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I. *On Isometrical Perspective.*

BY WILLIAM FARISH, B.D.

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AND PRESIDENT OF THE PHILOSOPHICAL SOCIETY
IN THE UNIVERSITY OF CAMBRIDGE.

[Read *February 21, and March 6, 1820.*]

IN the Course of Lectures which I deliver in the University of Cambridge, I exhibit models of almost all the more important machines which are in use in the manufactures of Britain.

The number of these is so large, that had each of them been permanent and separate, on a scale requisite to make them work, and to explain them to my audience, I should, independently of other objections, have found it difficult to have procured a warehouse large enough to contain them. I procured therefore an apparatus, consisting of what may be called a system of the first principles of machinery; that is, the separate parts, of which machines consist. These are made chiefly of metal, so strong, that they may be sufficient to perform even heavy work: and so adapted to each other, that they may be put together at pleasure, in every form, which the particular occasion requires.

Those parts are various; such as, loose brass wheels, the teeth of which, all fit into one another: axes, of various lengths, on any part of which the wheel required may be fixed: bars, clamps, and frames; and whatever else might be necessary to build up the particular machines which are wanted for one Lecture. These

models may be taken down, and the parts built up again, in a different form, for the Lecture of the following day. As these machines, thus constructed for a temporary purpose, have no permanent existence in themselves, it became necessary to make an accurate representation of them on paper, by which my assistants might know how to put them together, without the necessity of my continual superintendence. This might have been done, by giving three orthographic plans of each; one on the horizontal plane, and two, on vertical planes at right angles to each other. But such a method, though in some degree in use among artists, would be liable to great objections. It would be unintelligible to an inexperienced eye; and even to an artist, it shews but very imperfectly that which is most essential, the connection of the different parts of the engine with one another; though it has the advantage of exhibiting the lines parallel to the planes, on which the orthographic projections are taken, on a perfect scale. This will be easily understood, by supposing a cube to be the object represented. The ground plan would be a square representing both the upper and lower surfaces. And the two elevations would also be squares on two vertical planes, parallel to the other sides of the cube. The artist would have exhibited to him, three squares; and he would have to discover how to put them together in the form of a cube, from the circumstance of there being two elevations, and a ground plan. This method, therefore, giving so little assistance on so essential a point, I thought unsatisfactory.

The taking a picture, on the principles of common perspective, was the next expedient that suggested itself. And this might be adapted to the exhibition of a model, by taking a kind of bird's-eye view of the object, and having the plane of the picture, not as is most common in a drawing, perpendicular to the horizon, but to a line, drawn from the eye, to some principal part of the object. For example: in taking the picture of a cube, the eye might be

placed in a distant point on the line which is formed by producing the diagonal of the cube. But to this common perspective, there are great objections. The lines which in the cube itself, are all equal, in the representation are unequal. So that it exhibits nothing like a scale. And to compute the proportions of the original from the representation, would be exceedingly difficult, and, for any useful purpose, impracticable: there is equal difficulty too, in computing the angles which represent the right angles of the cube. Neither does the representation appear correct, unless the eye of the person, who looks at it, be placed exactly in the point of sight. It is true that, as we are continually in the habit of looking at such perspective drawings, we get the habit of correcting, or rather overlooking the apparent errors which arise from the eye being out of the point of sight, and are therefore not struck with the appearance of incorrectness, which, if we were unaccustomed to it, we should feel at once.

The kind of perspective, which is the subject of this paper, though liable in a slight degree, to the last-mentioned inconvenience, till the eye becomes used to it, I found much better adapted to the exhibition of machinery; I therefore determined to adopt it, and set myself to investigate its principles, and to consider how it might most easily be brought into practice.

It is preferable to the common perspective on many accounts, for such purposes. It is much easier, and simpler in its principles. It is also, by the help of a common drawing-table, and two rulers*,

* It is unnecessary to describe the drawing-table any further than by observing that it ought to be so contrived, as to keep the paper steady on which the drawing is to be made.

There should be a ruler in the form of the letter T to slide on one side of the drawing-table. The ruler should be kept, by small prominences on the under side, from being in immediate contact with the paper, to prevent its blotting the fresh drawn lines, as it slides over them. And a second ruler, by means of a groove near one end on its under side, should be made to slide on the first. The groove should be wider than the breadth of the first ruler, and so fitted, that the second may at pleasure be put into either of the two positions

incomparably more easy, and, consequently, more accurate in its application; insomuch, that there is no difficulty in giving an almost perfectly correct representation of any object adapted to this perspective, to which the artist has access, if he has a very simple knowledge of its principles, and a little practice.

