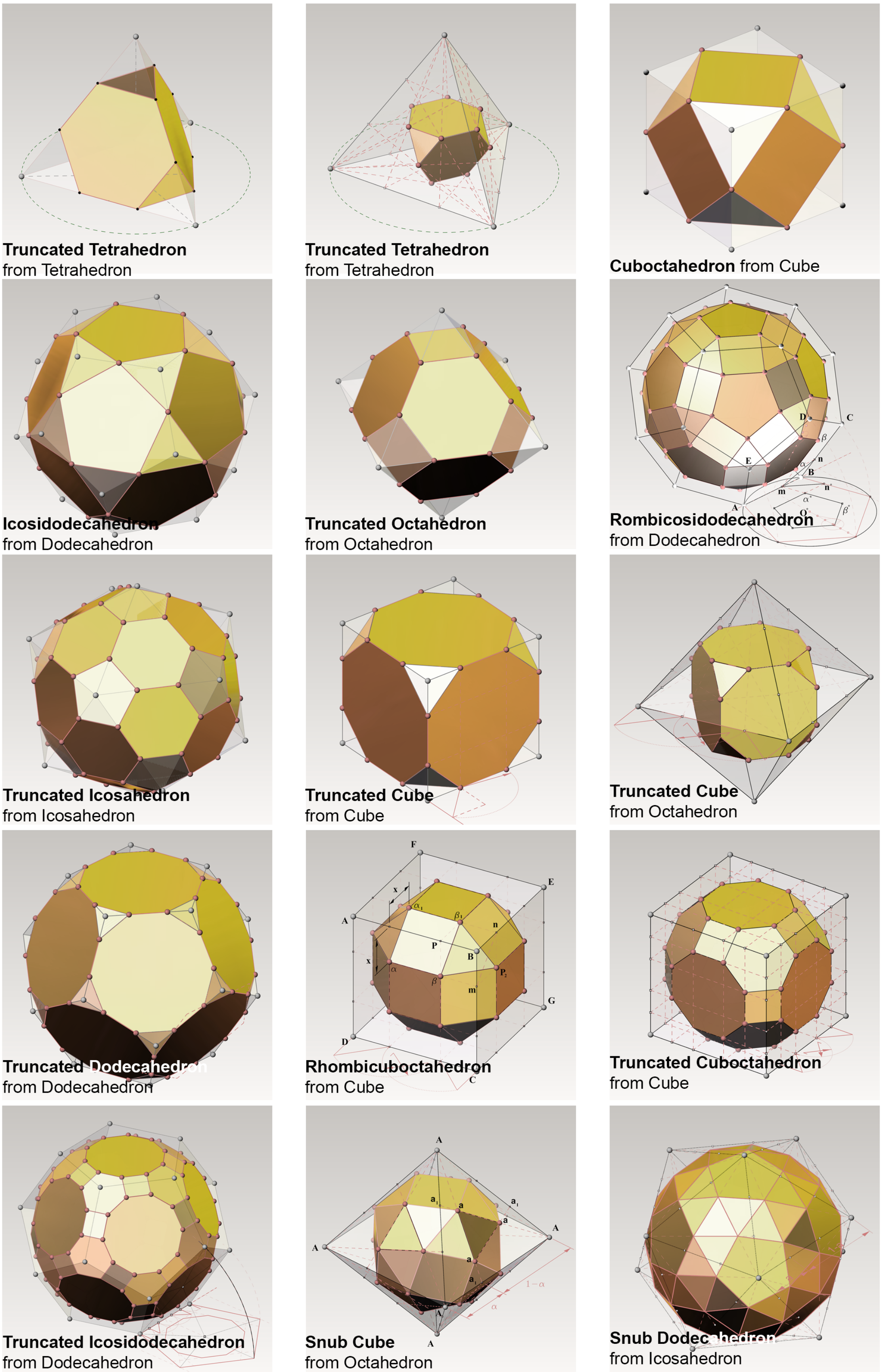
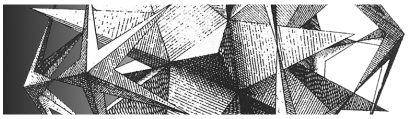


Fig. 2 - Construction of Archimedean Solids starting from Platonic Solids.



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It is not possible to construct the last two chiral solids via SE&C [05: 121-133]. The authors show, however, that it is possible to construct the snub cube using the properties of *origami* controlling and using an analytical language. We propose the same Weissbach and Martini's demonstration, through synthetic representation. In fact, it is possible to construct the vertex A' of the snub cube, starting from the cube that envelops it. We call the top edge g and the bottom left corner, P (Fig. 3). We divide the edge of the base into four equal parts and call the second point from the left Q. We trace the two vertical lines from the other two points and name the second h. To find the point A' it is sufficient to fold the plane in two so that the points P and Q, respectively, fall on the lines g and h. This fold is exactly the sixth axiom of Humiaki Huzita. This fold allows us to find the points P' and Q' which give the coordinates of the vertex A' of the snub cube.

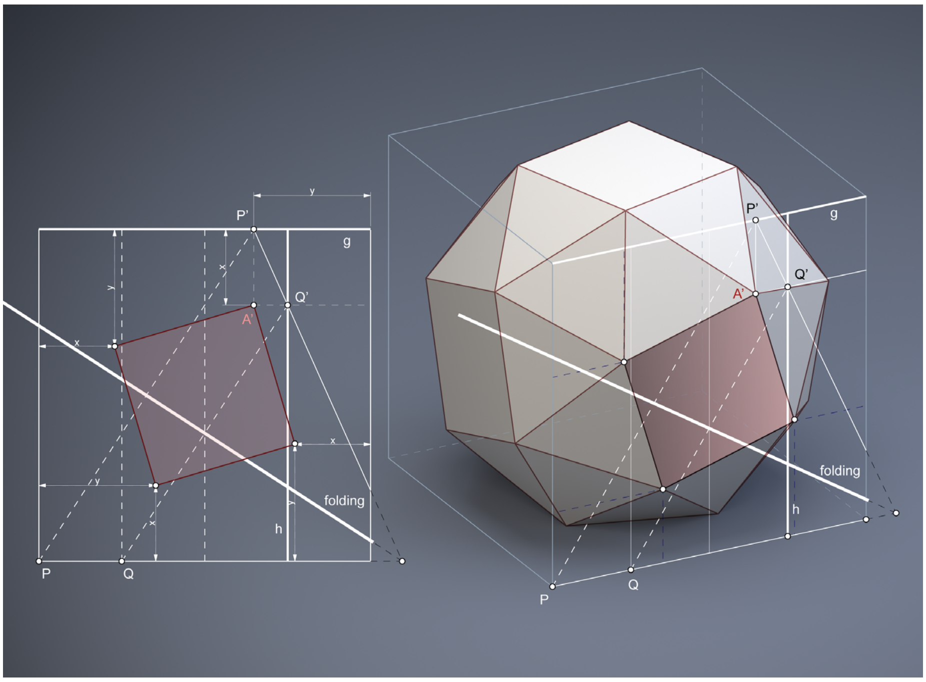

Fig. 3 - Snub Cube construction via paper folding.

It is possible to simulate the first fold correctly, by inserting constructive constraints in the polylines vertexes. Once point A' has been found, through symmetry, it is possible to find all the missing vertices and edges of the snub cube. This synthetic method allows us to obtain the accurate construction of the chiral AS. The property of this sixth fold, as is known, is used to solve two other problems of classical geometry that are not possible to solve with SE&C, such as the trisection of the angle and the duplication of a cube.

The last synthetic method presented here allows us to construct any regular and semiregular polyhedra from its flat development (we developed this method with Riccardo Foschi, a Research Fellow of Bologna University). This method, therefore, is the most flexible one and allows us to accurately construct even the snub cube and snub dodecahedron (Figs. 4 and 5). The example shows the construction of the snub cube (and snub dodecahedron) starting from its flat development. The construction principle is very simple and is based on the physical simulation of its solid construction. As Loria explains, if you actually try to build a regular polyhedron in the real world, the easiest thing to do is to draw a possible flat development and rotate the faces until the free edges touch. In this way, the only possible final solution is the one sought. It is evident that another final combination of faces is not possible unless the faces are rotated in the opposite direction: some clockwise and the others counter clockwise. In addition, in the latter case, the final result would not be a closed polyhedron.
In order to apply this simple method we have employed a live physics engine within a mathematical modelling environment (*Kangaroo*, a *Rhinoceros* plug-in). The procedure consists of these steps:

1) plan a possible flat development of the regular or semi-regular polyhedron to be built (Fig. 4 depicts a snub cube);
2) now, set some constraints. Some vertices must remain fixed in the transformation (in the case study, we chose the five vertices of the central face); and all the edges must remain the same length.
3) apply active forces to some vertices; the vertices must be chosen with consistency and, of course, it is not necessary to apply these forces to all the vertices of the figure.

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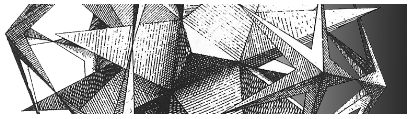

4) to start the simulation, it is necessary to apply an initial thrust otherwise the points will all remain in the same plane. We applied an initial down thrust to the vertices of the central face. In the mathematical virtual space, self-intersections are possible, so there is no danger that inconsistent rotations of the faces will be generated; if this happens, the algorithm just takes more time to reach the result.

The delicate issue of this physical algorithm is the balancing of forces. We have noticed that when the individual weights of the forces are not well balanced, the algorithm generates imploded solids and fails to close the figure in an adequate time. In order to overcome this problem, it is necessary to experimentally find the right balance by calibrating the forces involved (Fig. 4). Once the right balance is found, the figure begins to form and in a short time the solid is obtained (it is also possible to calibrate the time and speed of construction of the figure).

In the algorithm, we have included a check that automatically allows us to understand when the figure is correct, i.e. when the solid is perfectly closed. Kangaroo is based on the reiteration of the algorithm up to the optimization of the procedure after a certain number of attempts; the limit we have established is the tolerance of the modeller in recognizing a closed solid; therefore, when the program recognizes the presence of a closed polysurface, it stops the process. The method was tested with regular and semi-regular polyhedral, but we believe that by well balancing the forces and the attractive weights of the selected vertices, it will be possible to solve the construction of more complex polyhedra.

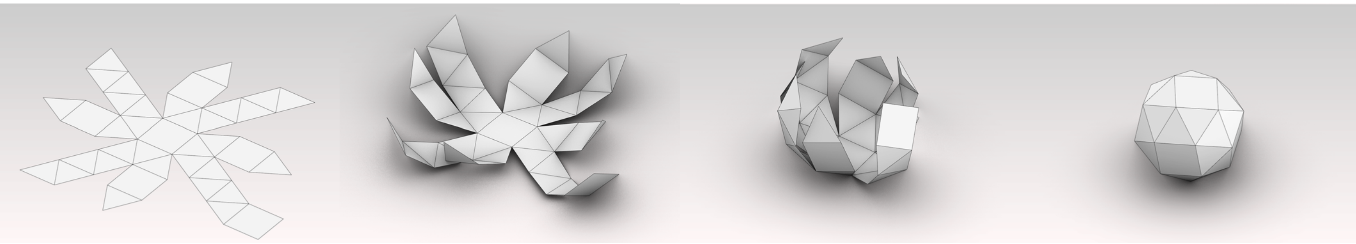

Fig. 4 - Construction process of snub cube starting from its flat development.

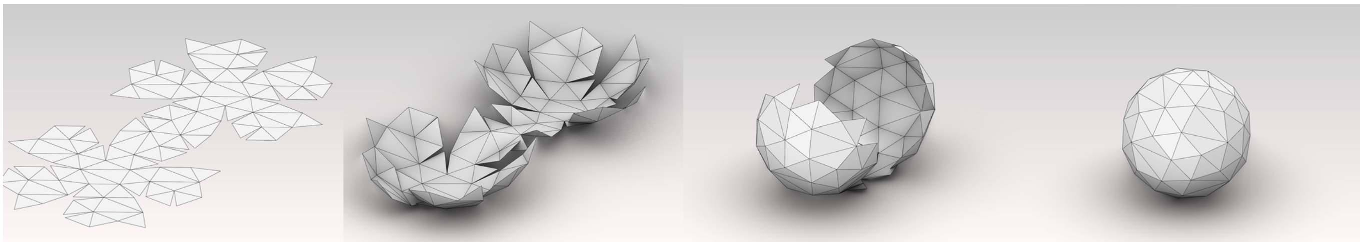

Fig. 5 - Construction process of snub dodecahedron starting from its flat development.

CONCLUSIONS

The study of regular and semiregular polyhedra allows us to deeply understand the constructive potential of the synthetic method. For example, the construction of the snub cube with the method of paper folding, without the advent of the digital revolution, would have been inaccurate, because it would have been linked to the approximation of the paper or the material folded. Today, thanks to digital representation methods, we are able to experiment in space with an accuracy that was unthinkable only a few years ago. Scientific and technological progress has widened the frontiers of the synthetic method and has found in computer graphics a means of interchange between different scientific disciplines. The digital image has taken on a fundamental role in research. Drawing becomes a tool for research and discovery of the properties and relationships of forms in space, increasing the heuristic capacity of the synthetic method.

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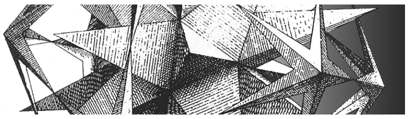

PLENARY SESSION
KEYNOTE SPEAKER

HENRY SEGERMAN

Henry Segerman received his masters in mathematics from the University of Oxford in 2001, and his Ph.D. in mathematics from Stanford University in 2007. After post-doctoral positions at the University of Texas at Austin and the University of Melbourne, he joined the faculty at Oklahoma State University in 2013, where he is now an Associate Professor.

His research interests are in three-dimensional geometry and topology, working mostly on triangulations of three-manifolds, and in mathematical art and visualization. In visualization, he works mostly in the medium of 3D printing, with other interests in spherical video, virtual, and augmented reality.

He is the author of "Visualizing Mathematics with 3D Printing", a popular mathematics book published by Johns Hopkins University Press in July 2016.

Spherical photo taken by Matt Parker at the Gathering 4 Gardner 12 conference, digitally edited by Henry Segerman.

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PLENARY SESSION
KEYNOTE SPEAKER

ARTISTIC MATHEMATICS: TRUTH AND BEAUTY

I'll talk about my work in mathematical visualization: making accurate, effective, and beautiful pictures, models, and experiences of mathematical concepts. I'll discuss what it is that makes a visualization compelling, and show many examples in the medium of 3D printing, as well as some explorations in virtual reality and spherical video.

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PLENARY SESSION
KEYNOTE SPEAKER

MANUEL ARALA CHAVES

Manuel Arala Chaves was Full Professor of Mathematics at the Faculty of Science of Porto from 1973 until 2003, when he decided to retire, in order to work full-time in the Atractor Association. Since the beginning, he has been the president of the board of this non-profit association, created in 1999 with the aim of raising public awareness and attracting to Mathematics.

In 2000, Atractor was invited to create a large exhibition, entitled *Matemática Viva*. The exhibition lasted from November 2000 until August 2010 in *Pavilhão do Conhecimento* (Lisbon) and it had more than 2 million visitors.

In more recent years, Atractor focused on producing interactive virtual contents and exhibits, and many of them can be found on its homepage[1]. Other activities include the DVD "Symmetry the dynamical way" (in 6 languages), several movies in the *Atractor's YouTube Channel*[2], a large collection of interactive stereoscopic 3D mathematical contents in several different formats (3D TVs, side-by-side, anaglyphs, etc.) and some free software, like *GeCla* and *AtrMini* (both in several languages).

[1] https://www.atractor.pt/index-_en.html
[2] https://www.youtube.com/channel/UCymZYeiV6b-86ZpDM3_HspQ

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PLENARY SESSION
KEYNOTE SPEAKER

SYMMETRIES IN PORTUGUESE *AZULEJOS* (AND WHAT THIS HAS TO DO WITH GEOMETRIES)

The abundance and variety of *azulejos* in Portugal can be used as a form of cultural tourism to illustrate the mathematics of frieze and wallpaper patterns. As part of an ongoing project which we will describe, Atractor has developed tools and created a large database of existing *azulejos*, which will be easily accessible to visitors.

It is well known that the number of types of symmetry for frieze and wallpaper patterns is not unlimited: in fact, there are only 7 + 17 different possibilities. However, not even all of these are to be found in houses, and there is a good reason for this. This is not impossible to overcome, and it would be interesting if a town decided to include all possible patterns among its houses.

It is not obvious why there is a limit to the number of patterns, and we will give an intuitive idea of the reason. It is then natural to ask what happens in non-Euclidean geometries. We will see why the argument fails in this case and that the result, in itself, is not true.

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PAPERS' SESSION
PAPER 13

AN INTRODUCTION TO SOLID TESSELLATIONS WITH STUDENTS OF ARCHITECTURE

João Pedro Xavier[1], José Pedro Sousa[2], Alexandra Castro[3] and Vera Viana[4]

KEYWORDS: Architectonic Tessellations, Three-dimensional Software, Architectural Design.

INTRODUCTION

This research intends to describe a didactic experiment on the exploration of solid tessellations, accomplished in the Geometry course of the 1st year of the Architecture program's Master in the Faculty of Arquitecture of Porto's University (FAUP). In this didactic experiment, digital three-dimensional modelling was explored as a recurrent tool to study and research the topics proposed.

