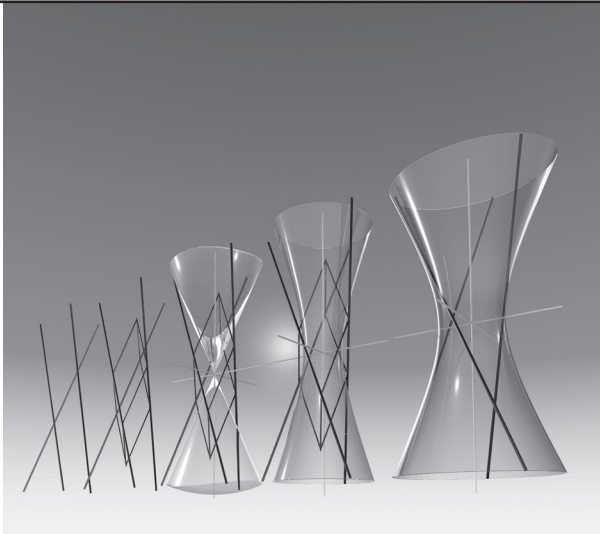


Michela Rossi (editor)

Descriptive Geometry and Digital Representation: Memory and Innovation

General Investigator Prof. Riccardo Migliari



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I Descriptive Geometry and Digital Representation: Memory and Innovation I

General Investigator

Riccardo Migliari (University of Rome "La Sapienza")

2008_Research Project of National Significance

Methods and Applications

Research Unit of Rome

Modeling, Graphic and Experimental Characterizations

Research Unit of Genoa

The Relation Shape/Color.

Digital Procedures for Color Management and Representation

Research Unit of Milan

Natural Interfaces for the Shape Genesis and Improvement in Artefact Design

Research Unit of Venice

Intelligent User Interfaces to support Modeling and Navigation in 3D Graphics Application for Architecture and Design

Research Unit of Udine

Research of national relevance granted by MIUR (Ministry of Education, University and Reserach)

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Foreword

Towards a new Descriptive Geometry

Riccardo Migliari

Although Descriptive Geometry owes its name to Gaspard Monge (1794), it is a very ancient science that has given scientific support to engineers and artists of every time. It allows, in fact, the control of the three-dimensional shapes within a virtual space, both if merely graphical, as it happened in the past, as well as if digital, as it happens nowadays. Descriptive geometry exercises this control in two phases, which are not subsequent but interactive: the project of geometric operations; the representation of such operations. The construction of a ruled surface, for instance, happens first as construction of the straight lines that lay on three oblique directrices, then as representation of the construction operations that bring to the representation of the same surface. Once represented the surface, it is possible to model it, for instance extending it or sectioning it, and the cycle restarts. This process is typical of the project, both for the architecture, as well as for the design, and it can in fact be considered an abstract exercise of these arts. Descriptive geometry possesses, so, a heuristic potential that unfolds not only in the project but also in the research. Gino Loria (1935)¹ counts among the Mathematical Methods "La costruzione come metodo di dimostrazione esistenziale" (Construction as existential demonstration method). And, likewise, René Thom² acknowledges in the model an essential tool of the human mind even in the most abstract research.

This stated, it is necessary also to observe how during the last years this science, once alive and vital, has suffered a serious crisis, induced by the advent of the computer-aided drawing and modelling applications. These applications allow, in fact, the automatic realization of two classes of operations: the construction of simple and derived figures (as surfaces and their intersections) and the dynamic visualization of the same and, as it can be observed, these are the same performances of the ancient science of the representation. We could therefore conclude, but only at a first superficial examination, that descriptive geometry has de facto converted itself into the CAD.

We believe, instead, that it is not so. We think that the CAD is a tool, new and powerful, which is added to the traditional tools (rule, compasses and other devices of the technical drawing). But the CAD doesn't replace, because it doesn't include, that large group of theoretical principles and consequent procedures that allow the control and the modelling of the shapes of the space.

Nevertheless, being a powerful tool, much more powerful than those traditional, the CAD can provide to descriptive geometry the means to carry out procedures always more simple and general, and the means to face those problems that, before, were ignored because of their complexity. For instance: the solution of the Apollonian problem in the space³ finds in the accuracy of the digital representation a solution absolutely general that was impracticable with rule and compasses. The digital representation could therefore be, for descriptive geometry, what the telescope was for the astronomy in Galileo's times and lead to an analogous process of renewal.