It further represents the straight lines, which lie in the three principal directions, all on the same scale. The right angles contained by such lines are always represented either by angles of 60 degrees, or the supplement of 60 degrees. And this, though it might look like an objection, will appear to be none on the first sight of a drawing on these principles, by any person who has ever looked at a picture. For, he cannot for a moment have a doubt, that the angle represented is a right angle, on inspection.

And we may observe further, that an angle of 60 degrees is the easiest to draw of any angle in nature. It may instantly be found

represented in the plate, fig. 1, so as to contain with the former ruler, in either position, an angle of 60 degrees. The groove should be of such a size, that when its shoulders *a* and *d* are in contact with, and rest against the edges of the first ruler, the edge of the second ruler should coincide with *de*, the side of an equilateral triangle described on *dg*, a portion of the edge of the first ruler; and when the shoulders *b* and *c* rest against the edges of the first ruler, the edge of the second should lie along *ge*, the other side of the equilateral triangle. The second ruler should have a little foot at *k* for the same purpose as the prominences on the first ruler, and both of them should have their edges divided into inches, and tenths, or eighths of inches.

It would be convenient if the second ruler had also another groove *rs*, so formed that when the shoulders *r* and *s* are in contact with the edges of the first ruler, the second should be at right angles to it.

For representing circles in their proper positions the writer made use of the inner edge of rims cut out from cards, into isometrical ellipses as represented in the figure; of these he had a series, of different sizes, corresponding to his wheels. Such a series might be cut by help of the concentric ellipses in fig. 5, but he thinks that it would be an easier way to make use of that set of concentric ellipses as they stand, by putting them in the proper place under the picture, if the paper on which the drawing is made, be thin enough for the lines to be traced through, as by help of them the several concentric circles will go to the representation of one which might be drawn at once. It is difficult to execute them separately with sufficient accuracy, to make them correspond. For this purpose a separate plate of fig. 5, should be had, and one edge of the paper on the drawing table, should be loose to admit of the concentric ellipses being slid under it, to the proper place, as described, page 9.

by any person who has a pair of compasses, and understands the First Proposition of Euclid. The representation, also, of circles and wheels, and of the manner in which they act on one another, is very simple, and intelligible. The principles of this perspective which, from the peculiar circumstance of its exhibiting the lines in the three principal dimensions, on the same scale, I denominate "Isometrical," will be understood from the following detail:

Suppose a cube to be the object to be represented. The eye placed in the diagonal of the cube produced. The paper, on which the drawing is to be made to be perpendicular to that diagonal, between the eye, and the object, at a due proportional distance from each, according to the scale required. Let the distance of the eye, and consequently that of the paper, be indefinitely increased, so that the size of the object may be inconsiderable in respect of it.

It is manifest, that all the lines drawn from any points of the object to the eye, may be considered as perpendicular to the picture, which becomes, therefore, a species of orthographic projection. It is manifest, the projection will have for its outline an equiangular, and equilateral hexagon, with two vertical sides, and an angle at the top and bottom. The other three lines will be radii drawn from the center to the lowest angle, and to the two alternate angles; and all these lines and sides will be equal to each other, both in the object and representation: and if any other lines parallel to any of the three radii should exist in the object, and be represented in the picture, their representations will bear to one another, and to the rest of the sides of the cube, the same proportion which the lines represented, bear to one another in the object.

If any one of them, therefore, be so taken, as to bear any required proportion to *its* object, *e. g.* 1 to 8, as in my representations of my models, the others also will bear the same proportion to

their objects; that is, the lines parallel to the three radii will be reduced to a scale.

I omit the demonstration of this, and some other points, partly for the sake of brevity, and partly because a geometrician will find no difficulty in demonstrating them himself, from the nature of orthographic projection; and a person, who is not a geometrician, would have no interest in reading a demonstration.