From the vast subject of solid tessellations and its possible applications, students were guided to focus their attention on uniform tessellations, in which regular-faced polyhedra fit together "to fill all space just once, so that every face of each polyhedron belongs to one other polyhedron" [01: p. 68] In uniformly tessellated polyhedra, all the vertices are equally surrounded and superimposable under symmetries onto any other. For the sake of concision and feasibility, only six of the thirteen "architectonic tessellations" [02] were explored by the students in collaborative assignments.

THE RESEARCH

For many years, the Geometry course in the Faculty of Arquitecture of Porto's University was entirely concerned with projective geometry based on the traditional systems of representation in architecture. However, since 2015, its syllabus has been updated with the introduction of three-dimensional modelling software. Besides allowing students to address a broader range of geometric themes (whose exploration through the traditional representational systems would be harder to accomplish) 3D modelling allows students to focus more on geometry itself, than in the representational procedures and its predicaments. When a student creates a virtual model, his/her attention focuses mostly in the geometry of the objects, rather than in the representational system. The fact that digital tools automatically provide an accurate representation of geometric objects, by itself, strongly enhances the students' abilities for spatial visualization, mental rotation and geometric reasoning.

The introduction to digital 3D modelling is structured in a 5-6 weeks teamwork, in which students explore Computer Aided Design (CAD) processes to study a selection of geometric subjects that might have a sturdy impact on architectural design. Each year, a different topic is proposed, aiming to introduce students into themes that are generally unexplored within the course and whose complexity clearly justifies the use of digital media.

On the academic year of 2016/2017, the teamwork focused on solid tessellations, with the following as main purposes:

- to introduce students into polyhedral geometry and in the subject of space-filling polyhedra and solid tessellations, and their possible relations with architecture;
- to overcome the characteristic abstraction of their underlying geometrical structures, by regarding them from an architectural - spatial - standpoint, and trying to explore, in the design process, their intrinsic spatiality;
- to explore 3D computer modelling as a support (and driving force) to study, represent and communicate a geometric design assignment.

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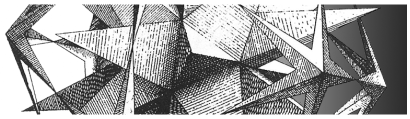

POLYHEDRA AND SOLID TESSELLATIONS

Studying and modelling polyhedra in the educational context stands as one of the most interesting subjects through which geometric concepts can be explored, given not only their tangibility, but the myriad of mathematical contents and branches they allow students to be introduced into [03: 135]. For higher education courses in which modelling and materializing space is a topmost concern such as Architecture, researches on polyhedra might be regarded as a valuable subject matter. This is particularly true for under-graduate students, especially if they are given the possibility to model polyhedra virtually with three-dimensional modelling software, through which students can gain further knowledge on geometric concepts and operations, while exploring them as if "directly in space" [04, 507]. The full extent through which students' geometric reasoning can be stimulated and developed in a CAD educational environment is yet to be determined, and the experiment we are currently presenting is but a minor contribution to this discussion.

Understanding the conditions through which polyhedra can fill space around a vertex, and exploring the concepts of "plesiohedra" [05], space-filling polyhedra, primary paralelohedra and the twenty-eight possibilities [06, 07] through which polyhedra outline uniform solid tessellations or honeycombs, is of particular interest for students of architecture, given their potential for architectural design, for instance, as constructive modules or spaceframes. In this regard, six (Fig. 1) of the thirteen honeycombs considered as analogues to the semiregular plane tessellations, that Conway et al. designate as "architectonic tessellations" [02: 292-298], were selected as subject matter for a teamwork.

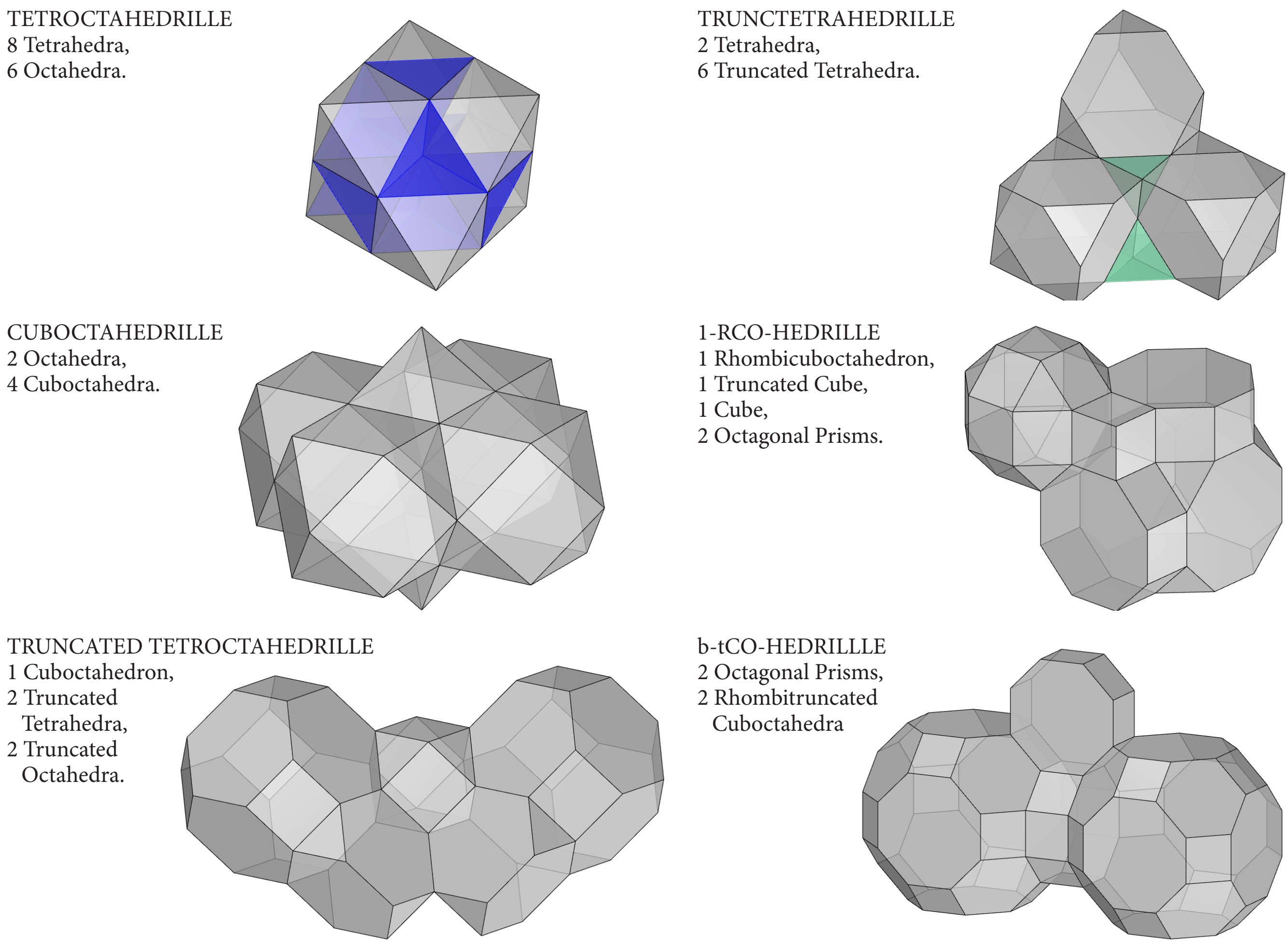

Fig. 1 - The six Architectonic Tessellations selected for the teamwork (designations according to [02]).

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THE DIDACTIC EXPERIMENT

The subject of Solid Tessellations was proposed as leitmotif for a teamwork, in which students, divided in groups of four, were asked to explore one architectonic tessellation, with the purpose of creating a spatial structure to be placed at one of the exterior spaces of the Faculty.

The time scheduled for the assignment was six weeks, and involved six classes with an average of 24 students each. The work was developed according to a methodology that comprised the following steps:

- The presentation of the theme and theoretical framework;
- A brief introduction to the software and demonstration of the commands that would be most relevant for the assignment;
- An understanding of the geometric concepts involved, through preliminary modeling experiments;
- The design and development of the Project itself,
- The presentation of the projects.

From the 28 different sets of convex uniform tessellations, six of the thirteen architectonic tessellations were selected, and one of them assigned to each of the six groups of the different classes. Each group was then proposed to manipulate virtual models of the different polyhedra of their tessellation, aiming to find out, by themselves, how each set would fill space. The task proposed afterwards aimed for students to diverge beyond the abstracting nature of the geometric structure under analysis, considering different possibilities of conceiving a habitable structured space to be installed in the garden's setting of the Faculty.

Following a theoretical introduction, the development of the work in CAD environment was structured sequentially in the following phases:

Phase 1 - Tessellation's modelling and research for its possibilities of multiplication through translation and rotations, exploring different operations of geometric transformations;

Phase 2 - Adapting the geometrical structure conceived to the location chosen for the assignment, by adjusting its scale and orientation, and considering the pos-sibility of including a cutting plane, as a disruptive and creative element, that would trim the entire structure or only part of it;

Phase 3 - With the geometric structure outlined, conceiving the materialization of the spatial structure, considering the following as rules: (1) structural bars in correspondence to the polyhedron's edges, leaving its faces open; (2) total or partial closure of some of the faces, using its vertices as a drawing reference;

Phase 4 - Preparing the communication of the design project, through a poster and a short movie or, alternatively, a mock-up.

The accomplishment of this task by the six classes resulted in the production of 33 geometric structures with six different convex uniform tessellations. The multiplicity and richness of the results, explicit on the different habitable structures imagined, and their spatial diversity are a clear testimony of the vast potential that solid tessellations (and, particularly, the architectonic) might bring as a creative leitmotif for students of architecture. This statement is particularly evident and interesting to look at, when we analyze different works developed from the same tessellation, that originated completely different spatial concepts, as shown in Figs. 2 and 3.

CONCLUSION

Aiming to describe the whole experience in further detail, the full paper that this extended abstract intends to propose, will discuss and acknowledge the potential that the described didactic experiment embodied in the context of the Geometry's course to which it was proposed. This potential was confirmed, in many levels, by the students engagement with the challenge itself; through the contribution of the selected studies and researches on polyhedral geometry for the development of the students' geometrical knowledge and spatial intelligence; the potential of digital tools to support the development of the design process; and, more importantly, the way in which an improved knowledge on the geometric concepts and operations involved was achieved.

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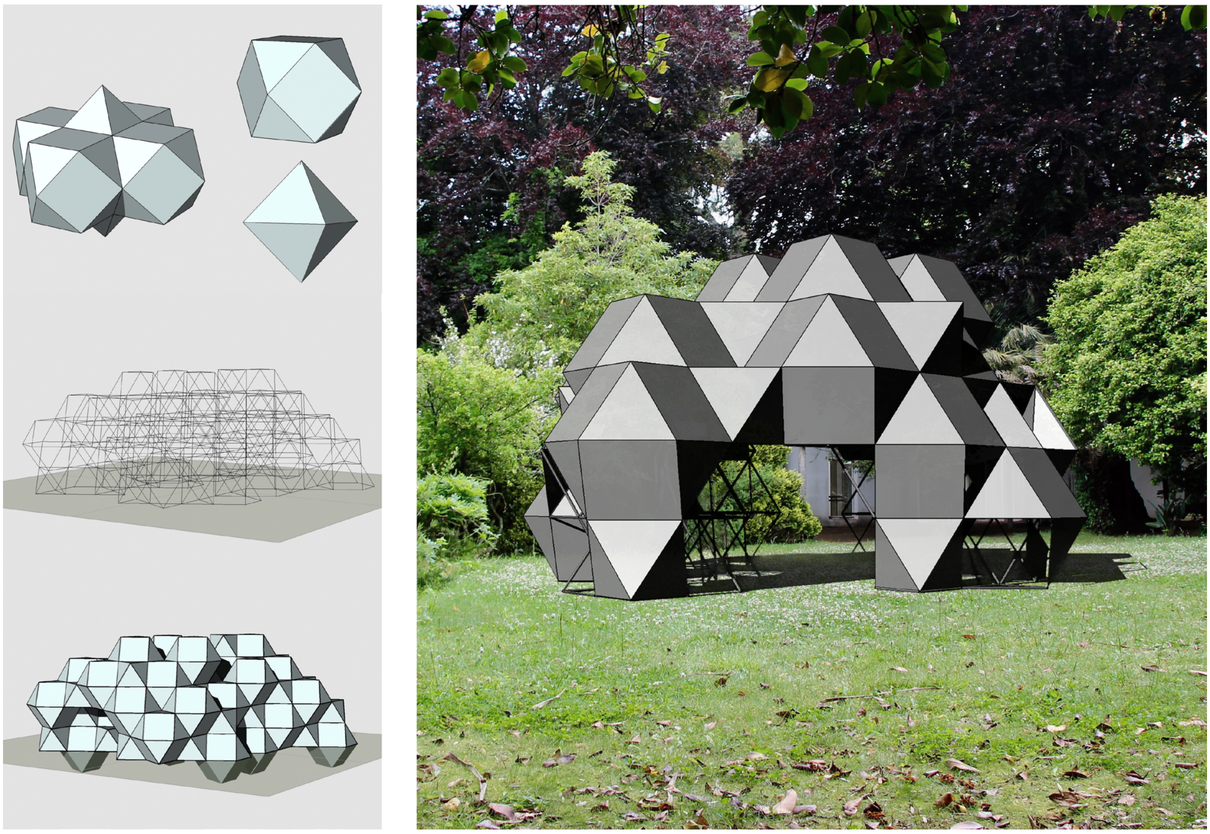

Fig. 2 - "(In)Tangível", Carolina Correia, Cynthia Machado, Nuno Delgado, Pedro Gouveia, 2016-17.

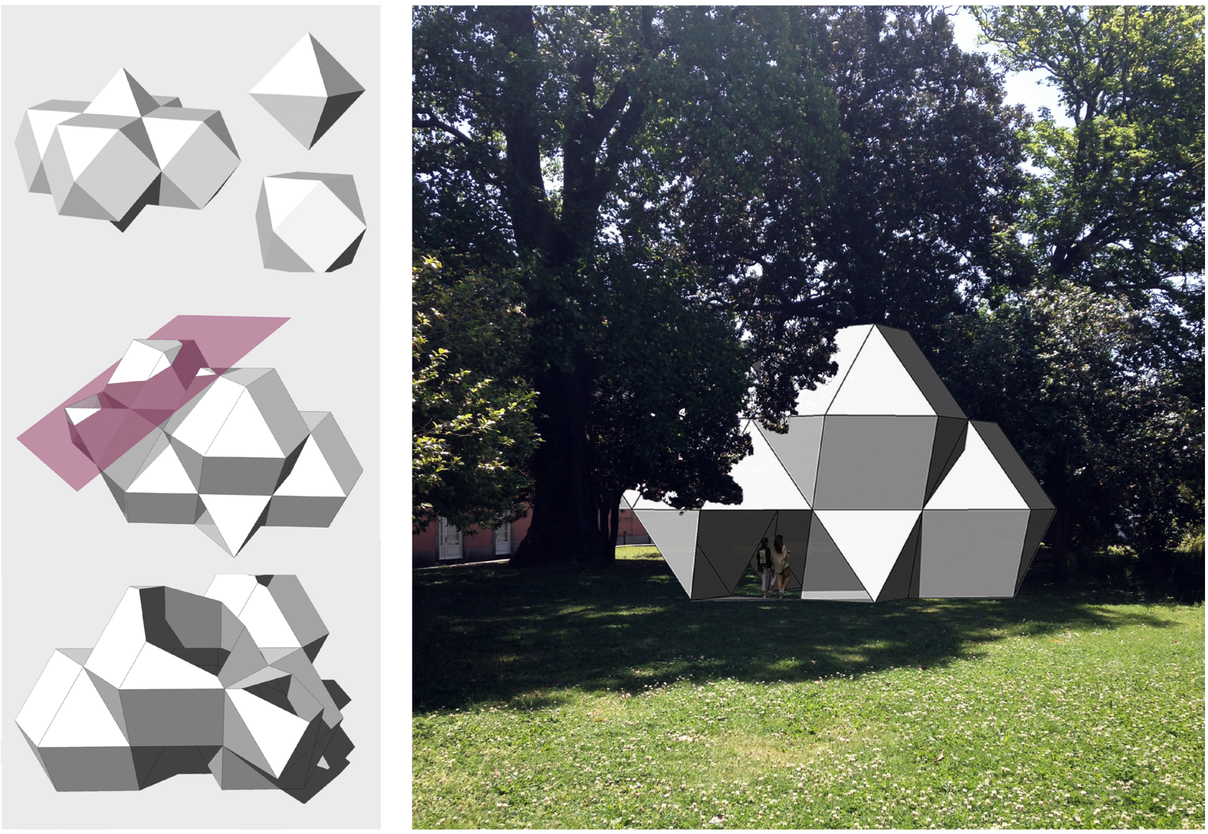

Fig. 3 - "Ponto de Paragem", Eliana Santos, Inês Mateus, Joana Gonçalves, Matheus Aliseda, 2016-17.

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PAPERS' SESSION
PAPER 14

SOURCES AND FEATURES OF THE SMALL STELLATED DODECAHEDRA PAINTED IN GENOA

Cristina Càndito[1] and Ilenio Celoria[2]

KEYWORDS: Stellated Polyhedra, Small Stellated Dodecahedron, Five-Pointed Star, Architectural Perspective, Perspective Treatises.