In conclusion, the crisis of descriptive geometry is not due to obsolescence, but rather to the lack of a strong connection or better of integration with the digital technologies, software and hardware. This integration process has already begun in the applications of descriptive geometry, even though it is far from being completed; as far as the basis research is concerned it is still in the years to come.

This book presents some first results of a research project⁴ that was carried out at Italian universities during the period 2010 - 2012 and that aims to make a contribution towards the above said integration and development process, in order to reach a renewal of the studies of the scientific representation for the architectural project and the design. The spin-offs of the results are various and include the development of new and more efficient solutions to the classic problems of descriptive geometry, the definition of standards for the interfaces of the computer-aided modelling systems and also the realization of innovative applications.

The interest for the problem of the renewal of the studies of descriptive geometry is emphasized in the papers that many researchers have dedicated to this theme during the last ten years, most of all out of Italy. It is necessary to observe, nevertheless, that most of these papers regard the field of the didactics, as if an academic teaching can exist without the contents and the forms of the related science. We ask ourselves, in synthesis, how to reconcile the teaching of the descriptive geometry and the CAD, instead of wondering how to re-found the science that will be taught.

And the difference between these two ways of seeing is not of little importance.

The volume written by Standiford, K. and D. Standiford⁵, *Descriptive geometry: an integrated approach* using AutoCAD is a manual that has a noteworthy editorial success in the USA. This manual illustrates the elementary problems of descriptive geometry employing a well-known application for the drawing and the computer-aided modelling. Also considering the minimalist objective of this work, and namely to educate university students, we cannot avoid criticizing this approach, which ends up humiliating the descriptive geometry and also the CAD and, in fact:

- descriptive geometry, with its theory, its history, its many applications, is reduced to a pretext to learn the techniques of representation programmes within this or that commercial product;
- the training of the ability to control the three-dimensional forms passes through the code of the combined orthogonal projections, as in the past, rather than to take advantage of the potentialities offered by the virtual space in three dimensions, simulated by means of the software;
- the student is brought to believe that his work will consist in constructing the projections of an object and, secondly (as inevitably will happen), to discover that said projections can automatically be obtained from the three-dimensional model.

It seems evident to us that this issue should be tackled in a totally different manner, at the root, that is, at the scientific research level, and namely before the flow of knowledge pours, as it shall, into the formation. It is necessary to place the problem in the historical perspective, to come back to the keen science, not yet crystallized in the contents of the discipline, to accept the idea of a metamorphosis, namely the idea of a new descriptive geometry that integrates, yes, the computer among its tools, but that uses it without being subdued of it.

Besides, this different way of seeing the things also finds some comparison in the base of international research, for instance in the work of C. Jiannan (1998), *Kernel problems of the modernization of engineering graphics education* (Journal for Geometry and Graphics, Volume 2, No. 1, 65-70)⁶ who, already ten years ago, pointed out the problem of the renewal of descriptive geometry, as a science even before as a teaching subject; and, again, in the clear-headed analysis of Hellmuth Stachel⁷ who, particularly, reassumes the situation like this: stated in advance that descriptive geometry is still, and a great deal, the best mean to form the capability of mentally controlling the space, as the researches made by K. Suzuki (2002) show; to be considered obsolete are: the complicate hand-made constructions, the difficult theoretical demonstrations, the theory that teaches to construct the images of particular three-dimensional objects; whereas still necessary are: the control of the space, the capability to understand spatial configurations as from bi-dimensional images, the capability of orientation with particular reference to the systems of local coordinates, the knowledge of the solid geometry, the creative capability to find solutions, the applications of the descriptive geometry, the capability to produce convincing images; and, finally, furthermore are required: the capability to use modelling software, to construct new geometric forms, as the surfaces of free form are (NURBS), the mastery of the exchange formats and of the dynamic visualizations.

Finally we have to cite those Authors who recently have in various ways denounced the necessity of the renewal of descriptive geometry, as science, beyond the problems and the logics of the formation⁸.

An exception, which therefore absolutely deserves a separate mention, is that of Helmut Pottmann. Pottmann abandoned descriptive geometry and focused his attention on studies of the geometry applied to the tessellation of the surfaces, with important outcomes for the execution plans (engineering) of transparent free form structures. He believes that the problems relating to the representation of three-dimensional shapes on a plane, dealt with in the first chapters of Descriptive Geometry of Mongian tradition, are nowadays totally outdated thanks to the digital representation. Consequently, the graphical study of three-dimensional shapes is developed analytically and, then, represented using information systems. In

this overall picture it does not make sense anymore to talk about descriptive geometry, whereas it makes sense to talk about geometry (tout-court).