For the same reason, it is unnecessary to shew that the three angles at the center, are equal to one another, and each equal to 120 degrees, twice the angle of an equilateral triangle; and the angle contained between any radius and side is 60 degrees, the supplement of the above, and equal to the angle of an equilateral triangle. All this follows immediately from Euclid, B. IV. Prop. 15, on the inscription of a hexagon in a circle.

In models, and machines, most of the lines are actually in the three directions parallel to the sides of a cube, properly placed on the object. And the eye of the artist should be supposed to be placed at an indefinite distance, as before explained, in a diagonal of the cube produced.

DEFINITIONS.

The last-mentioned line may be called the *line of sight*.

Let a certain point be assumed in the object, as for example *C*, fig. 2, and be represented in the picture, to be called, *The regulating point*. Through that point on the picture, may be drawn a vertical line, *CE*, fig. 2, and two others, *CB*, *CG*, containing with it, and with one another, angles of 120°, to be called *the isometrical lines*, to be distinguished from one another by the names of the *vertical*, the *dexter*, and the *sinister* lines. And the two latter, may be called by a common name,—the *horizontal isometrical* lines. Any other lines, parallel to them, may be called respectively by the same names. The plane passing through the dexter, and vertical lines,

may be called the *dexter isometrical plane*; that passing through the vertical, and sinister lines, the *sinister plane*; and that through the dexter and sinister lines, the *horizontal plane*.

By the use of the simple apparatus described above in the Note, the representation of these lines in the objects may be drawn on the picture, and measured to a scale, with the utmost facility: the point at the extremity being first found, or assumed. The position of any point in the picture, may be easily found, by measuring its three distances, namely, first its perpendicular distance from the *regulating horizontal plane*, (that is, the horizontal plane passing through the regulating point) secondly, the perpendicular distance of that point, where the perpendicular meets the horizontal plane, from the regulating dexter line; and thirdly, of the point, where that perpendicular meets the dexter line, from the regulating point; and then taking those distances reduced to the scale, first, along the dexter line, secondly, along the sinister line, and thirdly, along the vertical line, in the picture. These three may be called the *dexter distance* of the point, its *sinister distance*, and its *altitude*. And it is manifest they need not be taken in this order, but in any other that may be more convenient to the artist: there being six ways in which this operation may be varied.

If any point in the same isometrical plane, with the point required to be found, is already represented in the picture, that point may be assumed as a new regulating point, and the point required found by taking two distances; and if the new assumed regulating point is in the same isometrical line with the point, it is found by taking only one distance. And this last simple operation, will be found in practice all that is necessary for the determination of most of the points required. Thus any parallelopiped, or any framework, or other object with rafters, or lines lying in the isometrical directions, may be most easily and accurately exhibited on any

scale required. But, if it be necessary to represent lines in other directions, they will not be on the same scale, but may be exhibited, if straight lines, by finding the extremities as above, and drawing the line from one to the other; or sometimes more readily in practice, by help of an ellipse, as hereafter described, page 11.

If a curved line be required, several points may be found sufficient to guide the artist to that degree of exactness, which is required.

The method of exhibiting the representations of any machines, or objects, the lines of which lie, as they generally do, in the isometrical directions; that is, parallel to the three directions of the lines of the cube, as has been already shewn; and likewise the mode of representing any other straight lines, by finding their extremities; or curved lines, by finding a number of points.

But in representing machines, and models, there are not only isometrical lines, but also many wheels working into each other, to be represented. These, for the most part, lie in the isometrical planes. And it is fortunate that the picture of a circle in any one of these planes, is always an ellipse of the same form, whether the plane be horizontal, dexter, or sinister; yet they are easily distinguished from each other, by the position in which they are placed on their axle, which is an isometrical line, always coinciding with the minor axis of the ellipse.

This will be obvious from considering the picture of a cube with a circle inscribed in each of its planes, fig. 3, and considering these circles as wheels on an axle. The two other lines (or spokes of the wheel) in the ellipse, which are drawn respectively through the opposite points of contact of the circle with the circumscribing figure, are isometrical lines also; for the points of contact bisect the sides of the circumscribing parallelogram, and therefore the lines are parallel to the other sides. They give likewise the true diameter of the wheels, reduced to the scale required. It further

appears from the nature of orthographic projection, that the major axis of the ellipse, is to the minor axis, as the longer, to the shorter diagonal of the circumscribing parallelogram, that is, (since the shorter diagonal divides it into two equilateral triangles) as the square root of three, to one; as appears from Euclid, Lib. I. Prop. 47. And since the sum of the squares of the conjugate diameters in an ellipse, is always the same, if we put $\sqrt{1}$ for the minor axis, the $\sqrt{3}$ for the major, and i for the isometrical diameter, we shall have $2i^2 = 1 + 3, = 4$, and $i = \sqrt{2}$.