INTRODUCTION

The present study investigates the history, the significance, and depiction of a polyhedron painted in the *Room of Leda* as part of the decorative plan, started in 1655, of Palazzo Balbi Senarega in Genoa, Italy. The illusory golden oval vault is the main element in the *Room of Leda*, painted by Valerio Castello with *quadratura* by Andrea Seghizzi. It integrates various art forms in its painted architectural structure and figurative mythological insertions. In this same room, one can observe six small stellated dodecahedra. Johannes Kepler, in 1619, was the first to describe this solid figure comprehensively, even if there is an earlier representation in a Venetian mosaic attributed to Paolo Uccello. The study formulates a hypothesis for the meaning of these representations by drawing on scientific and symbolic sources as well as objective evidence that the authors observed on site, revealing that the representation of the polyhedron was altered, in order to achieve an ideal shape.

THE FRESCOES OF PALAZZO BALBI SENAREGA

The subject of the study is a room inside Palazzo Balbi Senarega, constructed by Bartolomeo Bianco from around 1616 for the brothers Giacomo and Pantaleo Balbi. From 1655, after a renovation by Pietro Antonio Corradi[3], the second noble floor underwent the realisation of its decorative plan. The rectangular *Room of Leda* belongs to the original nucleus of the building and is covered by a cloister vault. The decoration of the vault by Valerio Castello with *quadratura* by Andrea Seghizzi (1613-1684) displays a golden oval cupola resting on four sections, each with four ionic columns, standing upon marble pedestals alternated with polylobate cornices (Fig. 1).

Above the illusory impost, there are six painted circular windows alternated to six stellated polyhedra. The myth of Leda and Zeus, depicted in the form of a swan, is represented in a portion of sky inscribed in the central oval window, so to fully integrate various forms of art, in accordance to the Baroque style. The reason behind the representation of a sensual but composed Leda together with the presence of the four deities Venus-Amor, Minerva-Prudence and Wisdom, Diana-Chastity, Mercury-Peace and Fortune [01: 277] could be that Barbara Ayrolo, wife of Francesco Maria Balbi, probably used this room. It is, in fact, located next to the *Room of Hercules*, which was Francesco's room [01: 13].

THE SMALL STELLATED DODECAHEDRON: SCIENTIFIC AND ICONOGRAPHIC HISTORY

The polyhedron represented six times on the vault of the *Room of Leda* is a small stellated dodecahedron, which is a geometric figure obtained by extending the faces of a regular dodecahedron until twelve pentagonal pyramids are formed (Fig. 2). It is a regular polyhedron with regular identical faces, and edges of the same length - although it is not convex.

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[2] Architecture and Design Department, Italy (ilenio.celoria@unige.it)
[3] For the building's history with reference to the other sources, Magnani, Rotondi Terminiello 2003, pp. 57-58

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As it is known, the first systematic reference to the five regular polyhedra (tetrahedron, hexahedron, octahedron, icosahedron, and dodecahedron) is in the *Timaeus* by Plato (V-IV century BC). Many scholars have studied this topic since then, but only a few showed an interest in the stellated variants of these polyhedra. Among these, it is worth remembering Luca Pacioli in *De divina proportione* (Venice, 1509), dated 1498 in the inscription of his manuscript. The text includes 60 woodcut illustrations by Leonardo Da Vinci, representing regular solids and their stellated and sectioned variants, in the solid and *vaqua* (frame) version. Pacioli also described a stellated dodecahedron, which, unlike the regular solid previously described, is a generic stellated construction, where the pyramids show a reduced protrusion (charts XXXI and XXXII) (Fig. 3a). Similar observations can be made with other authors as well, i.e.

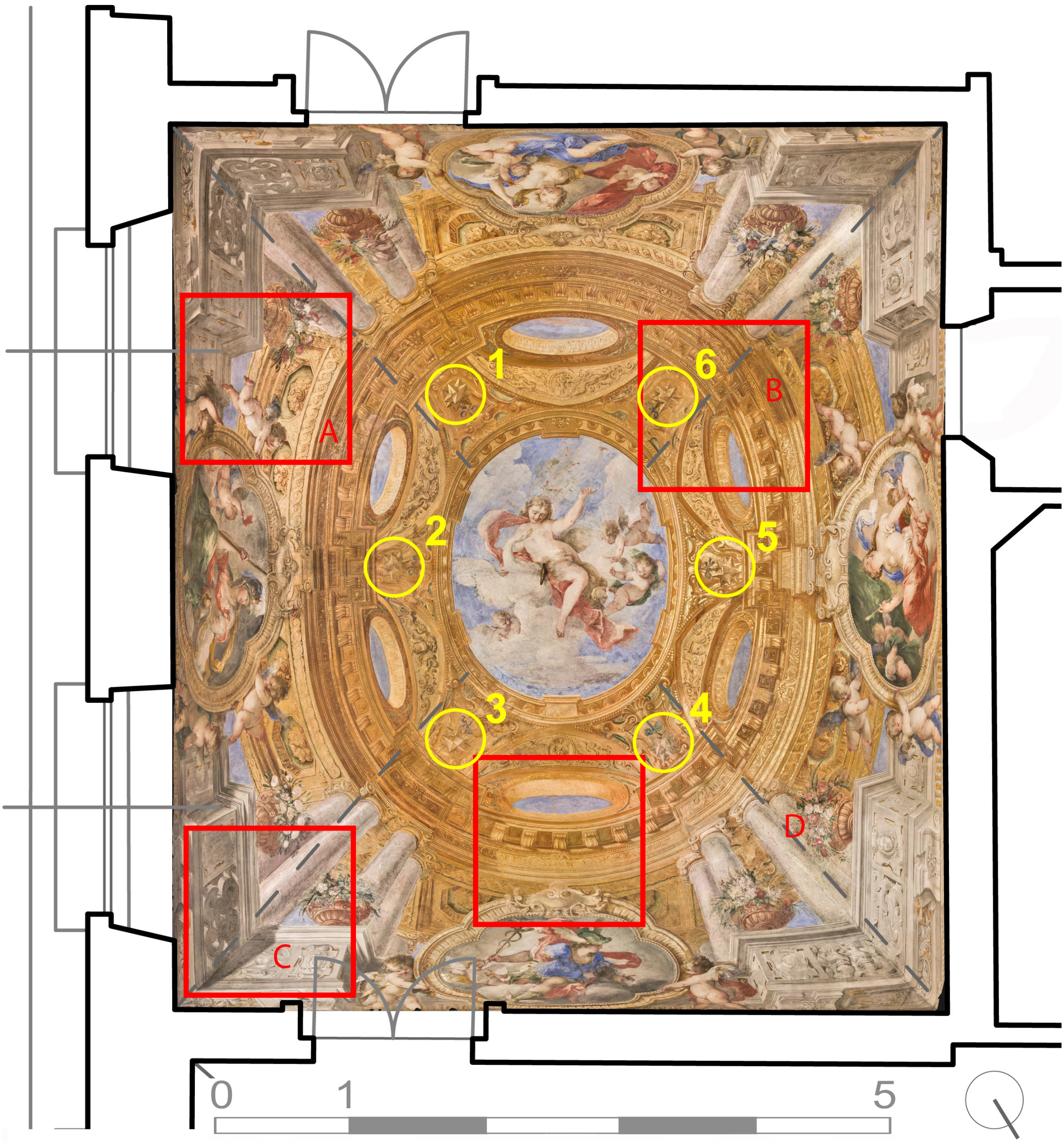

Fig. 1 - Palazzo Balbi Senarega (Genova, Italy), *Room of Leda*, orthophoto of the vault: the six polyhedra (1-6) and details of the shadows (A-D; see Fig. 5).

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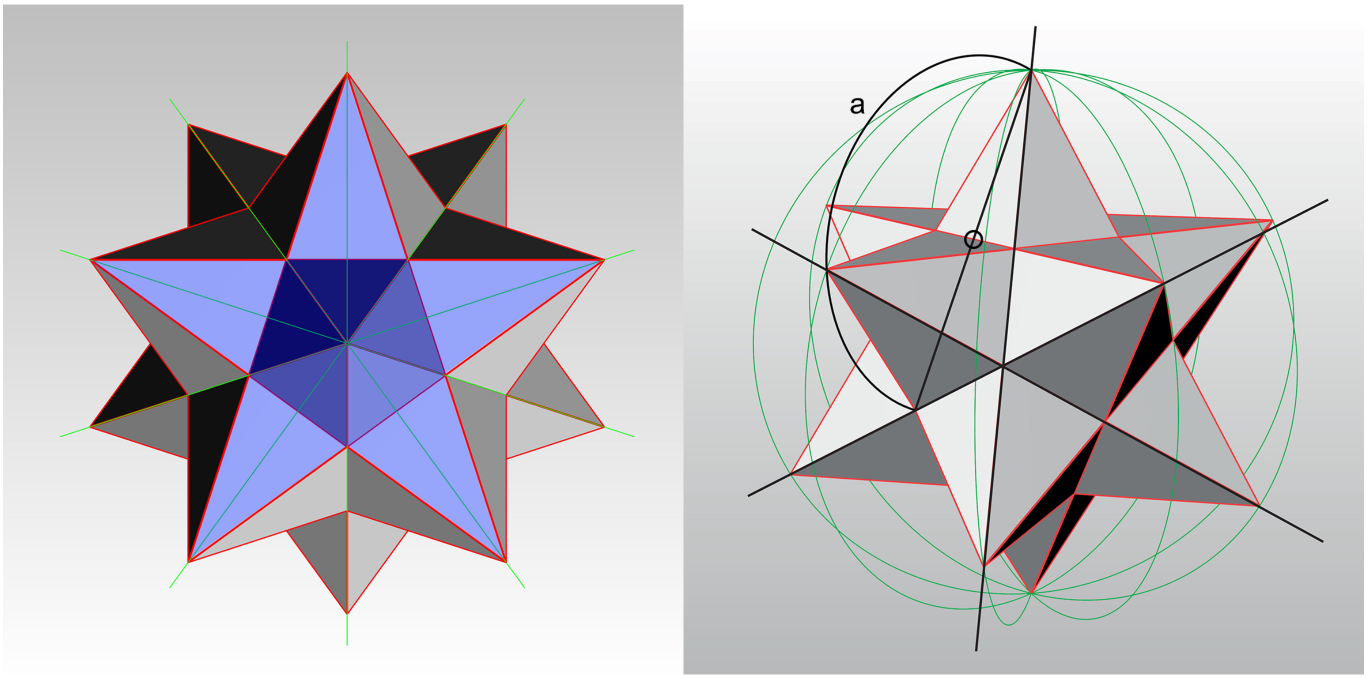
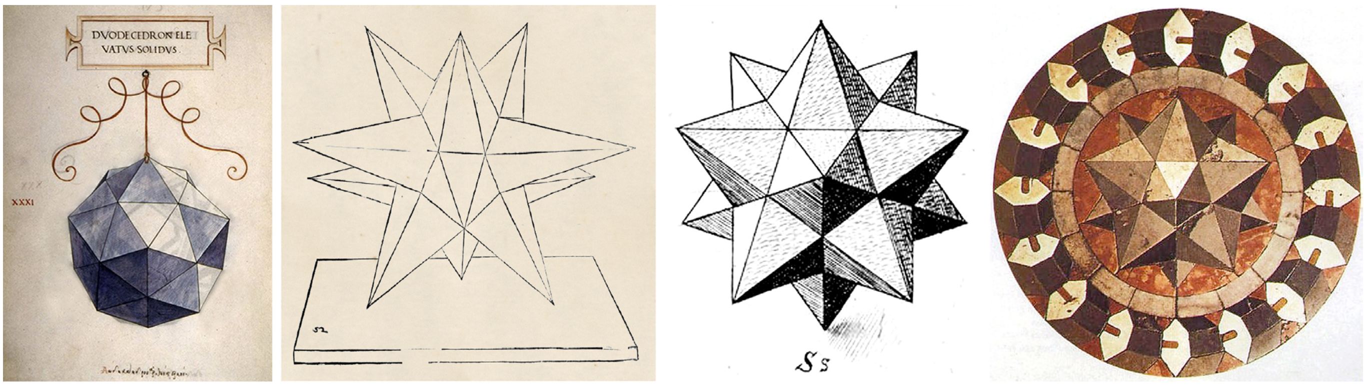


Fig. 2 - The small stellated dodecahedron: orthogonal and perspective view of the virtual model. The construction via extension of the face of the regular dodecahedron (arch *a* for the rabattement of the pentagon's height on the axis of the adjacent triangular face, with indication of the apothem of the pyramid). The five-pointed star is highlighted in blue.

Daniele Barbaro (*La pratica della perspettiva*, Venice, 1569, p. 49) who described a stellated dodecahedron with much higher pyramids than those of the small stellated polyhedron (Fig. 3b).
A comprehensive description of the small stellated dodecahedron was later formulated by Kepler (*Harmonices Mundi*, 1619). With his work, Kepler wanted to provide a systematic treatise on plane and space tessellation, but developing this study also led him to the description of the small stellated dodecahedron (book II) (Fig. 3c)[4], although it seems Kepler did not realise that this was in fact a regular polyhedron, even if not convex [02: 111].
Paolo Uccello (1397-1475), known for his interest in geometry and perspective, appears to have probably preceded Kepler (Fig. 3d) in a mosaic (Venice, Basilica di San Marco, 1425-1431), where he represented the solid in axonometry, with the exception of the central pyramid, which is slightly shifted to one side.



a) b) c) d)

Fig. 3 - The representation of the stellated dodecahedra: a) Pacioli, 1509; b) Barbaro, 1569; c) Kepler, 1619; d) Uccello (?), 1425-1431.

[4] Kepler also described the great stellated dodecahedron (12 stellated pentagons, 20 vertices, 30 edges), and in 1809, Louis Poisot demonstrated the duality of two other non-convex regular polyhedra: the great dodecahedron and the great icosahedron. Even Archimedes (287-212 a.C.) provided his contribution by introducing 13 semiregular solids, which can be obtained by truncating the platonic solids or by substituing some elements.

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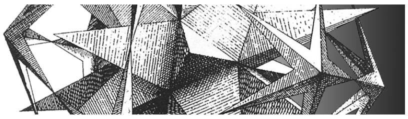

THE SMALL STELLATED DODECAHEDRON IN THE *ROOM OF LEDA*: GEOMETRIC CHARACTERISTICS AND SYMBOLIC MEANING

The representation of the geometric elements in the *Room of Leda* can be attributed to the *quadratura* artist Andrea Seghizzi. It is interesting for the purpose of this study to research his training, and the possible sources he might have used. There is a clear collaboration between the figure painter and the *quadratura* artist [03: 73] that could explain the numerous revisions made to figures and objects during the project's execution, to refine this complex composition[5].
Andrea Seghizzi learned the art of *quadratura*, while working in Bologna, Italy, with Agostino Mitelli (1609-1660). Regarding the sources that Seghizzi might have drawn on for the polyhedra, it is worth considering that artists and scholars were in contact and could have shared the latest findings published by Kepler or the representation of its Venetian predecessor. Further clues might come from the Jesuit College, adjacent to Palazzo Balbi Senarega and that was partly paid for by the Balbi family itself. This connection led to the identification of potential sources used by Seghizzi, which were available at the time in the Jesuit library, and to the mathematics lessons by the Jesuit Giacomo Bonvicini, who left his written notes from that time.
Based on this research, a number of hypotheses can be put forward taking into consideration that the polyhedron in the *Room of Leda* is the regular small stellated dodecahedron, which becomes visible through the superimposition of the outline of one of its perspective images (Fig. 4). This graphic elaboration highlights some particular features of the polyhedron. Some of its parts were wilfully altered to obtain a perspective image that was closer to an ideal shape of the polyhedron: however, these distortions cannot be explained solely by the slight curvature of the cylindrical surface onto which they are projected. In fact, there are two added pyramids that should not be visible in a frontal view (in green in Fig. 4), but one can also see another alteration, i.e. the modified position of the other pyramids (the most evident in yellow in Fig. 4). Despite these alterations, it is still possible to recognise the continuity between the lateral faces of the pyramids with the pentagon of the adjoining base (blue line in Fig. 4), derived by the construction previously described (Fig. 2). Thanks to this geometric construction, it is possible to visualise the five-pointed star and the apparent decagonal contour of the solid.

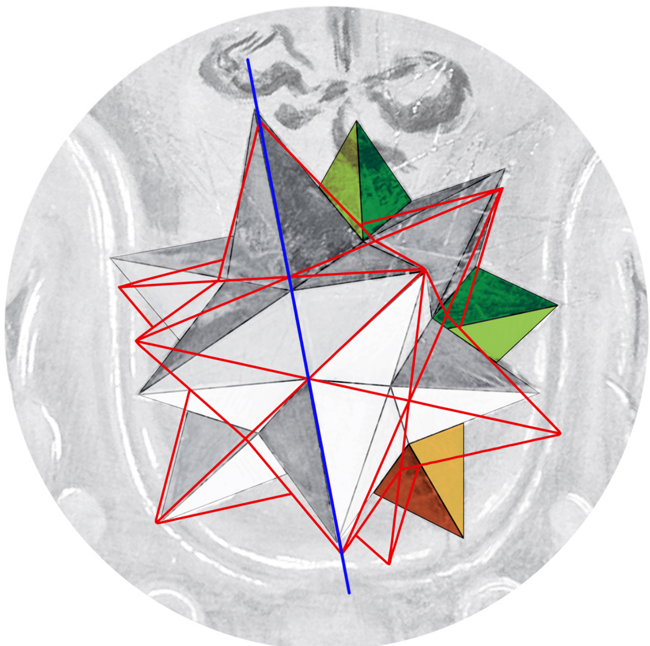

Fig. 4 - On the right, the superimposition with the perspective image of Fig. 2 (with the Point of View placed at the centre of the room), with the two pyramids added in the painting (in green) and the one that deviates the most from the regular pattern (yellow). The blue line reveals the continuity between the lateral sides of the pyramids with the pentagon of the adjoining base, so to show the presence of the five-pointed star.