We, instead, believe that the second and most important purpose of descriptive geometry, enunciated by Monge himself, should not be neglected, namely the graphical study of the properties of the surfaces and, more in general, of the shapes, of the 3D space. This graphical study, which integrates and improves the analytical study, can nowadays be carried out, using digital representation methods, in the same way as it once was done within the Mongian method.

Descriptive geometry should therefore seize and master the methods brought in by the information-technology revolution. There is still one question to answer: should we renounce this term 'descriptive geometry' which seems so obsolete today, to the advantage of another term considered more appropriate?

On this subject Pottmann proposes 'Architectural geometry'⁹.

We will not renounce the old name coined by Monge, because taking a closer look at the flow of History, this name does not refer, nowadays, only to the contents of the Mongian lessons, but rather to the wealth of the scientific representation, which includes the origins that can be found in the Greek and Latin world, the extraordinary revival during the centuries XV and XVI, and all that scientists, artists, engineers and architects have added during the following centuries, before and after Monge.

This is why we do not want to give up the term 'descriptive geometry'. We have, though, proposed a distinction: that we say *Géométrie Descriptive* when we refer to Monge and to his École Polytechnique, and simply 'descriptive geometry' when we allude to the culture of the scientific representation of the shapes in space.

Much attention has been dedicated in Italy too, during the last years, to the issue regarding the renewal of descriptive geometry: this is evidenced by numerous papers on the topic, presented at International Congresses, both in Italy (Lerici UID from 2000 to 2008) and at several Italian universities, as well as more recently abroad (Tokyo ICGG 2011 and Montreal ICGG 2012). Important steps forward have been taken in the integration of the methods and in the experimentation of advanced geometric constructions.

As for the methods it has been observed that those traditional and those computer-aided are only apparently different, but not substantially. They are different since the first ones characterize themselves in the way of visualizing the objects of the representation, whereas the second in the way of treating the data and elaborating them with linear geometries or of higher degree. But, in reality, the methods of representation are characterized above all in the use that is made of them: some, as the methods of the orthogonal projections and the solid modelling, are directed towards the metric control of the three-dimensional shapes, others, like the perspective and the polygonal modelling, are instead directed towards the formal control. It is therefore possible to unify the methods of scientific representation, both if graphical as well as digital, using a functionalist key. But beyond this course, which describes a possible path, is the fact that it is not possible to give descriptive geometry a future and, at the same time, to recognize the CAD its noble and antique history, without achieving this integration.

As regards the possibility to construct figures capable of accuracies once unrealisable, various experimentations have shown that it is possible to undertake with success new paths and to discover new properties of the considered figures. It has been verified, for instance, that two hyperboloids of two sheets that have one only focus in common, do reciprocally cut in a hyperbola and not in a curve of fourth order, as it was to be expected.

Other examples can be found in the recent studies of Baglioni¹⁰ on the Kasner's problem, of Fallavollita¹¹ on the ruled surfaces, of Salvatore¹² on the research of the sub-contrary sections of cones. More than ever present appears therefore the definition of Monge: "the second (aim of descriptive geometry) is to deduce from the exact description of the bodies all that follows consequently from their forms and from their reciprocal positions: in this sense it is a means for the search for the scientific truth and it offers perpetual examples of the passage from the well-known to the unknown".