Therefore the minor axis, the isometrical diameter, and the major axis may be represented respectively by $\sqrt{1}$, $\sqrt{2}$, $\sqrt{3}$, or nearly by 1, 1.4142, 1.7321; or more simply, though not so nearly, by 28, 40, 49.

These lines may be geometrically exhibited by the following construction:

Let AB , fig. 4, be equal to BD , and the angle at B , a right angle. In BA produced, take $Ba = to AD$. Draw aD , and produce both it, and aB . Then will BD , Ba , and aD , be respectively to one another, as $\sqrt{1}$, $\sqrt{2}$, $\sqrt{3}$ by Euclid I. 47. Therefore if $a\beta$ be taken equal to the isometrical diameter of the ellipse required, $\beta\delta$ drawn perpendicular to it will be the minor axis, and $a\delta$ the major axis. The ellipse itself, therefore, may be drawn by an elliptic compass, as that instrument may be properly set, if the major, and minor axes are known. If it is to represent a wheel on an axle, care must be taken to make the minor axis lie along that axle. In the absence of the instrument it may be drawn from the concentric ellipses, fig. 5, which may be placed under the paper, in the position above described, and seen through it; if the paper be not too thick, and in this method the smaller concentric circles of the wheel may be described at the same time, as they may be seen through the paper; or if they should not be exactly of the right size, it would be easy to describe them by hand, between

the two nearest concentric ellipses; and thus also the height of the cogs of a wheel in the different parts of it may be exhibited, longer and narrower towards the extremities of the major, and shorter and wider at the extremities of the minor axis. Their width may be determined from the divisions of the ellipse. In most cases, this may be done with sufficient accuracy from the circumference of the ellipse being divided into eight equal divisions of the circle, by the two axes, and two isometrical diameters, each of which parts may be subdivided by the skill of the artist; and not only the face of the wheel in front, may be thus exhibited, but the parts of the back circles also, which are in sight, may be exhibited, by pushing back the system of concentric ellipses on the minor axis, or axle through a distance representing the breadth of the wheel, and then tracing, both the exterior, and interior circles of the wheel, and of the bush on which it is fixed, as far as they are visible. Care should be taken to represent the top of the teeth, or cogs, by isometrical lines, parallel to the axle, in a face-wheel, or tending to a proper point in the axle in a bevil-wheel. And nearly in the same way may the floats of a water-wheel be correctly represented. If a series of concentric ellipses, such as are given, fig. 5, be not at hand, it will still be easy for an artist to draw the ellipses with sufficient accuracy for most purposes, by drawing through the proper point in the axle, the major, and minor axes, and the two isometrical diameters, thus marking eight points in the circumference, to guide him.

If in any case it should become necessary to represent a circle, which does not lie in an isometrical plane, we may observe that the major axis will be the same, in whatever plane it lies: and it will be the picture of that diameter, which is the intersection of the circle with the plane parallel to the picture, passing through its center. And the major axis, will bear to the minor axis, the proportion of radius, to the sine of the inclination of the line of sight, to the plane of the circle. We may observe further that the diameters of

the ellipse, which are to the major axis, as $\sqrt{2}$ to $\sqrt{3}$, when such exist, are isometrical lines*.

And the representation of every other line parallel, and equal to any diameter of the circle, may be exhibited by drawing it equal and parallel to the corresponding diameter in the ellipse.

If it should be desired to divide the circumference of an ellipse into degrees, or any number of parts representing given divisions of the circle, it may be done by the following method :

Let an ellipse be drawn, fig. 6, and on its major axis, *AG*, a circle described, with its circumference divided into degrees, or parts in any desired proportion, at *B, C, D, E, F*, &c.: from which points, draw perpendiculars to the major axis. They will cut the periphery of the ellipse in corresponding points. It would be difficult, however, in this way, to mark, with sufficient accuracy, the degrees, which lie near the extremities of the major axis. But the defect may be supplied by transferring those degrees in a similar way, from a graduated circle, described on the minor axis. In this manner, an isometrical ellipse, may be formed into an isometrical circular instrument, or an isometrical compass, which may shew bearings or measure angles on the picture, in the same manner, as a real compass, or circular instrument would do in nature.