[5] According to Gavazza [01: 13] this could prove that a collaborator of Valerio Castello participated in the decoration work.

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This aspect prompted further research into the possible meaning of the polyhedra represented in the *Room of Leda*. Generally, regular solids are connected with the concept of geometric perfection. In ancient times, the five-pointed star symbolised health and harmony, after Pythagoras identified its golden ratios. This figure is associated particularly with Venus, the symbol of beauty. Venus was also painted in one of the allegoric representations on the vault - perhaps a homage to the future guest of the room, Barbara Ayrolo.

OBSERVATIONS ON SHADOWS

Further interesting aspects come to light when analysing the shadows of the polyhedra, the importance of which had already been explained by Magnani [03: 73]. The first distinct characteristic is the presence of form shadows alone. The addition of cast shadows would have required drawing more projected pyramids, thus generating a superimposition of lines that would have compromised the artist's search for clarity, as demonstrated by the alteration of the perspective image.

The direction of the light that generated these form shadows is compatible with the room's orientation, only when taking into consideration the real sources of light as well as the fake openings painted on the vault. In fact, without this additional source of light, a comparison of the polyhedra would be contradictory and, furthermore, would fail to explain the other form and cast shadows generated in other parts of the painted vault as, for example, the pedestal of the painted columns (Fig. 5c) or the brackets of impost cornice (Fig. 5d).

CONCLUSIONS

The painted vault in the *Room of Leda* shows various interesting features, such as the first appearance in Genoa of the small stellated dodecahedron, the new geometric element introduced by Kepler in 1619. For this reason, the present study investigated a series of possible iconographic sources as well as the symbolic meaning of this polyhedron and its method of representation. It is here hypothesised that the distortions applied to the representation of the geometric solid in the *Room of Leda*, were executed in order to come closer to its ideal form and allegorical meaning. In fact, the perspective layout of the dodecahedron is altered by adding parts that should not be visible in a frontal view and by eliminating cast shadows. This signals the intention to place attention on the five-pointed star, revealed by the artist's use of shadows that strengthens the allegoric structure of the represented myths, and is a symbol of harmony and beauty, as a possible homage to the future guest of the room.

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Fig. 5 - The *Room of Leda*: detail of the shadows on the vault (see Fig. 1) showing that they are generated by the natural source of light integrated by that coming from the painted openings: a) the upper surface of the opening on the east impost; b) the oval window and the lunette at west; c) the pedestal at east; d) the brackets of the opening at north.

ACKNOWLEDGEMENT

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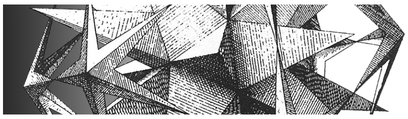

PAPERS' SESSION
PAPER 15

THE "HELE" MODULE OF RAFAEL LEOZ. RESEARCH AND PHOTOGRAPHIC DISSEMINATION OF MODULAR COMBINATION IN ARCHITECTURE

Joan Carles Oliver Torelló[1]

KEYWORDS: *Módulo Hele*, Rafael Leoz, Architecture and Photography, Photography and Geometry.

INTRODUCTION

The extensive iconic program promoted by the Spanish architect Rafael Leoz de la Fuente (1921-1976) to disseminate and deepen the aesthetic and constructive possibilities of the so-called "HELE module", is a significant example of the media strategy that accompanied his publications and public presentations. In parallel with the use of a language with symbolic geometric, social, and mystical connotations, Leoz relied on a powerful visual structure to legitimize his research, linked to contemporary artistic trends close to geometric abstraction and using a projective visual language inherited from experimental methodologies. The present work constitutes a historical and critical analysis of some of his photographs, in relation to the publication of the polyhedral module of his invention and subsequent development of the hyperpolyhedra, relating it, on the one hand, with the graphic and photographic tradition of modular representation in the field of artistic experimentation (especially prolific in the 1960s and 1970s) and, on the other, with the scenography and formal treatment of model photography and architectural projection in the indicated period.

RESEARCH

Rafael Leoz concentrated much of his research carried out between 1960 and 1965 in the publication *Redes y Ritmos Espaciales* [01]. Previously, he had worked with a team of architects in the construction of the *Poblado Dirigido de Orcasitas* (Madrid, 1958), this being the starting point of his concerns for social housing and architectural industrialization. From 1960 onwards, he devoted himself to developing a methodology that, through modular geometric solutions, could be used as a guide for the work of the social architect. Along with the analytical function of diagrams and other drawings of plans and elevations that he continued to make later [02], numerous photographs exemplify the most descriptive character of his method, visually anchoring the consecutive and combinatorial complexity of his discourse. As Leoz himself noted, photography had to become a mnemonic resource, a document to record the different variables that could be adopted during projective ideation.

Analyzed as a whole, among the photographs that accompany the publication of Leoz, some can be distinguished as: representations of solid geometric figures or combinations of these on neutral backgrounds (Fig. 1); sequences of images used to suggest interactions between figures (sections, deformations, displacements, etc.); images that use different angulations and frames of the same modular combination; and, especially, photographs from views and scenographic perspectives

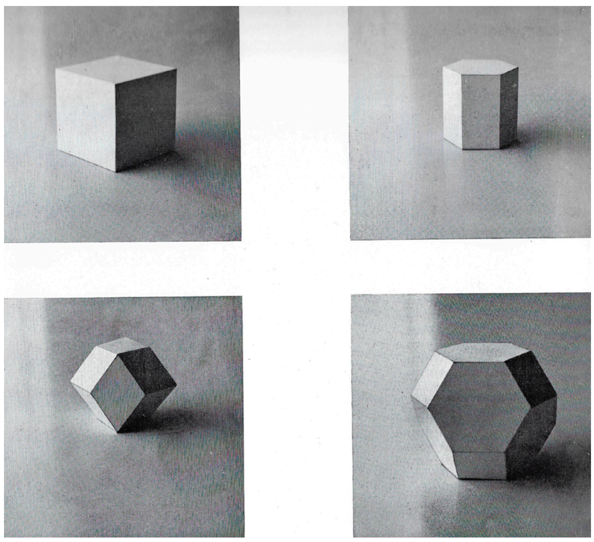

Fig. 1. Cube, Truncated Octahedron, Regular Hexagonal Prism and Rhombic Dodecahedron.

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mounted on landscape or pictorial backgrounds (Figs. 2 and 3), up to bird's-eye views that show the urban integration of his prototypes. Among these last views of scale models, stands out, due to its graphic presence, an untitled double-page image of a group of apartments and residential complexes on the coast (Fig. 4), which years later was attributed to a group of houses designed by architect on the island of Mallorca [03: 11]. In this, regard, it has been indicated that the project of a residence for workers in Mallorca, awarded in the *III Hispano-American Architecture Biennial* (1956), would have been one of the first to awaken his interest in work with modulation [04: 37], and although, judging by the images, it does not seem to be about this same project, Leoz himself commented, in 1969, that the "urgency" with which they had to work for a "quite complex urbanization of a summer center in Palma de Mallorca" (Spain, 1960), together with the aforementioned works in Orcasitas, had made it possible to "intuitively find" the volume that would serve as further modular investigations [05: 20].


Fig. 2 - Architectural model photography.

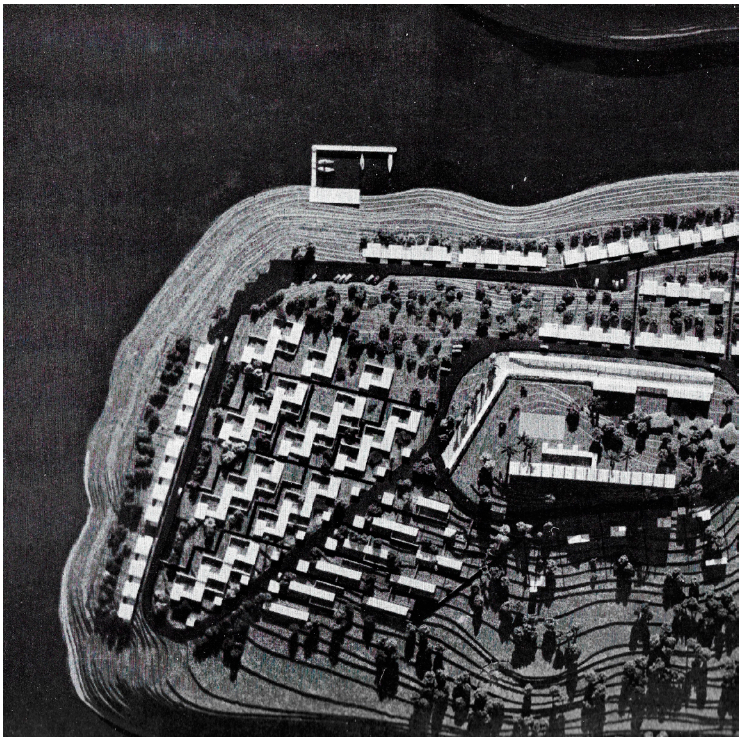
Fig. 3 - Project of a group of homes in Mallorca based on the L module (Architectural model photography).

The historical and architectural implications of the HELE module have been studied and contextualized in various publications, especially by Javier López Díaz [04, 06, 07], in previous monographic publications [08] and articles in the decade of the 60s. His role has also been referenced in the sphere of the geometric aspects of Spanish art of the 60s and 70s and especially in a certain resurgence of this movement in the *Sao Paulo Biennial* of 1961 [09: 136]. In our case, we want to focus not so much on the constructive consequences of Leoz's theories, but on the role developed by the visual and photographic corpus that integrates his work. In this sense, our contribution is linked to the studies that have dealt with the relationships between image and ideation or architectural projection [10] and especially those that show the relationship between the history of photography and architecture [11, 12]. In previous works, we have referred to certain factors existing in the photography of scale models specifically around the images of Leoz [13], so that now we can deepen in the set of appreciations of the author on the role of this medium in the visual particularization of his geometric-symbolic system.

Initially, Leoz's work stems from the plastic consideration of polyhedrons with central symmetry that can fill the space completely: the cube, the truncated octahedron, the regular hexagonal prism and the rhombic dodecahedron, all of which

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are capable of forming a complete spatial network, and whose sections or basic triangles allow to develop reticles that the author calls set square and hemi-pythagorean triangle. From these, several flat networks are derived where possible architectural applications are displayed, pursuing the discovery of a molecular, primary form, capable of linking geometry, aesthetics and functionality. At the end of the study, it includes the study of the tetrahedron, the octahedron and the irregular clog, the latter under the influence of Matila C. Ghyka, transformed into Moebius surfaces to obtain the "openwork polyhedra that continues to fill the space with its edges" [01: 288], suggesting a reflection on the confusion between exterior and interior architecture in the Modern Movement. He focuses on the role of photography in suggesting, "through the opportune changes of scale, a whole world of formal architectural projection" [01: 263]. This section of Leoz's publication constitutes a paradigmatic example of the use of photographic resources in architectural projection, both by the deliberate use of color and by the pictorial backgrounds, that contrast with the rectilinear accuracy of the modules and the close-ups of reflections of what would be their *Stained glass windows with L module* (1971). Among them, the so-called *Hyper-polyhedral Structuring of Space* (Fig. 4), which exemplifies its interaction with concrete art and considers

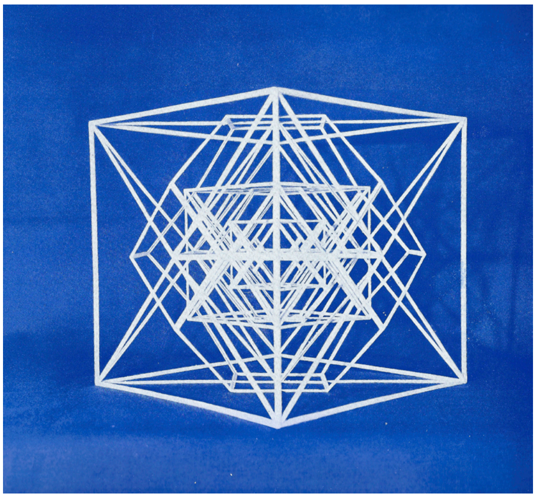

Fig. 4 - Hyper-polyhedral Structuring of Space.

> *"the most interesting symbolic and summarized volumetric representation that I know of Architecture, its content, its functions and the interrelationships between all its aspects and functions*" [01: 156].

Leoz highlights among these combinations the "HELE module" (Fig. 5), composed of four hexahedra arranged in the form of a letter *hele*, which the author exhibited successfully at the *São Paulo Biennial* of 1961. The practical application of this module was reflected in the 218 experimental dwellings in the neighborhood of Fronteras de Torrejón de Ardoz (Madrid, Spain, 1973), promoted by the National Institute of Housing. Faced with the scant architectural definition of his proposals, the terminological and conceptual richness of his research leads him to attribute symbolic, mystical and social connotations to the interpretation of his module, with allusions to the sphere of the symbolic tradition of geometry and with authors such as Luca Pacioli or Matila C. Ghyka:

> *"The already historical studies of Vitrubio, of Vignola, of Luca Pacioli de Borgo, of Matila C. Ghyka, of Le Corbusier, of Neuman, etc., on harmonic analysis, proportionality and regulatory plots, have a huge interest for us [...] We must return to a kind of new Pythagoreanism in our profession, to work and conceive with mathematical rigor, aided by the intuition of an almost mystical spirit."* [01:15]

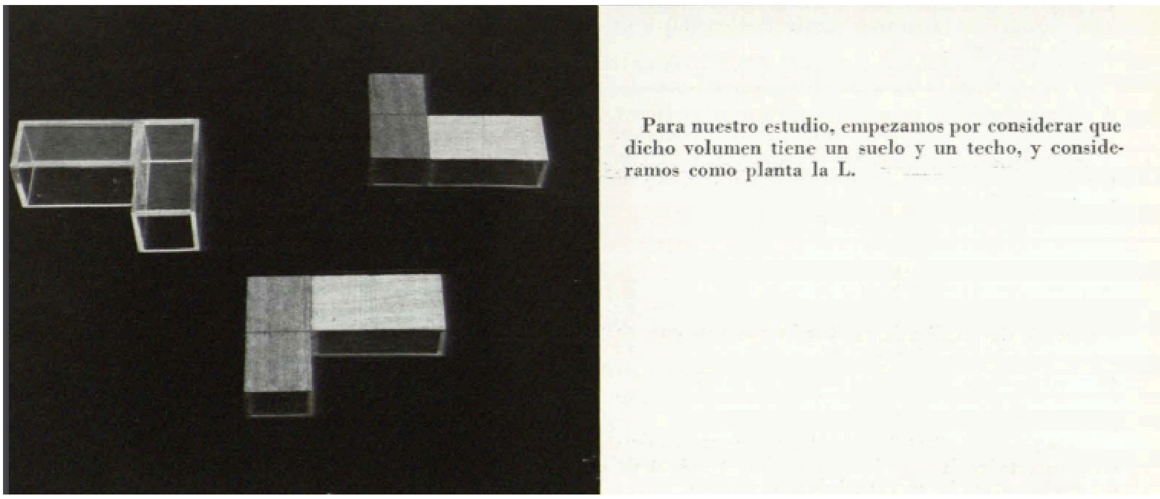

Fig. 5 - The Hele Module considered as an architectural element.

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On the other hand, this geometric investigation was understood as an act of continuity with the artistic and architectural processes of constructivist roots and rationalist claims, clearly linked to Le Corbusier and close to the rhetoric of the first avant-garde movements. In fact, another facet of the use of photographs in the articles of Leoz, although not directly in the case of *Redes* [01], connects with the editorial language of machinism and the New Objectivity, through the metaphorical or conceptual photographic vision of objects, structures and images of nature as cosmovision or iconographic maps of a chronological and vital paradigm. In the article "*Humanism, research and architecture*" [14], the author uses an extensive repertoire of references to scientific images, machinist elements, diagrams and photomontages on proportion systems to exemplify how the interdisciplinary connection of knowledge of what should be participates in the new architecture.