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After all, descriptive geometry has a historically consolidated relationship with the art and the construction in general, and it could therefore not fail to be affected by the technological evolution we mentioned earlier. The classical corpus of texts on the discipline, based first of all on the representation methods, understood as the theories of the construction of the encoded image, appeared to be completely inadequate compared with the contemporary project procedure and, what is worse, it seemed unrelated to the new representation techniques of the space, while these last, at the same time, did not seem to have a basis theory of general character, but only the algorithms that permit to solve this or that particular problem. In the academic circles, the architects who, due to changing historical events, today are the repository of the discipline of descriptive geometry, have finally seized the wish for renewal that came above all from the youth, faced, on the one hand, with a theory that does not seem to have any more applications and, on the other, with a technique that is incomprehensible, precisely because it is lacking the general concepts of a theory; now, finally, is ongoing a process of revision and renewal of the classic descriptive geometry, which is based on new definitions of the fundamentals and fulfilled through integrations and transformations of the corpus of texts on the discipline. As we will see in a while, the integrations concern, essentially, the representation methods, while the transformations involve above all the construction procedures of the geometric shapes. The representation methods, in general, are distinguishable for two essential reasons: the first, and the most important, is that each of the methods is able to record the characteristics of the shape and the dimension of an object in the space and, at the same time, it is able to transfer the object back into the space once it has been represented. A method, to be considered as such, must be able to perform this path, in both directions, autonomously, that is, without turning to other methods. The second reason that permits to distinguish the methods, the one from the other, concerns the use of each of them within the planning activity: the metric control, as in the case of the representation in plan and elevation, or the formal and perceptual control, as in the case of the perspective. The information systems make use, basically, of two digital representation methods that have been called: mathematical representation and numerical or polygonal representation. The mathematical representation can use both the classic equations of the analytic geometry, when it describes the geometric locus, and the parametric equations called NURBS (Non Uniform Rational B-Splines), when it describes graphic lines and free-form surfaces. These last equations generate the shape at the varying of two parameters u and v , and they can represent graphic lines and free-form surfaces as well as geometric loci. The mathematical representation, thus, is continuous and able,

therefore, to precisely define the geometric shapes in every point of the space. The simplest and most common of the operations that are typical of a designer's work, namely the drawing of sections of the represented object, highlights the most important characteristic of this description: the result of the cutting a surface is, as a matter of fact, a throughout defined line, which inherit the continuity from the surface to which it belongs. The measure of the length of this line is, in turn, precise to the extent that the accuracy of the system allows. The numerical or polygonal representation, instead, describes the bodies and the surfaces that define them by means of plane triangular or polygonal faces that form, in fact, an irregular polyhedron. The description consists of some lists that gather: the coordinates of the points that constitute the vertices of the polyhedron; the vertices that determine the sides of the triangles; the vertices that determine the sides of the polygon. The list of the coordinates identifies the vertices with an index and shows the Cartesian coordinates of these with respect to the local reference system, namely relative to the considered polyhedron. The lists of the polygons and the n -angles assign an index to each polygon and name the points that are part of these, using the index that distinguishes each of them. It is clear that, the higher is the number of the sides (also called, improperly, polygons), the more the represented shape will be able to approximate the reality. Nevertheless, this same operation of creating sections, that we mentioned above, produces, in this case, a broken line, which is defined precisely only in the vertices, and a measure of the length which accuracy is proportional to the number of the sides of the broken line. In short: the numerical representation is discrete and not continuous, approximate and not precise, as instead is the mathematical representation. It is interesting now to note that the above said representation methods are able to record the shape of an object and to render it to the space, even if using different procedures and different application potentialities. It should also be noted that the digital drawing applications, or better the modelling applications, since we are dealing with three-dimensional shapes, always use both of these two representation methods, even if in different measure and with different aims. In particular, the mathematical representation is used when a correct metric control is essential, whereas the numerical or polygonal representation is used when a perceptive control is useful. To the four traditional representation methods, which are the perspective, the representation in plan and elevation, the axonometric projection, the topographic projection, are therefore added the mathematical representation and the numerical or polygonal representation. In this first theoretical distinction, the two digital methods do not differ from the already well-known graphical techniques: the continuous tracing of a curve, with a pair of compasses for example, and the construction by points, obtained through the intersection of projective fasciae. For that reason, one part of the scientific world thinks that the digital representation methods are techniques and not methods in the true sense of the word. But, since the two systems are based each on its own apparatus of theoretical principles, and not of empirical rules, it seems right to consider them, in all respects, new methods of descriptive geometry. Another objection concerns the fact that, while the classic methods are easily distinguishable by the kind of images that they produce (plan, elevation, perspective), the new methods are both able to generate, real-time, whatever kind of image the designer asks for. Nevertheless, as we already said, the distinguishing character of the representation method, is the use that the designer makes of it and from this point of view there is no doubt that the above said methods have different application fields, which do not depend on the image that they are generating, but on the functional characteristics of the method. Finally, one last doubt concerns the genesis of the images employed by each method; in the classic methods, the image is generated by means of a projection and section process, which is nothing but the 'intersection of the visual pyramid' already theorized by Leon Battista Alberti (1435). In an apparently very different way, the applications that allow to draw in the space, generate the two-dimensional image that appears on the screen, simply eliminating the depth of the three-dimensional object, or better, setting the coordinate z of its vertices equal to zero, with respect to a reference system that have the axes x and y parallel to the screen. As a matter of fact, the plane section of the sheaf of lines and planes that project the geometric shapes onto a plane surface, creating the image used in the method, is nothing but a particular case of the process employed in the digital representation. It is easy to understand this if we generalize the concept of projection, using the space as a support, instead of the plane. And this widened concept is not new at all, it was already recognized as general view of the representation methods by Wilhelm Fiedler (1871) who thinks that every graph is a specialization of the solid perspective. A theory of the genesis of the image that is common to all the methods of descriptive geometry is therefore identified, both when they are graphical as well as when they are digital.