It may be often useful to have a scale, to measure distances, not only in the isometrical directions, but in others also. And this may

* We may remark, that if a cone be described, having its vertex at *C* which lies in the line of sight, fig. 2, and passing through the three radii *CB, CE, CG*, all the straight lines in the superficies of that cone passing through *C*, and all other lines parallel to any of them, are isometrical, as well as those parallel to the three principal isometrical lines, *CB, CE, CG*; and no other lines but these can be on the same scale. But though this multiplies the number of isometrical lines infinitely, it is of little practical use: because it is only those, which are parallel to the three principal lines, that can be easily distinguished at sight, to be isometrical.

We may further remark, that if a line be drawn through the point *C* parallel to any given line whatever, and that line be made to revolve round the line of sight, at the same angular distance from it, so as to describe the surface of a cone, all other lines parallel to it, in any of its positions, will be isometrical, as they respect one another.

be done, by a series of similar concentric ellipses, as in fig. 7, dividing the isometrical diameters into equal portions. The other diameters will be so divided, as to serve for a scale, for all lines parallel to them respectively.

Thus, in the isometrical squares, exhibited in fig. 2, distances measured on the longer diagonal, or its parallels, would be measured by the divisions on the major axis, those depending on the shorter diagonal, by the divisions on the minor axis.

To describe a cylinder, lying in an isometrical direction, the circles at its extremities, should be represented by the proper isometrical ellipses, and two lines touching both, should be drawn: and in a similar way, a cone, or frustum of a cone, may be described. A globe is represented by a circle, whose radius is the semi-major axis of the ellipse representing a great circle.

It would not be difficult to devise rules for the representation of many other forms which might occur in objects to be represented. But the above cases are sufficient to include almost every thing which occurs in the representation of models, of machines, of philosophical instruments, and indeed, of almost any regular production of art.

Buildings may be exhibited by this perspective, as correctly, in point of measurement, as by plans and elevations, under the advantage of having the full effect of a picture.

A bridge, or any circular, or gothic arch, consisting of portions of circles lying in isometrical planes, may be represented by portions of isometrical ellipses, which will easily be adapted and drawn, upon the principles already explained, by which wheels are exhibited on their axles. The centers of those circles must be found, with which the centers of the ellipses must be made to coincide, their minor axes lying along the lines drawn from those centers perpendicular to the planes of the circles. The shaft of a pillar consists of a frustum of a cone, and a cylinder united; or perhaps of a cylinder

alone, or a congeries of cylinders: and we have already shewn the method of exhibiting these, as well as their bases. And on the same principles, the position, and size of the volutes and ornaments of the capital, may be found, and such guiding points, as will make it easy to trace their forms. Thus the different courts, and edifices of a Cathedral, a College, or a Palace may be correctly depicted; and even the rooms, and internal structure, though less in the form of a picture, may be exhibited in such a way as to enable an architect, or his employer, to contemplate their situation, their ornaments, furniture, or any other circumstance belonging to their appearance; and to mark down exactly what he would have done, in such a way, as could hardly be misunderstood by an attentive agent, though at a distance.

But in thus exhibiting buildings as transparent, and their interior laid open, there is a danger of being confused by a multiplicity of lines; which is a difficulty in a building containing many rooms, that would need some address to get over. It is better adapted to exhibit the inside of a single room, of a Cathedral, for instance, the aisles, and transepts of which would not cause any great perplexity.

In the same manner a plan of a city might be given, which would not only represent its streets, and squares, as well (by the help of the scale above described fig. 7.) as a common plan, but also a picture of its churches, and public buildings, and even its private houses, if such were the design contemplated by the artist, as they would almost all become visible, when looked down upon, from the commanding height which this perspective supposes. And such a single exhibition, if well executed, might give a better idea of a distant capital, than a volume of description.