Although Leoz mentions few names as responsible for the design and visual work of his works, and most of his photographs of models or modules of architectural ideation appear without signature or as anonymous among the work of other authors, this does not prevent the ascription of some of them to professional photographers related to the publication of photographs in architecture journals such as Francisco Gómez, especially those collected in *Redes* [01] but previously published in the journal *Arquitectura* [05: 110], which suggest closer approximations to the usual practice of photography of scale models in architecture. On the other hand, we know that Juan Miguel Pando had also made, on behalf of the Comisaría General de Ordenación Urbanística, images of the construction process of the town of Orcasitas, and it is likely that some of the images of the urbanization model in Mallorca published in *Redes* could be ascribed to this same author. In another sense, it is worth mentioning the references of Leoz to the artist Gustavo Torner for the help given in the editorial design of *Redes* [01], as well as the role of the model maker José Alcoba as his main collaborator in the realization of drawings and models. In any case, it is obvious that the research images respond to a team effort and stem from various sources or previous publications compiled by the author. The use of photographic series in these investigations implies a form of dissemination, philosophical and visual anchoring of their theories and, above all, a tool for organizing and documenting their achievements:

> *"We can predict that the large number of different compositions that will come out of our hands will suggest so many solutions that only a system of "memory", such as photography or unambiguous nomenclature, can make us remember everything we have composed."* [01: 207]

The intuition that photography could capture scenes that would have had a conceptual or graphic prefiguration is present implicitly in the author's language. Along with the rest of the graphics and forecast of technological possibilities of other media such as television or computer, photography seemed to recover its prosthetic character, discourses on the ability to capture scenes not visible to the naked eye, or confidence in the role of the medium itself to generate a range of formal possibilities.

CONCLUSIONS

Apart from breaking down the different functions and characteristics assumed by the photographic image in the diffusion of the theoretical work of Leoz, we associate this practice with a previous visual corpus and a "graphic life" of combinatorial and topological diffusion. In this representative tradition, from the first constructivist avant-gardes, the photographic series and similar formal structures are used, especially in the publications of previous authors in the 1950s and 1960s who disseminate their research in the field of experimental architecture and "combinatorial structuralism" [15: 17]. In Spain, Emilio Pérez Piñero had developed a very similar photographic language [16]. Knowledgeable and precursor in Spain of this experimental line, Leoz extrapolates it to a more general system of architectural application and imbues it with sociological and spiritual components. In the specific field of architectural projection of photography, given the variety of proposals and artistic resources used, his body of work occupies an exceptional place.

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FIGURES

Fig. 1 - Published in Rafael Leoz, *Redes y ritmos espaciales*, 1969, p. 63.

Fig. 2 - Published in Rafael Leoz, *Redes y ritmos espaciales*, 1969, w.p.

Fig. 3 - Published in *ABC newspaper*, 03/01/1971, p. 114

Fig. 4 - Published in Rafael Leoz, *Redes y ritmos espaciales*, 1969, p. 157.

Fig. 5 - Published in Joaquin Ruiz Hervás & Rafael Leoz, Un nuevo modulo volumétrico, in *Revista Arquitectura*, 1960, n.15, p. 21.

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PAPERS' SESSION
PAPER 16

INVENTION AND ORDER: THE PROPORTIONAL CONTRIBUTION IN JOÃO MENDES RIBEIRO'S ARCHITECTURE

Joana Maia[1] and Vítor Murtinho[2]

KEYWORDS: Architecture, Proportion, Geometry, Order.

INTRODUCTION

On a journey marked by alternation of disciplines (architecture and scenography [01, 02, 03]) João Mendes Ribeiro (JMR) stabilizes a particular methodology worthy of analysis, fertile in interdisciplinary relations. *Essence*, *efficacy*, *abstraction*, and *elegance* are characteristics that Graça Dias underlines in defining the work of JMR, qualities naturally provided by "refinement, a delicate way of feeling and establishing proportions […], [setting] an accurate drawing"[3] [04: 14]. Geometry is the instrument in a process of high management capacity where debugging, clarity of language, economy of means, are the result of surgical interventions capable of producing signification, flexibility and adaptability. The word *proportion* dominates this equation, where *consistency* and *rigour* extend from the prior study to the completion of the constructive process, in a salutary relationship between parts understood as a whole. The *invention* of the project is sustained in the *order* component, intellectualizing a method that revives a forgotten theme in contemporary theory: proportional value.

RESEARCH

The aim of this research is to understand the role of proportion as a tool in contemporary project, in the methodology of the Portuguese architect JMR (1960-)[4]. Graduated in Porto (1986), with a teaching practice linked to Coimbra's School, he had the opportunity to work closely with figures such as Fernando Távora, a fact that consolidated his way of understanding architecture. The necessary care of project management, controlling a powerful focus of social/personal transformation; the sensitive attention to the site; the recognition of history value; the importance of drawing; the solidity and enhancement of the construction process; the sensitivity to the potentiality of materials, are just a few assumptions shaped by an intense intellectual sharing with larger names of Portuguese architecture, consolidating a *praxis* improved with experience. His method follows a minimal drawing of an assumed rationalist bond, where the influences of Mies van der Rohe or Donald Judd are admitted by the author [05: 190]. This fact, sometimes touching abstraction and enhancing openings of experience and meaning makes, whenever possible, the bet of an understanding of space in metamorphosis, capable of introducing flexibility and adaptability as imperative requirements of contemporaneity. If architecture tectonics seem to demand it more and more, the transience of scenography emerges as an experimental field, promoting the transfer of ideas between disciplines. The notion of scenographic ephemerality (and representation) forays the field of architecture in an attempt to make the perenniality of architecture more flexible, incorporating also a certain dominant "magic" of the theaters atmosphere: if the case is verifiable in projects made from scratch, it is underlined in architectural heritage interventions subject to successive (because necessary) readaptations (Figs. 1 and 2).

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[2] Centro de Estudos Sociais, Departamento de Arquitectura, Universidade de Coimbra, Portugal (vmurtinho@uc.pt)
[3] Free translation from: "refinamento, um modo delicado de sentir e estabelecer proporções […], [configurando] um certeiro desenho".
[4] This abstract is part of a broader research in progress, focused on understanding the value of proportion (in its classical Euclidean version) in the practice of the contemporary project in Portugal. With a selection of nine architects configuring case studies, several works and methodologies are analysed aiming to perceive, within a broad observation framework, not only the systemic types of practices and their relation with the current cultural context, but also the definition of the concept in its full capacity. This research summarizes one of these cases.

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Fig. 1 - Scenography "As orações de Mansata" (2013); Interior perspective of the Center of Visual Arts, Coimbra (1997-2003).

Fig. 2 - Scenography "Vermelhos, negros e ignorantes" (1998); Interior perspective of the Chimico Laboratory Museum, Coimbra (2001-2006).

Despite the search for a synthesis that condenses only the essential, practice doesn't end in the mathematical abstraction revealing a humanistic concern extensible to a methodology rich in its personalized complexity. From the definition of the program to the analysis of the site; from topographic directives to guidance premises; from edified spaces to non edified ones; from the relationship building/body, several are the routes of exhaustive knowledge able to establish bonds with the new intervention. Regulating lines (of material or immaterial nature) occur as connecting channels to establish precise links for the construction of an ordered design, with particular emphasis in an initial phase. Simple geometric figures, starting from the most perfect of quadrilaterals[5], arise as gauge for the construction of accurate definitions during the composition (Fig. 3)[6]. Modular systems appear as a stabilizing rule of particular importance, regarding the constructive matter and stereotomy of materials. Different proportional systems are applied at different stages of the process, communing with the creative strand as well as a required constructive rationalization. However, it is important to note a continuity and interpenetration of the different phases and systems, not being the choice of a dimension that supports the development of the project a mere abstraction in relation to the: site, construction, concept, Man. It is up to the constructive phase to rigorously fine-tune the pre-established dimensions, with a particular focus on aligning the various systems adopted, now adjusted to detail without prejudice of the conceptual structure. Although, in this case, the *geometric proportion* largely surpasses the *arithmetical proportion*[7], with no tradition in its numerical logic and corresponding multiple and sub-multiple, this fact doesn't result in contempt for the measure. A particular anthropometric concern is played in first instance in the intuitive/sensory capture of the experience of the space, assimilated in knowledge by the fixation of the exact dimension. The measuring-tape is used daily by JMR for continuous surveys of tested experiences, in order to obtain a database that is adequate to the comfort of his

[5] "An obsession I have is the shape of the square [...]", free translation from: "Uma obsessão que eu tenho é a forma do quadrado [...]"[06].
[6] JMR keeps returning through the drawing to these diagrams, namely in the black notebooks of daily use, searching for the stabilized dimensions for rectangular spaces: despite the minimum difference, the change of scale in length is considered relevant.
[7] "I give the fundamental dimensions and the rule, and deep down these rules are what interpret the very concept of the project", free translation from: "Eu dou as cotas fundamentais e dou a regra, e no fundo essas regras são o que interpreta o próprio conceito de projecto" [06].

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own body (Fig. 3). The criteria is, thus, that of the study of standard measures, not on a universal sense, but on the architect/character's own tested life setting proportions by way of dimensional registration. From the fascination with the minimal dimension or comfort of relationships, to the tense provocation between bodies in space in restraint/liberation, he shows that proportion exists, although it does not function in the traditional pursuit of stable harmony, but in the search for emotional intensity. An investigation not detached from the experience with scenography[8]. It is proportion that supports the idea, even if destined to escape a preconceived notion of balance: both harmony and conflict can and should follow a path of rigour. The notion of globality also boosts the development of proportional exercise in both plan and altimetry, in the conjugation of pure volumetry and its real materiality, promoting the organism to an integrated whole.

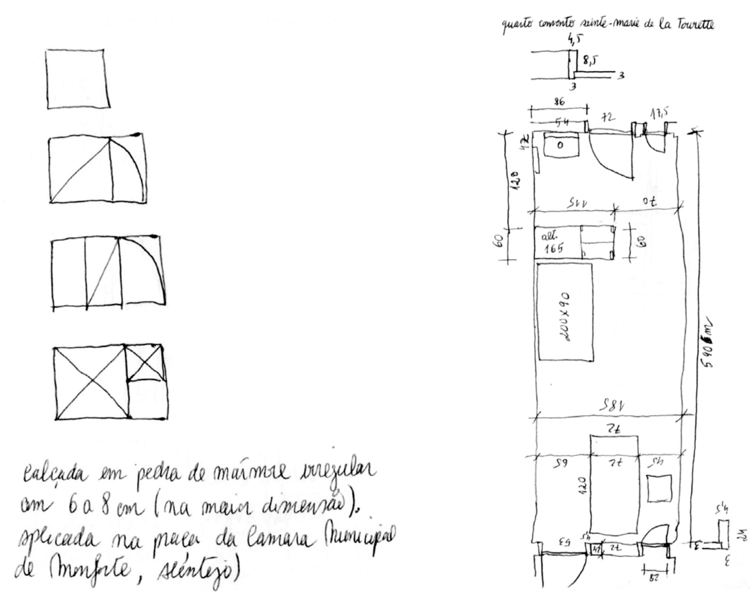

Fig. 3 - Study drawings: Sequence (square/√2 rectangle/golden rectangle/square and a half rectangle); Measurements (room of the Convent Sainte Marie de La Tourette, Le Corbusier).

In order to deepen this methodology where proportion plays a crucial role, two works by the architect of Coimbra are proposed for analysis, exemplifying his theoretical thought.

The expansion of the house and the construction of the swimming pool in Chamusca da Beira are clear examples of a practice devoted to a permanent proportional sense. A wall draws the separation of "two eras", a kind of an organic axis of conceptual symmetry. Topography emerges as an initial theme, with other pre-existing parameters (natural and constructed) contributing to the germination of the volumetric definition. The existing/proposed connection takes place in contracted axes, enhancing relations close to the body, taking the new volume defined not only by the ratio 3:1 of the organism in plan (seized in the elevation drawings), but also by the dimension 0.105m (of the constructive scope). The modular game also arises on the pool platform and its attached dependence (along with regulating lines and 2:1 transverse ratios, from the module to the overall volume and platform), where volumetric versatility based on mathematical systematization owe a lot to the scenographic experience (Fig. 4).

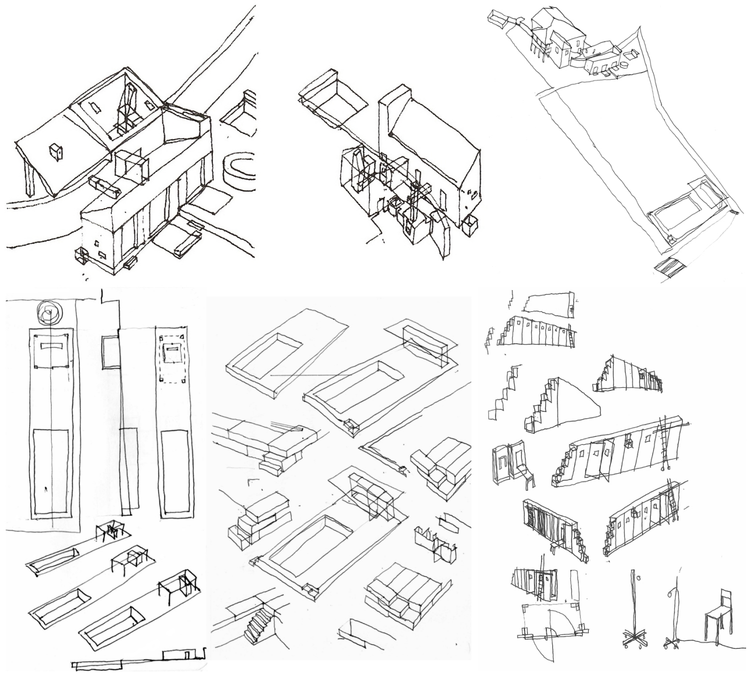

Fig. 4 - Study drawings: Chamusca da Beira's house (expantion), global perspective of the intervention; Chamusca da Beira's swimming pool.

[8] In the work of scenography, JMR starts from the actual body of the actor as a generator of space, for a reciprocal interaction between interpreter and physical environment, conforming a highly personalized process. In the practice of his architecture, that is to say in the "stage of life", given the impossibility of measurability, he assumes (himself) the role of actor/interpreter, projecting in the comfort of true users, his personal comfort in the living of spaces [05, 291-302, 7, 8]. In this way, his research emphasizes a sensorial aspect, extendable to the interaction with its direct agent (the Man), showing its humanist inclination.

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The reflecting facades of the surrounding natural environment manifests itself in as an extra-systemic strategy, extending proportional control to other spheres.
The Tea House in Montemor-o-Velho appears in a context of intervention in the architectural heritage. With all the symbology associated with the tea ritual, the project does not neglect geographic/orientation care. Nested within the ruins of the Alcáçova's Palace (intra-castle walls), the new volumetry is drawn on an elevated platform configuring interior and esplanade in a √2 relationship, whose character of detachment to the ruins follows the theoretical directions of *critical restoration* of Cesare Brandi [09]. In a scenario where scenography forays the real space, the modulating work, along with the accuracy of simple geometric figures or subtle symmetries, promotes the systematization of an ordered composition, transferring protagonism to the pre-existence. As in the previous example, the centrality and precision in the alignments (elements, materials) competes for cohesion manifested in a clean, unitary drawing (Fig. 5).
The analysis of the practice requires a necessary revision of the theory. In order to perceive the totality of JMR's method, improved by permanent self-criticism, it is necessary to intersect different scales by apprehending a proportional work to: the territory and contextual environment (physical, historical, social), the scale of the building, the constructive work, the relationship with the body. The need to implement an intellectual logic that justifies the choices made (promoting the construction of a coherent design), is the motto required in a discipline where the inevitable inventive strand leads a conceptual narrative, yet properly controlled.

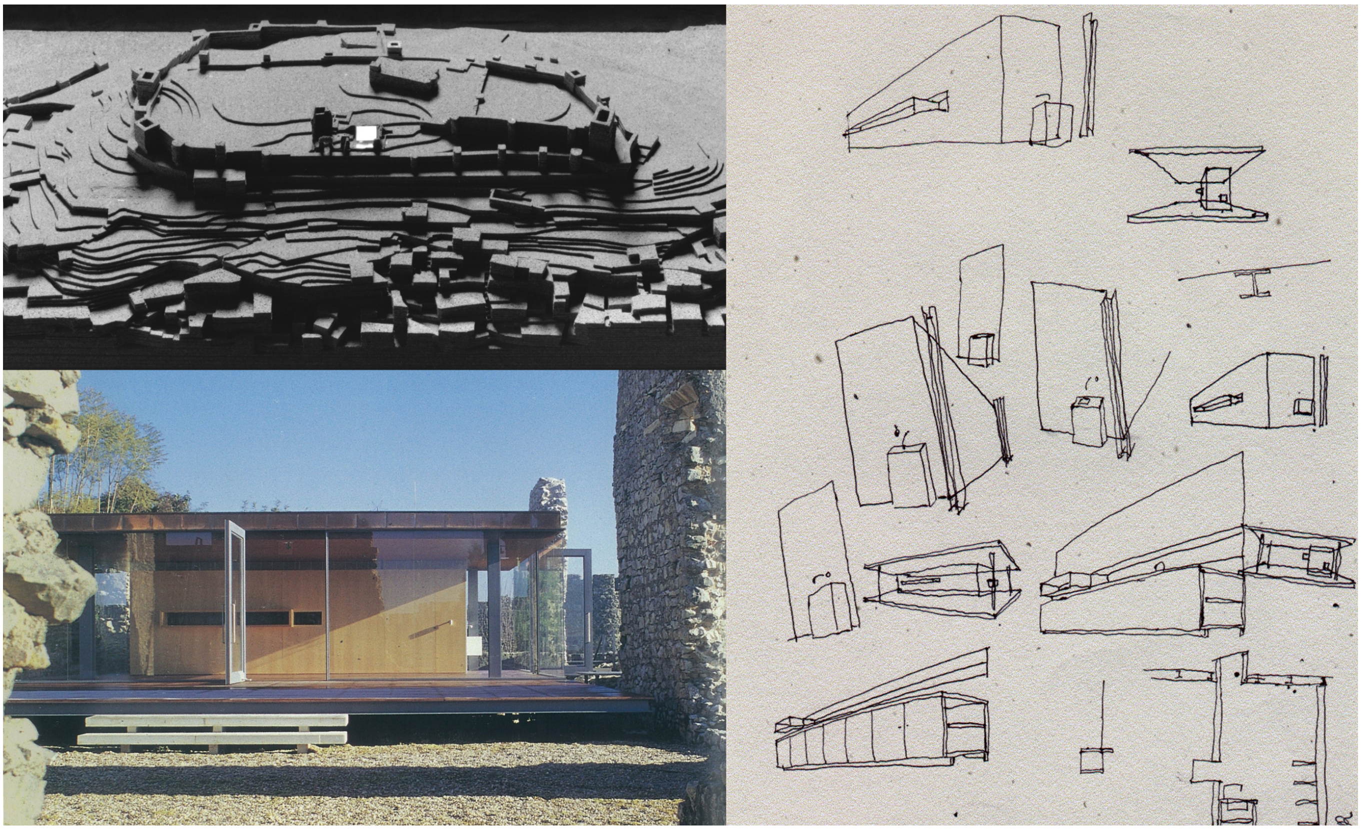

Fig. 5 - Model; External perspective of the Tea House; Study drawings.