From: "The Geometric Fundamental of Design", by R. Migliari, Enciclopedia Treccani, 4th attached to XXI Century, Roma: 2010

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Certainly, the language of representation—or the language of images, understood as nonverbal communication—is potentially universal, as it is able to overcome cultural and linguistic barriers. Of course, the more complex an encoded image is, the more knowledge of the code is required to understand its message. Conversely, the more developed the iconic aspect of a figure is, the more immediately and "instinctively" viewers will understand its message.

So, how to apply descriptive geometry is a prime concern of nonverbal communication projects, particularly in consideration of the purposes, the target, and the content of the communication.

Among the elements characterizing descriptive geometry, those having to do with optics and color attract the most research, as they are commonly recognized as the signs that enable connotation and identification.

They become communication and send information via the transmission and reception of content—that is, via perception intended as consciousness of an experience or an intuition (see G. Devoto and G. C. Oli, *Dictionary of the Italian language*, Le Monnier, Florence 1971) that goes beyond the boundaries of verbal transmission. Therefore, it is necessary to study the laws that regulate this communication in order to apply optical and chromatic treatments in the way that is best for the pursued aim.

Our purpose is to investigate the effects of light in representation, with particular attention to the influence of the colors and lighting effects applied to descriptive geometry. The topics are developed in their theoretical foundations (physical and projective) and their application outcomes (in traditional and computer-aided drawing).

Lighting phenomena are very interesting for descriptive geometry. Because it can be misleading not to apply certain effects of rendering, consideration must be given to factors such as reflection and aerial perspective when doing so. In the representation of highly reflective surfaces, the theory of bright points, for example, facilitates the detection of shiny or semi-shiny spots on a surface that, in the eye of the observer, reflect the image of the light source.

The themes are treated in terms of two complementary issues: the relationships between digital modeling techniques and the recovery of tradition, and the unification of the graphical interfaces common to the various software packages, with a view to improved reading.

Importance is also given to the books written in the nineteenth and twentieth centuries, when the

knowledge of optics first made it possible to propose a scientific formulation of phenomena previously observed in an empirical way. The application of rendering in software creates a wide range of expressive possibilities which, in mass communication, uses the language of images exclusively for practical purposes. The heritage of the historical scientific tradition can, perhaps, renew the importance of image-based communication.

The research proposed here analyzes and compares the graphical elements present in the "libraries" of software used for 3D representation, with a view to exploring their common features, their limits and their potential for communication in greater depth. Analyzing, and overcoming the limits, of the graphical output of the "optical and chromatic characterizations" of objects and finishes included in the abovementioned libraries for the "house container" permit the representation of more personalized interiors. In addition, the issues linked to representation and design are examined from the practical point of view. The nineteenth-century industrial revolution, which provided a new model of habitation and proposed interiors employing innovative materials, colors and furnishing compositions, is tied to the search for housing solutions where mass-produced objects are still, today, exemplary models for production.

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The Relation Shape/Color in Design. Digital Procedures for Color Management and Representation

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Descriptive Geometry plays a fundamental role in spatial investigation and object measurement. It therefore became the main instrument of design representation in the past. Also, geometry is the essential component of any formal art, and it is the main reference of design, which means the creation of shapes, and design drawing, which concerns their representation. If geometry is the substance, the line is the communication element of graphic language. The sign is the being of drawing just as color is the substance of paint. The designer uses drawing, and then signs, to express concepts and describe shapes and rooms.