In the instances which have been given, most of the lines are isometrical. But the art is applicable to many cases, where there are few, or none such. It may be necessary, in many of them, to

draw isometrical lines, or isometrical ellipses, by way of a guide, to determine the position of certain lines, and points, to enable the artist to describe with accuracy what he has in view. And there is scarce any form so anomalous, as to preclude the artist from taking advantage of these methods of ascertaining such lines, or points in it, as will give him much assistance, in representing it with precision. If the intention be merely to make a picture, the guiding lines may be obliterated as soon as they have served the purpose designed, or they may be retained, in some cases, and their lengths or diameters noted down in figures, if it be wished, to give ready information. And often, if the artist wishes to provide materials to enable him, at his leisure, to give accurate descriptions, or exact drawings, the rudest exhibition of such lines may completely serve his purpose, provided he notes down on the spot, such measurements with accuracy, however unexact the lines may be on which they are recorded. In many cases it may be expedient to take liberties with this perspective, or with the picture, which will make it suit the purpose designed. And this will produce no confusion, provided those liberties are explained: for instance, it may often be expedient to make the scale, in the vertical direction, larger, sometimes very considerably so, than in the horizontal. It may in some cases be necessary to represent on paper, what is hid in nature. What has been said on the internal structure of buildings, in p. 13, is an instance of this as well as what we shall observe on the exhibition of subterraneous objects. We shall proceed to give some examples of these observations.

To give such a representation of an Etruscan vase, as would enable an artist to model it exactly, would be exceedingly easy. Let a vertical line be drawn to represent the axis of the vase, fig. 8, and let points be taken in that axis, corresponding to the centers of the principal circles of the vase; through which the horizontal isometrical lines may be drawn representing the radii of those

circles, by the help of which the isometrical ellipses representing them are easily drawn. These will become a complete guide to the artist. He may assist himself by looking at the object along the line of sight, and then, if he has any skill in drawing, he will find no difficulty in tracing the outline from one of these to the other, with sufficient correctness. If he is unskilled in the art, of course he must be at the trouble of finding a larger number of ellipses to guide him. And in a similar manner, any solid, formed by the revolution of a plane figure round one of its sides, may be represented.

The laying down the timbers of a ship, or making a picture of one, shall be another example.

Let a vertical isometrical plane be conceived to pass through its keel, and to be intersected by the perpendicular planes passing through the ribs, and by planes parallel to the decks. The isometrical lines, which are the intersections of these, may be measured in the ship, and represented, with their proper measures noted down, in the picture; which will afford the means of representing the ribs, and laying them down in their proper places.

If this should be designed for the purpose of constructing a ship from a given model, it might be sufficient to represent the ribs only on one side; those on the other side being the exact counterparts. If the purpose should be to make use of these lines for a drawing, they need be marked but very faintly, and the artist will have little difficulty, when guided by them, to fill up the representation by hand.

In a similar manner, this perspective may be applied to the exhibitions of animals, for the illustration of Natural History. All the leading points may be thus accurately designated, and a good artist will find no difficulty in making, by their help, a picture from the animal, which will shew its proportions distinctly.

By this means, those agriculturists, who of late years, have so

much improved the breeds of our cattle, might explain their ideas with precision, on the points, to which they wish to call the attention of their readers.

A regular fortification, which we will suppose to have eight bastions, will afford another example.

A person not conversant in such a subject, is in general puzzled with plans, and sections, and has very little idea of what is meant to be conveyed.

But he would easily understand it, if he should see every thing exhibited in a correct picture, especially where he has the view of his object varied, as in a fortification, such as has been proposed. Let an isometrical ellipse be drawn expressing the internal circumference of the place; and another concentric one, which marks the salient angles of the fortification, on the principles already explained. Draw other guiding lines to every necessary point; the lines of the fortification may be easily transferred from a common plan, to the isometrical, by the help of the scale of concentric ellipses described above, fig. 7, which will serve also to lay down the length of the bastions, and curtains, &c. in whatever direction they lie. Find the elevations of every part on the isometrical scale; and thus the body of the place, the ditches, counterscarpe, covered way, glacis, ravellins, and all the outworks will be represented to the eye as they appear in reality, and in every varied position; with the advantage of having all the admeasurements laid down with geometrical precision. If the artist should think the vertical lines, in such an exhibition, too small to give a correct idea of all the minute elevations, there would be no harm in his increasing the scale in that dimension in any desired proportion.

The face of a hilly, or mountainous country, like Switzerland, or the district of the Lakes in the north part of England, will afford another example.