CONCLUSIONS

The establishment of norms in the work of JMR is imposed not as a limitation but a guiding tool, leading to rooted results that not only enhance procedural safety but also simplicity and clarity of justification, facilitating communication between the various agents (project and construction). A "security matrix" ensures the "eternal return" to the resolution of problems, where the appearance of accidents eventually occur as a fortunate event, due to the malleability they introduce into the system, contributing to an equilibrium of assumptions. If the word

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invention, in its etymological root, embraces both *freedom* (creation, discovery) and *confusion* (fakery, lie), *order* emerges as an inevitable key capable of restoring "truth" in this interplay of independent, yet relatable, meshes promoting the restitution of the real (because total) balance. In a current scenario where the complexity of contemporary life proliferates, *proportion* arises with renewed significance to the encounter of new harmonies.

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FIGURES:

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Fig. 2 - Credits: [5,p.149]; Fernando Guerra, retrieved October 14, 2017, from: http://ultimasreportagens.com

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PAPERS' SESSION
PAPER 17

POLYHEDRONS IN THE WOODEN TEMPLE ARCHITECTURE OF ANCIENT RUSSIA

Olga Melnikova[1] and Svetlana Shuvalova[2]

KEYWORDS: Wooden Temple Architecture; Architecture of the Russian North; Ancient Russian Church.

INTRODUCTION

The author analyzes the compositional and tectonic features of religious buildings of old Russian architecture between the X and the XVII centuries. During many centuries of Russian history, wood was the main construction material, even when there were buildings of natural stone and brick. This abstract denotes that the nature and physical properties of wood as a building material largely determined the construction techniques and tectonics of architectural forms of both ancient and modern churches. The originality of each temple building of that time was the result of new compositional combinations of a relatively small number of traditional forms. It was in the wooden architecture that the main construction and compositional techniques were developed, which subsequently influenced the formation of stone architecture [01]. Some of the studied buildings are preserved, but others do not exist at the present time. The existing buildings are: Nikol'skaya Church in the village Lavle, Arkhangelsk region (1587); the Church of Dmitri Solunsky, which is situated in the village of Verhnya Uftuga, Krasnoborsky district of the Arkhangelsk region (1784); the Ascension Church in the village Kusherek (1669); the Church of the Transfiguration of the Kizhi Pogost, (1714). The non-preserved building is the Church of Michael the Archangel, built in the village Yuroma in 1686.

RESEARCH

Architectural archeology allows us to look into the early period of the development of wooden architecture of the old Russian state, and the preserved monuments give us the opportunity to trace the evolution of wooden architecture until the present day. The most preserved monuments of wooden architecture are located in the Russian North. As they are located in a relatively favorable natural environment, they were not affected by the Mongol-Tatar yoke and the later post-reform reconstruction [02]. It is here that you can explore the formation and development of the main types of religious buildings, to study the basic composition and planning techniques, as well as to retrace the obvious principles of the relationship between nature and architecture.

Obviously, the organization of ensembles of residential, commercial and religious buildings in villages was caused by the unity of the building material: the average length and diameter of the logs set the vertical and horizontal rhythm and module of all buildings of that time. To determine the size of future buildings, Russian masters have developed a system of simple relations, which were based on the Russian system of measures *sazhen* associated with the average size of the human body. The main element of almost any building of that time was a square crate. The ratio of the square side to its diagonal formed the basis of conjugation principle of Russian measures: *measuring moving sazhen* - side of the square (176,4 cm) and the *great slanting sazhen* - its diagonal (249,5 cm).

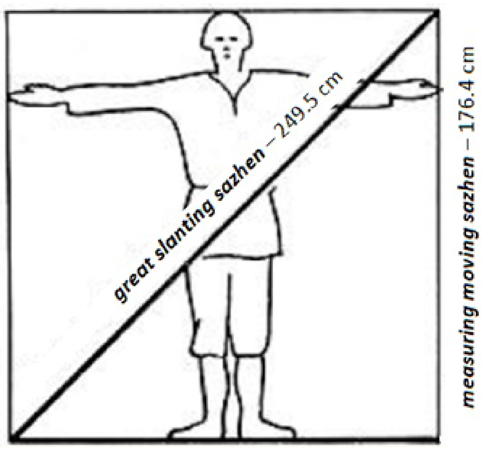

Fig. 1 - Russian system of measures (the stylized drawing) [03].

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And the rest of the length measures used in the construction related to each other as a side to the diagonal of the square. Old Russian measures of length allowed the constructors to feel all the time the size of a man during the construction. The same cannot be said about modern design, because it operates with the abstract meter. The methods of proportionating measures developed in wooden architecture were later passed into stone architecture.

It should be noted that the old Russian system of proportions was the result of a close harmonious relationship between the size of the structure and the nature of the architectural form [03]. But, in addition, another flexible system of proportions was used, based on the principle of geometric similarity of the facades of the building and its details, which allowed one to feel the uniqueness of each structure with the repeatability of its elements. At the heart of this system, is the property of our vision, which allows us to recognize various objects which are distant from the observer or unequal in size and have a common structure (composition); the similarity of the elements helps to memorize this composition.

In the construction of wooden religious buildings masters used different compositional techniques. So, constructions, square in the plan, were often overlapped on *eight slopes* (with pediments on each side) or the crossed *barrel* roofs ("*cruciform barrel*"). Centric building (*quadrangle*; *octagonal*) of small height were overlapped with four-sided pyramid, which was called a "*cap*", and taller ones were called polygonal pyramid, which had the name "*the tent*".

Sometimes, the building was crowned with another type of coating: the "*cube*". It is a kind of *tent*, but with the outlines reminding a profile of a *barrel*, and differing in good resistance to wind loading. In the temples, both *tents* and *cubes* ended with the *cupola*, which was often faceted.

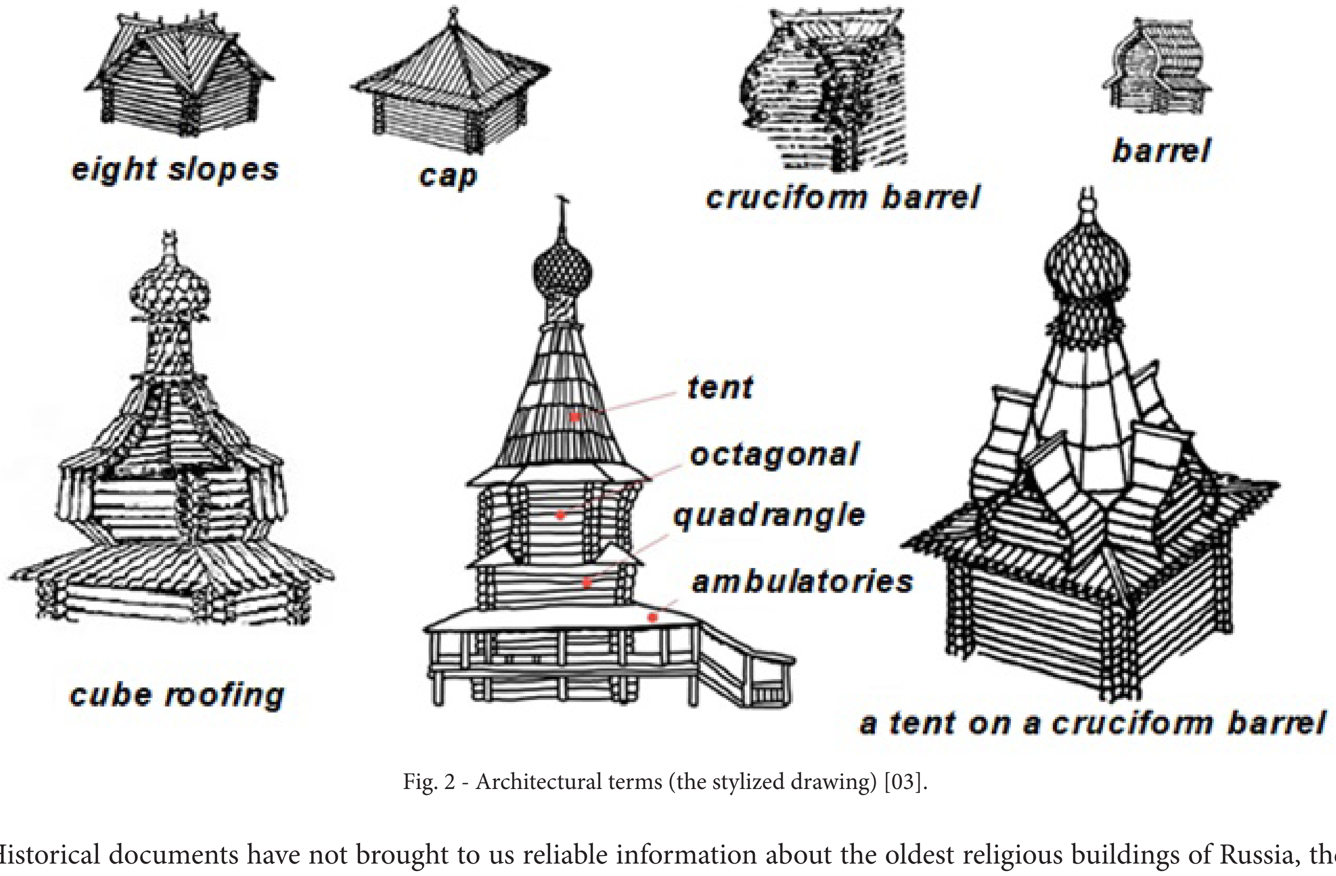

Fig. 2 - Architectural terms (the stylized drawing) [03].

Historical documents have not brought to us reliable information about the oldest religious buildings of Russia, the pagan temples, since, in the annals, there are only references to them. But there is no doubt that the scheme of church buildings, borrowed along with Christianity in Russia has undergone some changes. These changes concerned not only the arrangement of additional premises (for example, the refectory), but also led to the search for new images and forms for the structures and new methods of planning. So, to increase the area of the room, while maintaining the length of the log, builders began to use the shape of the *octagonal* structure, and the logic of overlapping such rooms led to the appearance of the *tent*.

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Tent churches perfectly matched the multifunctionality of religious buildings as the only public buildings of the Russian Middle Ages. They represented an aspiration to God and the dream of height implementation as the ideal of beauty. In cities and villages, they served as the main buildings, where many temporal matters were solved. In addition, they were the landmarks by which the village was recognized from afar [03].

Vertical tent churches emphasized the calm outlines of the Central and Northern Russia landscape as well as possible. But, it is worth noting that the external and internal volumes of high temple buildings do not always coincide. To achieve greater proportions, the interior premises of the temples were reduced to 1/5 - 1/3 of their external volume by means of seam ceilings. The heights of the temples were designed for perception from large distances, and often reached 25 - 45 m.

There are not many tent churches-towers, built as the *octagon* from the ground, which survived to our time. The oldest of them is the Nikolskaya Church in the village Lavle of the Arkhangelsk region, built in 1587. It is the oldest type of construction in which the tents, the necks and the cupolas have the shape of polyhedrons. In the XVII century, octagonal temples were crowned with the same *curvilinear cupolas* on the same *curvilinear necks*, which had a direct impact on the overall proportions. Refectories changed as well: if before they were relatively small, later on, they developed into frame *refectory-ambulatories,* encircling the *octagon* on three sides.

Fig. 3 - The Nikolskaya church.

It should be noted that, with a single well-established scheme of octagonal temples, the harmony and proportions of each of them are deeply individual. The ultimate fusion of structural and compositional components, their emphasis on height and monumentality contributes to the figurative expressiveness of octagonal tent temples. This also contributes to the symmetry of all volumes relative to the longitudinal axis. A sense of solemnity is generated by a gradual increase in the premises of the temple as you move inside: the *porch - ambulatory - refectory - narthex* - the very room of the *church*, which ended with a painted iconostasis with tiers of icons that are illuminated through the side windows.

The interior of the Peter and Paul Church in Puchuga village on the Northern Dvina river, built in 1698, but not preserved to our time, was very interesting. Presented in contrast to its harsh appearance, it was one of the highest achievements of Russian wooden architecture. Walking up the porch and passing the seni (narthex), one would reach the spacious refectory. The middle beam was supported by two massive pillars. There were carved benches along the walls of the refectory. On the axis of the refectory, was the entrance to the church in the form of a double portal, decorated with carvings and paintings. The long windows-slots of the refectory were arranged so that the service could be heard, and were decorated with carvings and wrought iron grill. The high volume of the church seemed even higher in comparison with the refectory, thanks to two slender carved pillars supporting the second beam. Next there was an iconostasis with tiers of icons, illuminated by four windows.

Tent-like temples built as the "*octagon* on *quadrangle*" became very common in the second half of the XVII and XVIII centuries. An example of this type of temple is the Church of St. Demetrius in the village of Upper Uftuga in the Krasnoborsky district of the Arkhangelsk region, built in 1784. The high *quadrangle* organically enters the *octagonal* volume and is completed with a powerful *pyramidal tent* with a small *cupola*. The *barrel-covered* apse from the East, and the small frame gallery from the West, do not disturb the vertical aspiration of this severe and majestic temple.

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The search of the silhouette and three-dimensional expressiveness of Russian temples by national architects has led to the emergence of the original school in the steepled church architecture in the XVII century [03]. It created a type of a church, with a *cruciform barrel* covering, onto which an octagonal tower was put. This coating was called "*a tent on a cruciform barrel*." In addition to the peculiarity of the silhouette, this coating (legalized by the time, and in accordance with the reforms of Patriarch Nikon in the 1650s), allowed to construct *five-cupolas* churches and, at the same time, maintain the traditional national form of *tent-like* temples.

Fig. 4 - The church of Dimitri Solumsky.

In conclusion, we would like to consider the multi-cupola churches, the extensive composition of which was developed in the early XVIII century, based on the traditional *octagon* with four additional structures or twenty-walled *log construction*. There are two temples with such a composition, the construction of which, in fact, completed the development of Russian wooden temple architecture. An early structure, built in 1708 but, unfortunately, not preserved, was the Pokrovsky Cathedral in the village of Ankhimova, the Vologda region. The second one is the Transfiguration Church of the great Kizhi churchyard, which was built six years later, in 1714. At the first glance at the multi-cupola pyramid of the Transfiguration Church, it becomes clear that this structure represents the art of the Northern Russian architects.

The first two tiers (eight *cupolas*) are formed by the hipped–roof method with the two *barrels*, each barrel has four wooden additions. The third tier (eight *cupolas*) is formed by the *barrel* completion of all eight sides of the main *octagon*. The four *cupolas* of the fourth tier are placed on *barrels* adjacent to the second *octagon*. And finally, the whole composition of the 22-*cupolas* temple is crowned by the central cupola, standing on the second *octagon*. On the diagonal faces of the main *octagon* at the level of the upper *barrels*, overlapping the wooden additions, decorative "*kokoshniks*" are placed, echoing the rhythm of the second tier's *barrels*.

Despite the compositional complexity of the Church of the Transfiguration, its silhouette pyramidal volume is single and very clear. Best of all, the temple is perceived from a long distance, which was very convenient at a time when the lake Onega was part of the ancient trade route. From nearer perceptions, it can be seen that the complex multi-cupola composition of the temple seems as if constantly changing as you move, fully conforming with the tendency towards the decorative effect typical for that time. The appearance of this memorial

Fig. 5 - The Church of the Transfiguration.

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church is, undoubtedly, connected with the rise caused by Russia's victories in the Northern war, which ended the Swedish claims to the Northern Russian lands. The Transfiguration Church in Kizhi is one of the greatest achievements, not only of the Russian, but also of the world architecture.

CONCLUSIONS

Our study showed that folk architects, observing the canonical scheme of religious buildings, were able to give the planned techniques, much diversity and originality, which was due to, among others, the constructive possibilities of wood material. Each of these techniques has been aimed at creating the maximum diversity in both volume and silhouette decisions to the main public buildings of towns and villages, from harsh ascetic octagonal temples of the XVI century, to the temples that are rich and diverse in plastic and silhouette decisions, built in the XVII and XVIII centuries. Empirically taking advantage of one of the most important properties of the human eye, which is an acute perception of the silhouette outlines of objects and structures alike, the church architects paid great attention to the search for a unique kind of coating forms for the temple buildings, reaching outstanding success. All of these factors allowed the wooden architecture of Russia to take a special place in the world architecture. Currently, there is a growing demand for the construction of wooden churches in Russia [04]. This means that the reconstruction of lost buildings and the preservation of the existing wooden churches of the Russian North will contribute to the preservation of the Russian temple architecture identity, which has a particular value during the domination of the global international style.

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WORKSHOP 01
07 . 09 . 2019 | SATURDAY, 14h30 - 18h00

JAVIER BARRALLO
DIDACTIC EXPERIENCES WITH POLYHEDRA

This workshop relates closely to the lecture of the plenary session with the same title.
It will consist in the assembly of some polyhedra, featuring different materials and techniques. We pretend to recover didactical experiences with handcrafted polyhedra constructions, that have been displaced in the last years by the explosion of three-dimensional printed models. Handcrafted polyhedra allow the introduction of mathematical and geometrical concepts in a very enjoyable and accessible way for the classroom.
Both individual and group constructions, during the workshop, will feature connections with topics like Fractals, Topology or Tensegrity, and will exercise the scientific and manual skills of the participants. No previous knowledge in the field is required, just the desire to play and experiment with geometry and take home beautiful polyhedra as souvenir.

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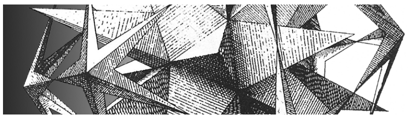

GEOMETRIAS'19: POLYHEDRA AND BEYOND
FCUP | PORTO | 05 . 06 . 07 SEPTEMBER 2019
GEOMETRIAS'19: BOOK OF ABSTRACTS
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WORKSHOP 02
07 . 09 . 2019 | SATURDAY, 14h30 - 18h00

RINUS ROELOFS
MAKING PAPER POLYHEDRA MODELS

INTRODUCTION

Making paper models of regular polyhedra was, as far as I know, first described by Albrecht Dürer in his book *Unterweissung der Messung*, first published in 1525. One of the pictures in this book is the folding plan of the icosahedron (Fig. 1a). Most probably Dürer was not aware of the fact that this folding plan also could be used to make another uniform polyhedron, the tetrahelix. One folding plan, two different polyhedra models!

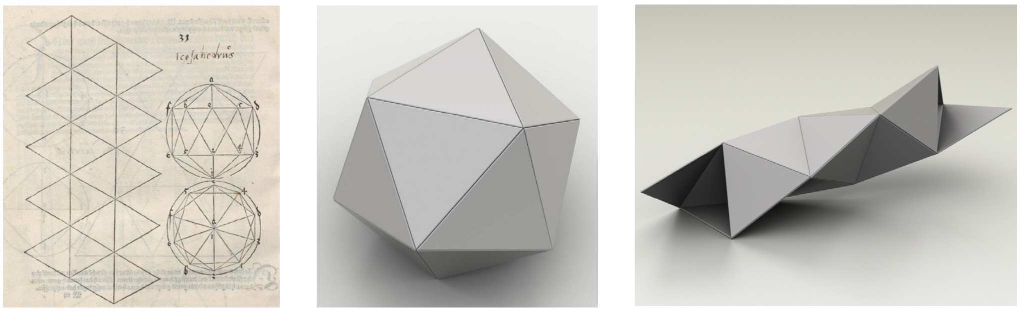

Fig. 1

STARTERS

A few simple exercises to continue with: two polyhedra combined in one model.
In the first one we see the tetrahedron combined with a cube. The total model is made from 4 equal pieces. The second model is a so-called "double tetrahedron", one model with a double skin. Also the third model has a double skin: the "double-cube". For both models we first need to weave the 2 basic parts.

Fig. 2

MAIN COURSE - FIRST DISH

In this part of the workshop a new way of making double models is introduced. In the introduction we started with just folding. In the starters we have seen a combination of weaving and folding, and now the complete procedure is "folding-weaving-folding". Again we will see combinations of 2 polyhedra in one model. The first model is the tetrahedron within the cube.

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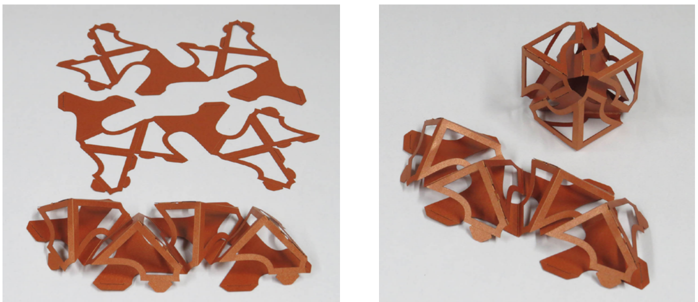

Fig. 3

The second model is the cube within the dodecahedron. For this model two different versions can be made. The second one, with 3 equal parts has some connections with the Borremean Rings.

Fig. 4

In the third picture, we see the models in which the outer skin is opened, in this way we can understand the total construction. In the workshop Both models will be available opened and closed.

MAIN COURSE -SECOND DISH

We don't have to limit ourselves to the Platonic solids. Kepler and Poinsot introduced new regular polyhedra which themselves have a double skin. Stellation is introduced by Kepler and in our model we apply this technique onto the rhombic dodecahedron, a polyhedron with 12 faces. For our model the faces have to be extended to make the stellated version of the rhombic dodecahedron, which is also known as the Escher-star.

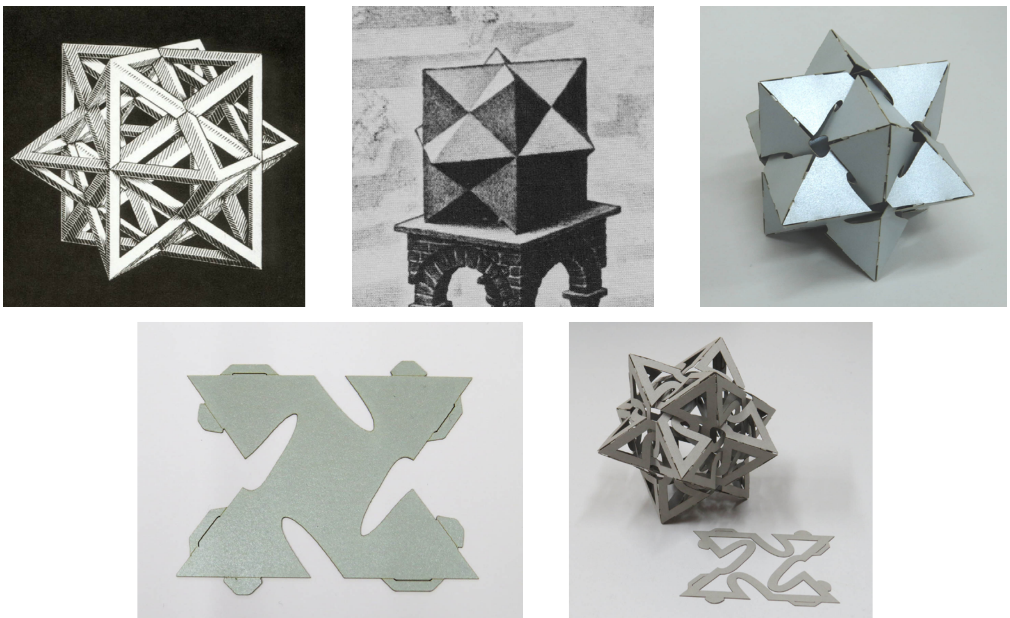

Fig. 5

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The second model is the great dodecahedron, also made from 12 parts, and first discovered by Poinsot. Models of Poinsot's polyhedra are very rare. With the technique we have used to make the Escher-star, which is "taking away parts of the faces where they intersect", we can also make a model of one of Poinsot's polyhedra, the great dodecahedron. With twelve parts, we can make this intriguing model.

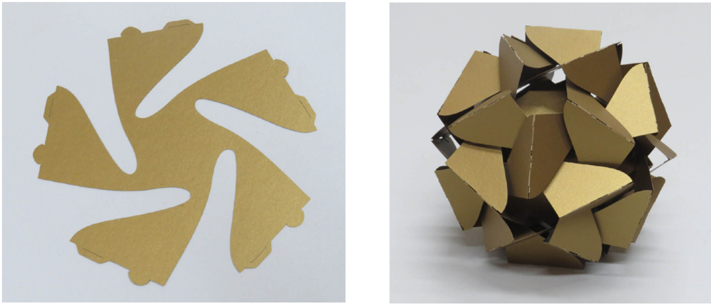

Fig. 6

DESSERT

We will finish the workshop with a nice dessert.

In 2003, Branko Grünbaum published some ideas with which new uniform can be created. One of his methods is called "doubling the faces". When we double the faces of a cube, we can rearrange the connections of the squares in such a way that the 12 square faces will again form a regular polyhedron. The way of connecting the faces is, in fact, the same as our "double cube" model.

In this model, only some of the faces are doubled. The total model, again is a uniform polyhedron.

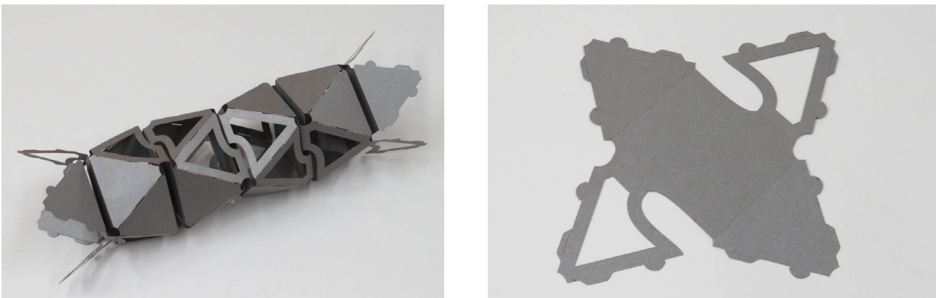

Fig. 7

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SHORT BIOGRAPHIC NOTES OF THE AUTHORS, MODERATORS AND SPEAKERS (listed by the surname's alphabetical order)

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Leonardo Baglioni, architect, since 2013, is a university researcher, for the scientific discipline ICAR/17 at Sapienza University of Rome where he teaches in the courses of Drawing and Fundamentals and Applications of Descriptive Geometry. Since 2017 he has been the scientific coordinator of the Laboratory of Innovation for the Detection, Representation and Analysis of Architecture - LIRALab at the Department of History, Representation and Restoration of Architecture. His research is mainly focused on descriptive geometry and its applications, with particular interest in its renewal by means of digital methods of representation.

BANJAC, BOJAN

PhD., is teaching assistant at Faculty of Technical sciences, University of Novi Sad. He obtained bachelor and master degree of Electrical engineering and computer science at School of Electrical engineering, University of Belgrade, and is currently finishing his PhD studies of Applied mathematics at the same faculty. His fields of research are software engineering, applied mathematics, computer graphics, computer geometry, artificial intelligence, computer vision and 3D modelling. He is author of more than 20 papers published at international journals and conferences. He is also technical editor of Journal of Applicable Analysis and Discrete Mathematics.

BARRALLO, JAVIER

47Javier Barrallo was born in Bilbao, where he studied and obtained a Ph.D. Degree in Computer Engineering at the University of Deusto. Javier is Professor at the UPV-EHU (University of the Basque Country) where he teaches in the Department of Applied Mathematics at the School of Architecture in Donostia-San Sebastián. Javier specialized in the relationship between Art & Science, dynamical systems including fractals and chaos, parametric programming of complex shapes and the scientific analysis for the structural maintenance and repair of heritage buildings. Javier Barrallo has written over 100 papers and books, collaborated in 15 interventions on historical heritage buildings, organized 12 International Conferences and events and arranged 30 international exhibitions.

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Marco Giorgio Bevilacqua is Associate Professor of Architectural Representation at the University of Pisa. His research interests are in the field of valorization of the historical architectural heritage, with particular attention to historical military architecture, architectural and urban survey and digital technologies for the communication of historical architectural heritage. He currently teaches Architectural Representation and Methodologies for Architectural surveying in the Master degree program of Building Engineering and Architecture at the University of Pisa.

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Mateja Budin was born in Ljubljana, Slovenia. She studied applied mathematics at the Faculty of Mathematics and Physics, University of Ljubljana. In 2003 she established Mathema, Institute for popularization of mathematics. She is the co-founder of the Foundation Mathema Art, she runs the House of Polyhedra and two national competitions in Slovenia.

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Cristina Càndito is Associate Professor in Drawing and Representation and member of the teaching body of the PhD program at the Architecture and Design Department, University of Genoa. She holds degree in Architecture (1991) and Modern Literature (1998). She obtained her PhD in Drawing and Surveying in 1999. She taught at the Polytechnic School of Milan and she was Architect at the Regional Board - Ministry of Cultural Heritage in Bologna and Genoa. Her teaching and research activity concerns Drawing, particularly Descriptive Geometry and History of Representation, through applications based on traditional and dig-ital methods, about Architectural Perspectives and cultural accessibility.

CASTRO, ALEXANDRA

Alexandra Castro is an Invited Assistant at the Faculty of Architecture of the University of Porto.
She is graduated in Architecture (FAUP, 2002), holds a master in Methodologies of Intervention in the Architectonic Heritage (FAUP, 2009) and she is currently developing her PhD research that is focused on the relationship between geometry and contemporary architecture, from the perspective of the impact that digital technologies have on architectural practice. Since 2013 she is a researcher at the Centre for Studies in Architecture and Urbanism of FAUP. On 2010 she founded with the architect Nicola Natali the architectural office "Castro Natali".

CELORIA, ILENIO

Ilenio Celoria following his degree in Architecture at the University of Genoa (2000), he developed his interest in teaching and research in photography. He has been teaching Photographic Technique since 2001 at Leardi's High School (Casale Monferrato). He is currently Contract Professor at the Polytechnic School, University of Genoa, in the course of Photography and Digital Imagine (Master Degree in Digital Humanities). His photographic projects have been exposed at the Venice Biennale, the Museum of Natural Sciences in Turin, Mu.Ma, Sant'Agostino Museum and Palazzo Ducale in Genoa, in the Istituto italiano di Cultura in Cologne, Vienna e Prague and in other Italian and foreign cities.

CHAVES, MANUEL ARALA

Manuel Arala Chaves was Full Professor of Mathematics at the Faculty of Science of Porto from 1973 until 2003, when he decided to retire, in order to work full-time in the Atractor Association. Since the beginning, he has been the president of the board of this non-profit association, created in 1999 with the aim of raising public awareness and attracting to Mathematics. In 2000, Atractor was invited to create a large exhibition, entitled Matemática Viva. The exhibition lasted from November 2000 until August 2010 in Pavilhão do Conhecimento (Lisbon) and it had more than 2 million visitors. In more recent years, Atractor focused on producing interactive virtual contents and exhibits, and many of them can be found on its homepage. Other activities include the DVD "Symmetry the dynamical way" (in 6 languages), several movies in the *Atractor's YouTube Channel*, a large collection of interactive stereoscopic 3D mathematical contents in several different formats (3D TVs, side-by-side, anaglyphs, etc.) and some free software, like *GeCla* and *AtrMini* (both in several languages).

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Arcady Dyskin is a Winthrop Professor at Department of Civil and Resource Engineering, School of Engineering. Arcady graduated in 1975 with academic distinction from Moscow Oil & Gas Institute and in 1980 from Moscow State University. In 1992 he has obtained PhD in Mechanics of Solids from The Institute for Problems in Mechanics, USSR Academy of Sciences. In 1991 he joined the Department of Civil Engineering of the University of Western Australia. His areas of expertise include topological interlocking, rock mechanics, wave propagation, fracture mechanics and the mechanics of solids.

EGEA, ADOLFO PÉREZ

Adolfo Pérez Egea holds a degree in Architecture from the Universitat Politècnica de València and a Master's Degree in Integrated Management of Building Projects from the Universidad Politécnica de Madrid. In addition to having professional experience within the private sector in the field of building construction, he is an expert in the use of software and methodologies for Building Information Modeling, as well as in Building Energy Efficiency. Since 2013, he has combined his professional work with a teaching career as an Associate Lecturer in the Department of Architecture and Building Technology, at UPCT's Technical School of Architecture and Building Engineering.

ESTRIN, YURI

Yuri Estrin received a MSc degree in Physics from the Moscow Institute of Physics and Engineering (1969), followed by a PhD degree from the Institute of Crystallography, Academy of Sciences of USSR (1975) and a habilitation from the University of Technology, Hamburg-Harburg (1986). He has held professorial positions in Germany and Australia, as well as adjunct appointments in several countries. Currently, Professor Estrin is an Honorary Professorial Fellow at Monash University in Melbourne and an Adjunct Professor at the University of Western Australia in Perth. His research interests are in a broad area of physical metallurgy and design of materials.

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Martino Peña Fernández-Serrano is Assistant Professor at the ETSAE at UPCT, having graduated in architecture at the Universidad Politécnica de Valencia. His PhD dissertation was entitled 'Energetic Artifacts. From Fuller to Piñero (1961-1972)', and he obtained his doctorate at the Department DPA at the ETSAM at the Universidad Politécnica de Madrid. He has been a guest researcher at the Technische Universität in Berlin, working with Professor Mike Schlaich, as well as at Dresden's Technische Universität. His recent publications are mainly about the Spanish architect Pérez Piñero and his deployable structures, and he has also written about utopian structures.