Isometrical horizontal lines may be drawn representing lines in the level from which the height of the mountains is to be reckoned, so that vertical lines drawn from the summits of the mountains may meet them, on which the heights may be marked; (as well as recorded in figures, if required). And the mountains themselves may be drawn in their topographical situation. Their bearings may be marked by the help of the isometrical compass described in p. 11. It would be easy to transfer them from a common map to the isometrical plan; and thus the face of the country might be represented, just as it would appear from the commanding height which the isometrical perspective supposes.

Yet, as the slopes of hills and mountains are seldom so steep as the line of sight, it might sometimes suit the purpose to represent the height of elevations as twice, or three times the reality, in order that mountains might project an outline on the plane behind; otherwise, the summit might be projected on the mountain itself; which would in a degree destroy the effect of a picture.

This art might be advantageously employed also, for tracing what is below the surface of the earth, as well as what is above it.

It may be applied to geological purposes, and give, not only the order of the strata, but their variations, and their geographical situations. And for this purpose it might be useful to increase the vertical scale, in a great proportion, above the horizontal. It would be easy to mark the dip, or rise of the strata, as well as of the earth above them: to represent their various disruptions, to shew the situation, and extent of fissures, and metallic veins, to mark the boundaries where the upper strata have been swallowed up, or cease to appear; or where the under strata push up towards the day. It would be easy to mark the variations in the thickness of the strata in different places, and to record the result of experiments made at any point, by boring, or sinking shafts: which might be done by drawing a vertical line downward, so as to represent the

thickness of the laminæ, which might be marked by different colours. By such a method, the geologist might obtain a map of the country, which might exhibit at one view, the general results of all the experiments, and enquiries, that had been made relative to that science. And the owner of an estate might record in a small compass, all that is known respecting its minerals, and be able, from a comprehensive view of them all, to judge of the probability of success in sinking a shaft, or driving a level. He might also make good use of this perspective, in tracing his shafts, and drifts, in all their windings, elevations, and depressions; and comparing them with the surface above: marking also the veins, and strata, in which they run. For if the artist knows what is beneath the surface, he has no difficulty in representing it as transparent. He must be careful however not to perplex himself by lines too much multiplied, and take advantage of his being able to paint the lines with different colours, for the purposes of distinction, and he must use a considerable address in throwing out such lines as would be of little use, and in retaining such as will produce the effect of a picture; which should be well preserved, in order to make the exhibition easily intelligible.

If he should wish to make a drawing of minerals, or crystals, this perspective would be well suited to the purpose.

The point, however, on which the writer of this paper can speak with the greatest confidence, is on the representation of machines and philosophical instruments: having been himself so much in the habit of practically applying to them the principles that have been detailed. And this he has exemplified in the plates.

The correct exhibition of objects would be much facilitated, by the use of this perspective, even in the hands of a person who is but little acquainted with the art of drawing; and the information given by such drawings, is much more definite, and precise, than

that obtained by the usual methods, and better fitted to direct a workman in execution.

The writer of this paper cannot help flattering himself, that what he has delivered in it, may be found of some use, in rendering more clear, and intelligible, communications to societies, such as that, of which he has the honour to be the President.



Fig. 1.

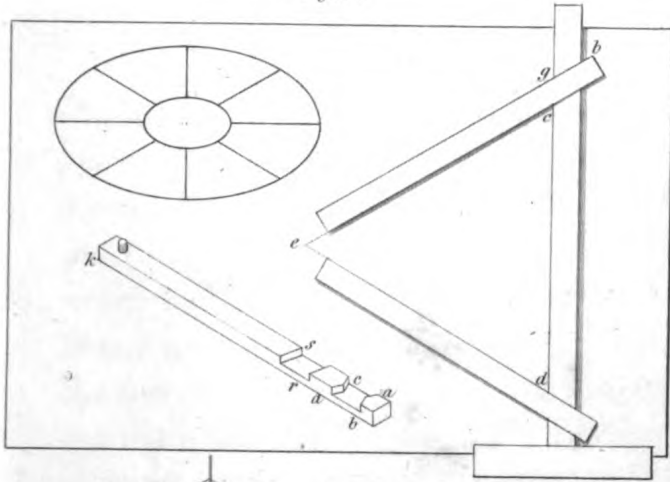


Fig. 2.