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Lorenzo Giordano is an architect, PhD student at the Department of Architecture of the University of Napoli "Federico II".

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Izidor Hafner was born in Ljubljana, Slovenia, in 1949. He received a bachelor's degree in 1972 and a master's degree in mathematics in 1974. He obtained PhD in computer science in 1984 from the University of Ljubljana. He had been a lecturer of mathematics at the Faculty of Electrical Engineering, University of Ljubljana, from 1985 until his retirement in 2013. He is a coauthor of 12 text books of elementary mathematics and logic. He has made scientific contribution in the theory of von Neumann regular rings, logic and geometry.

HANSMEYER, MICHAEL

Michael Hansmeyer is an architect and programmer who explores the use of algorithms to generate and fabricate architectural form. Recent work includes the design of two full-scale 3D printed sandstone grottos, the production of an iron and lace gazebo at the Gwangju Design Biennale, and the installation of a hall of columns at Grand Palais in Paris. He has exhibited at museums and venues including the Museum of Arts and Design New York, Palais de Tokyo, Martin Gropius Bau Berlin, and Design Miami / Basel. His work is part of the permanent collections of Centre Pompidou and FRAC Centre. Recently, he taught architecture as visiting professor at the Academy of Fine Arts in Vienna and at Southeast University in Nanjing, and as a lecturer at the CAAD group of the Swiss Federal Institute of Technology (ETH) in Zurich. He previously worked for Herzog & de Meuron architects, as well as in the consulting and financial industries at McKinsey and J.P. Morgan respectively. Michael holds a Master of Architecture degree from Columbia University and an MBA from INSEAD.

HUYLEBROUCK, DIRK

Dirk Huylebrouck worked at universities in the Congo for about eight years until a diplomatic incident interrupted his stay. He went to the University of Aveiro, Portugal, and the European Division of Maryland University, until the majority of his American (military) students were sent to Iraq. He returned to Africa, to Burundi, but for only three years, because of the genocide in neighboring Rwanda. Until his retirement in 2018 he taught at the Faculty of Architecture of the KULeuven (Belgium). He edited the column 'The Mathematical Tourist' in the journal 'The Mathematical Intelligencer' for 20 years. Having a weekly column called 'Professor Pi', in Belgium's largest newspaper, 'Het Laatste Nieuws', he is the author of four books in Dutch. He may flee abroad again having become (in) famous for his work in popularizing errors in, for example, the Belgian Atomium landmark, the work of Leonardo da Vinci, a Fibonacci artwork, etc.

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Filippo Iardella earned his Master Degree in Architecture and Building Engineering in 2018 at the University of Pisa. He's currently developing research in the field of computational design and digital fabrication.

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Alexei Kanel-Belov is an editor of the following academic journals: "Fundamentalnaya i prikladnaya matematika", "Chebyshevskii sbornik", "Kvant", "Matematicheskoye prosves-cheniye", "Matematicheskoye obrazovaniye". A.K.-B. graduated in 1985 with academic distinction from the Moscow State Pedagogical Institute. In 1992 he obtained PhD from the Otto Schmidt Institute for Earth Physics. In 1988-1990 Alexei held a position at the Moscow Mining Institute. In 2002 A.K.-B. defended Doctor of Physical and Mathematical Sciences thesis: "Algebras with polynomial identities: representations and combinatorial methods". A.K.-B. is interested in self-interlocking structures and their applications to creation of novel hybrid materials, particularly applications to superhigh pressure systems.

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PhD., earned both bachelor's and master degree in mathematics from Faculty of Sciences, University of Novi Sad. She earned PhD degree in applied mathematics at Faculty of Technical Sciences, University of Novi Sad. She is currently working at Faculty of Technical Sciences in University of Novi Sad at the position of assistant professor at topics of computer graphics. Her main research interest is in: image processing, pattern recognition, machine learning, computer visualization, 3D modeling for medical purposes.

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Pedro García Martínez is an architect and holds a PhD in Architecture obtained at the Universidad Politécnica de Madrid (UPM). He has collaborated with architectural practices, such as Foster & Partners and MVRDV, developing widely published and award-winning projects. Within his own professional career, he has developed projects that have been awarded in different national and international competitions. He has taught at various architectural schools, such as ETSAM (Madrid), ENSAPV (Paris), or the Institut für Architektur, at the Technische Universität in Berlin. Currently, he is professor of Architectural Design Projects in the ETSAE, and is a member of the GRAMMAR R+D Research Group at UPCT (Spain).

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Andrés Martin-Pastor [ORCID: 000-0002-5588-2886] is an architect with a Ph.D. from the University of Seville (Spain), where he currently works as a lecturer in the Department of Graphic Engineering. His research is focused on perspective and geometry in architecture, and covers a broad range from inherited graphic traditions until the digital tools of today. His research also includes the study of developable surfaces and their applications in lightweight architecture. He has lectured at several International Universities and published books, chapters and articles.

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Helena Mena Matos is a professor in the Department of Mathematics of the Faculty of Sciences at University of Porto and researcher at the Mathematics Center of the University of Porto. She holds a Ph.D. in Mathematics from the University of Porto and, in parallel with her teaching and research activities in fundamental mathematics, she has a strong interest in the connections between mathematics and the visual arts. She also dedicates part of her time to the dissemination of mathematics through lectures for regular and artistic high school students in various schools in the country, lectures for teachers and lectures and debates for the general public.

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MURTINHO, VITOR
Vitor Murtinho (1964). Architect, is Associate Professor at the Department of Architecture, University of Coimbra (UC), Portugal. Since 1988 teaches Geometry and in Theory and History areas of the Degree Course, Integrated Master's Programme and 3rd Cycles Studies in Architecture. Is senior researcher at the Center for Social Studies of the UC and has interests in the domain of Renaissance theory, architectonics of form and geometry. He was Vice-Rector (March 2011-February 2019) of the UC with responsibility for the heritage, buildings and sustainability. Is the author or co-author of about one hundred publications (Books, books chapters, papers in scientific journals and in conference proceedings). Has participated in several Aproged's conferences. More detailed publications in: http://ces.uc.pt/en/ces/pessoas/investigadoras-es/vitor-murtinho/publicacoes.

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Marija Obradović, architect, is an Associate Professor of Faculty Civil Engineering at the University of Belgrade, Serbia. She is teaching Descriptive Geometry, Computational Geometry and Visualization and presentation of 3D model in geodesy. She has participated in several national projects dealing with the development of technology and digitizing of national heritage, including the application and visualization of geometric contents. She authored two books, and over 70 scientific papers published in national and international journals and proceedings of scientific conferences. Her main interest concerns concave polyhedra of the second sort, introduced in her Ph.D. thesis: "Toroidal Deltahedra with Regular Polygonal Bases".

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PAIS, TERESA

Born in Alcantarilha, Silves, Algarve, Portugal. Graduated in Architecture from the Faculty of Sciences and Technology, University of Coimbra. Master in Drawing Practices and Theories by the Faculty of Fine Arts, University of Porto. PhD in the specialty of Plastic Expression and Architecture at the University of Coimbra, with the thesis "Contour drawing in the learning process of observing drawing". Assistant Professor of Drawing and Geometry in the Department of Architecture of the Faculty of Sciences and Technology, University of Coimbra. Author of several articles on the teaching of Drawing in the training of the architect.

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Elena Pasternak is a Professor at Department of Mechanical Engineering, Faculty of Engineering. Elena graduated in 1991 with academic distinction from Dnepropetrovsk State University, Department of Theoretical and Applied Mechanics. In 2001 she has obtained PhD in Geomechanics of Solids from The University of Western Australia. In 2002 she joined the Department of Civil Engineering of the University of Western Australia and then moved School of Mathematics and Statistics. Elena's areas of expertise include topological interlocking, mechanics of generalized continua, wave propagation, fracture mechanics and the mechanics of solids.

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Raul Piepereit is a doctoral student at Technical University of Berlin and a research assistant at Beuth University of Applied Sciences Berlin as well as University of Applied Sciences Stuttgart. His research is focused on the automated processing of virtual city mod-els for flow simulations. He studied mathematics at Beuth University of Applied Sciences and got his Master's degree in Mathematik – Computational Engineering in 2014.

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Konrad Polthier is full professor of mathematics at Freie Universität Berlin since 2005. His research focuses on discrete differential geometry, applied geometry, geometry processing, and mathematical visualization.

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Margitta Pries studied mathematics at TU Dresden and received her doctorate in 1991. Afterwards, she has worked as a software developer, project engineer and IT consultant in the CAD and CAM environment for several years. Margitta Pries has been a professor at Beuth University of Applied Sciences Berlin since 2003. There she teaches the courses "Mathematical Methods of CAD" and "Geometric Modelling in CAD" and others.

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ROELOFS, RINUS

After studying mathematics for a couple of years (applied mathematics at the University of Twente, Enschede), I decided to switch to the school of arts. In 1983, I started my career as a sculptor. Inspired by the works of M.C. Escher and Leonardo da Vinci, my works became more and more some kind of expression of my mathematical ideas. The main subject of my art is my fascination about mathematics - to be more precise: my fascination about mathematical structures. Mathematical structures can be found all around us. We can see them everywhere in our daily live. The use of these structures as visual decoration is so common, that we don't even see this as mathematics. But studying the properties of these structures and, especially, the relation between different structures can bring up questions. Questions that can be the start of interesting artistic explorations. Artistic explorations of this kind mostly lead to intriguing designs of sculptural objects, which are then made in all kind of materials, like paper, wood, metal, acrylic, etc. It all starts with amazement, trying to understand what you see. Solving those questions often leads to new ideas, new designs. Since I use the computer as my main sketchbook, these ideas come to reality, first, as a picture on the screen. From there, I can decide what the next step towards physical realization has to be. A rendered picture, an animation or a 3D physical model made by the use of CNC-milling, laser cutting or rapid prototyping. Many techniques can be used nowadays, as well as many different materials. But it is all based on my fascination about mathematical structures

REITEBUCH, ULRICH

Ulrich Reitebuch has a diploma in mathematics from Technische Universität Berlin. He works at the department of mathematics and computer science at Freie Universität Berlin and is interested in differential geometry, discrete geometry, tessellations, symmetries, and connections between mathematics and arts.

SEGERMAN, HENRY

Henry Segerman received his masters in mathematics from the University of Oxford in 2001, and his Ph.D. in mathematics from Stanford University in 2007. After post-doctoral positions at the University of Texas at Austin and the University of Melbourne, he joined the faculty at Oklahoma State University in 2013, where he is now an Associate Professor. His research interests are in three-dimensional geometry and topology, working mostly on triangulations of three-manifolds, and in mathematical art and visualization. In visualization, he works mostly in the medium of 3D printing, with other interests in spherical video, virtual, and augmented reality. He is the author of "Visualizing Mathematics with 3D Printing", a popular mathematics book published by Johns Hopkins University Press in July 2016.

SHUVALOVA, SVETLANA

PhD in Pedagogic sciences, the associate professor, the Head of the Department of Descriptive Geometry and Engineering Graphics of the Faculty of Architecture, Saint Petersburg State University of Architecture and Civil Engineering (SPbGASU), http://www.spbgasu.ru/Universitet/Vuz_v_licah_2/nameШ/id388/. Svetlana Semenovna Shuvalova graduated from St. Petersburg state Institute of Technology in 1979. At present she is a PhD in Pedagogic sciences, the associate professor, the head of the department of descriptive geometry and engineering graphics of St. Petersburg state University of architecture and civil engineering. The sphere of scientific and professional interests includes: creation of perspective projections, improving methods of teaching the discipline "descriptive geometry" for students of architectural and construction specialties; studying the basics of users' perception of the architectural environment. Permanent participant of the international scientific conferences.

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Jose Pedro Sousa is an Assistant Professor at FAUP (Faculty of Architecture, University of Porto) where he founded and coordinates the DFL (Digital Fabrication Laboratory) research group. His research interests are related with computational design, digital fabrication, robotics, digital culture and material innovation in architecture.

STASI, GIANLUCA

Graduated in Architecture from the Università la Sapienza de Roma (2005), and was awarded his Ph.D. from the Universidad de Sevilla (2018). One of the main axes of his research is the relation between geodesic geometry and community empowerment, and knowledge and technology transfer. In his professional life he has supported, contributed to, and developed collaborative, participatory, self-construction and low-tech initiatives and processes in various parts of the world. These initiatives, always in collaboration with local communities, activated long lasting social processes that have been recognized at the institutional level, as with the Curry Stone Design Prize, and, more importantly, by the communities they serve.

TEIXEIRA, SAMANTA ALINE

Samanta Aline Teixeira is Ph.D. student in industrial design from São Paulo State University UNESP. Master's degree from the same university in 2017. Origami researcher since 2011, with Scientific Initiation. Current research area: Origami design applied to medical instruments.

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PhD in Art History with a thesis entitled 'Photography and Artistic Thought in the Work of Juan José Gómez Molina (1943-2007)'. Professor of the Department of Historical Sciences and Theory of Arts of the University of the Balearic Islands. Member of the research group in Audiovisual Heritage, Mass-media and Illustration. He researches in the field of the history of photography in Spain and with the relationship between art and photography. Among the latest publications, we can quote: 'The Image of the Crucified in Salvador Dalí, José María Sert and Juan de la Cruz: Hypothesis of Realization of the Drawing of the Monastery of the Encarnación de Ávila' (Locus Amoenus, 2016); 'Relation of the Work of the Painter Antonio López and Photography.' (Art, Individual and Society, 2015) or the book chapter 'Optical Instrumentation and Scientific Image of Nature '(Cátedra, 2016).

ULIVIERI, DENISE

Denise Ulivieri is a History of Architecture university researcher at the University of Pisa, Department of Knowledge Civilization and Forms, where she teaches principles of architecture history and urban design, vernacular architecture and history of contemporary architecture. She is scientific coordinator of several research projects on historical seismic knowledge and technology in Tuscany. She collaborates with the European University Centre for Cultural Heritage, Ravello. Since 2015 she has been a member of ICOMOS Italia. She collaborates with Detroit University's Mercy School of Architecture. She is editor-in-chief of the academic book series Quaderni di ecostoria published by Pisa University Press.

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Emmanouil Vermisso is an Associate Professor of Architecture at Florida Atlantic University focusing on Design Computation. He holds degrees from the University of Westminster and Syracuse University. He has practiced in several architectural firms including Foster+Partners, AHMM and Porphyrios Associates. He is interested in analogue computation, architectural Organicism, and technology-driven design protocols which consider bottom-up thinking. His work has examined aspects of digital fabrication, analogue computational formfinding, responsive prototypes and self-organization in urban design methodologies. It has been published in venues including the International Journal of Architectural Computing, ACADIA, CAAD Futures, etc. and received recognition from ACSA, NCARB and AIA.

VIANA, VERA

Currently writing her Ph.D. thesis on polyhedra and solid tessellations at Trás-os-Montes and Alto Douro's University, Vera Viana studies polyhedral geometry and the relationships between architecture and mathematics as integrated researcher at the Centre for Studies in Architecture and Urbanism of the Faculty of Architecture of Porto's University.

Descriptive Geometry's Teacher, Director of Aproged (Portuguese Geometry and Drawing Teachers' Association), Viana has been engaged, since 2002, on the production of educational resources with dynamic geometry, three-dimensional modelling and algorithmic modelling software. having authored papers and presentations on the subject.

VICARIO, PEDRO MIGUEL JIMÉNEZ

Pedro Miguel Jiménez Vicario studied architecture at the University of Granada and obtained his PhD in the subject at the Technical University of Cartagena (UPCT). He is a member of the Department of Architecture and Building Technology, and teaches Graphic Thinking and Formal Analysis. He belongs to the GRAMMAR (Graphic Analysis and Methodologies for Architectural Research) R+D research group. His research also focuses on the study of Modern Architecture, as well as Generative and Parametric Design. He has published many papers in a wide range of journals and at international conferences. In addition, he has been awarded several prizes and distinctions, including the Iberian Prize for Traditional Architectural Research in 2016.

VUJANOVIĆ, MILOŠ

PhD., visual artist, born in 1965. year in Podgorica, Montenegro. Graduated, master and doctorate at the Academy of Arts in Novi Sad. He won several awards for art work, illustrations, and pedagogical work. He's a full-time professor at the same Academy. He started exhibiting in 1989, and since then has exhibited at one hundred collective and individual exhibitions at home and abroad. He experimented in various disciplines of the visual arts. Participated in making of several short films, video works and animations.

WEISS, GÜNTER

Günter Weiss is a retired professor for Geometry with former employments at the University of Technology Vienna, Austria, and Dresden University of Technology, Germany. He is interested in classical geometries and education. For information about his publications see http://www.math.tu-dresden.de/geo/weiss/index.html.

XAVIER, JOÃO PEDRO

João Pedro Xavier is an architect and associate professor at FAUP, where he received his degree in 1985, and his Pd.D. in 2005. Worked in Álvaro Siza's office from 1986 to 1999. At the same time, established his own practice as an architect. Member of CEAU's research teams – Architecture: Theory, Project, History – and – Digital Fabrication Laboratory. Correspondent editor of the "Nexus Network Journal" and member of the executive board of "Resdomus". Currently, FAUP's Dean.

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YOSHINO, TAKASHI

Professor of the department of mechanical engineering, Toyo University, Japan. Main interests include geometrical aspects of skeletal structures of marine plankton, patterns on curved surfaces, and hyperspace.

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