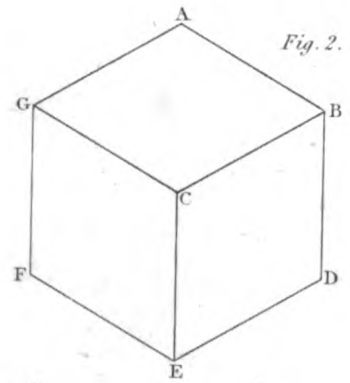


Fig. 6.

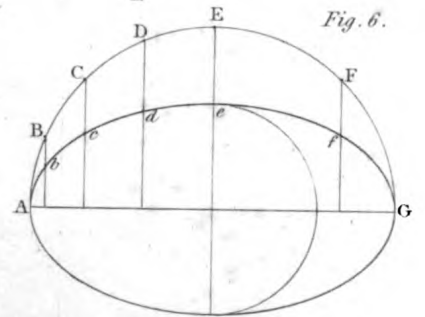


Fig. 3.

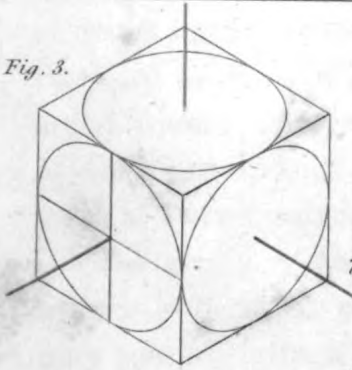


Fig. 4.

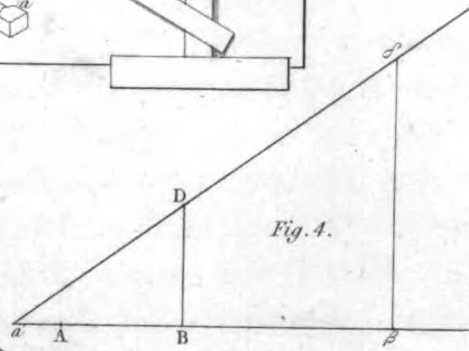


Fig. 5.

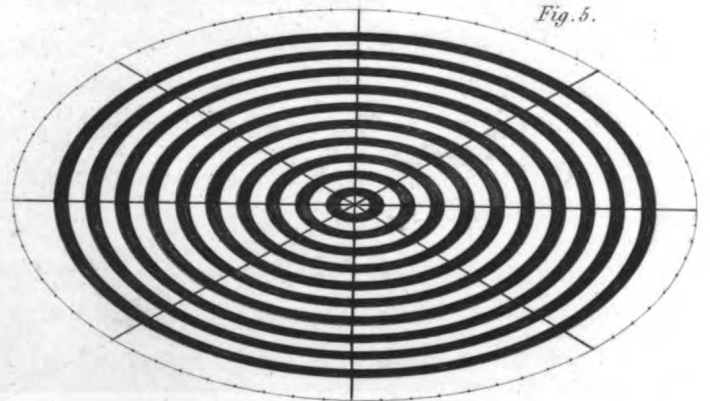


Fig. 8.

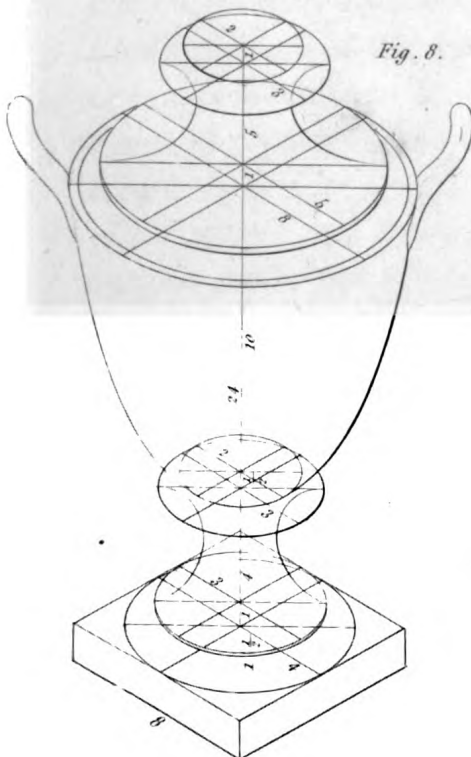


Fig. 7.