As is fully described in the general framework of this research project, the introduction of digital media (i.e., 3D modeling software) creates a new potential for studies of forms, their features and representation. Nevertheless, further investigation is required in order to establish language codes, standards and working methods, and to enable the development of new design tools with digital apps.

The new approach to representation has profoundly altered the design process in recent years. Like form and surface geometry, briefly discussed above, the study of representation and color and their relationship with forms and space has also changed. The introduction of digital representation has deeply affected the representational tools of design, shifting attention from the two dimensions of drawing to the three dimensions of virtual spaces. Modeling in virtual space has overtaken drawing, and color has substituted the sign as the main evidence of the image. As a consequence, color has gained new attention in architectural design, where it had been widely understated in the past. Because of the relevance tools have for the work they perform, the increasing importance of color in representation requires that attention be devoted to its representational devices, which have consequences for the design and, indeed, condition the whole pipeline.

The INDACO research unit contributed to the national project by testing software applications that play the same descriptive geometry role, making it possible to study and solve problems in a more innovative way. The team then focused its attention on the role of color in design, starting from its importance in the visualization of shapes, the perception of space and the characterization of objects or architecture,

based on scientific knowledge.

The research started with color theory and went on to approach the form/color relationship in the design pipeline and to experiment with standards of interior representation and product design.

The documentation of several color languages explains the relevance of geometrical elements in the measurement of color and in the explication of theory; architectural studies applied to the historical background show that traditional media of color representation lack effectiveness.

The binomial geometry/color can be represented and enhanced as a function of new geometrical forms: the chromatic sphere was a fundamental framework for knowledge and color representation in painting; now new design tools can be investigated setting out from geometry and digital representation. Our experience shows that a better knowledge of color may help the development of further facilitations for users and designers.

Our research also addresses color and its role within digital representation along the design pipeline of interior spaces and industrial products. The geometry of color models offers some ideas for the development of digital apps for color representation in design, which can be applied to design choices (color accordance).

A broad review of previous literature helps to underline important features that involve color design in:

- research into color in architecture in the last century;
- the perception of objects and urban space according to facade color;
- the importance of color survey in the analysis, conservation, representation and renovation of architecture and the urban environment;
- the "reconstruction" of the historical background of color design.

Starting out from the theoretical bases of color, the research offers a new contribution to color studies, addressing particularly:

- the importance of color management in design and in its representation tools;
- the intricate relation between formal features, cultural conditioning and perception of color;
- the role of geometry in updating the theory of color harmony in accordance with digital media;
- the setting of new code and apps to check the relations between color and shape and/or artificial space.

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Intelligent User Interfaces to support Modeling and Navigation in 3D graphics Application for Architecture and Design

Roberto Ranon, coordinator

In this project, the research activity of the Udine unit has focused on two main areas:

1. Innovative interfaces for modeling and navigation. In particular, we have developed novel methods to automatically position a viewpoint in a 3D scene such that certain visual properties are guaranteed on the rendered image (for example, size and position of certain framed objects, or absence of occluded parts). We have incorporated this capability into prototypes of modeling software, for example devoted to the production of virtual walkthroughs of architectural 3D models.
2. Novel tools for descriptive geometry. In collaboration with the other partners in the project, we have developed a touch-based application for tablets and smartphones to understand how an e-manual of descriptive geometry could be built.

In the first area, we have worked at two levels: fundamental problems, and novel applications.

At the fundamental level, we have tried to develop methods that can efficiently solve basic photographic composition problems such as “where should I put a camera to get an image that shows objects A, B, and C, each at a certain size and from a certain angle, avoiding their occlusion by other objects” (other, more complex problems can be formulated as well). Our method, which is based on evolutionary computational techniques, is able to solve such kind of problem in dozens of milliseconds for a complex architectural 3D scene (instead of seconds as reported by previous work in the literature).

We have then applied these ideas in two applications. The first one is devoted to the modeling of virtual walkthroughs and tours of architectural 3D scenes. Instead of modeling a viewpoint path, as is common in modeling software (e.g., building a spline by setting a number of control points), our approach requires the modeler to define a series of “views of interest.” For example, the modeler could want the virtual tour to start with a view of the virtual building entrance or surroundings; another view should visualize a certain room; and so on. Our software takes these high-level requirements and computes viewpoints that satisfy them and that can later be manually refined. The modeler then graphically “connects” the generated viewpoints, and realistic walkthroughs are automatically produced. For more details, we refer the reader to the paper:

Chittaro L., Ieronutti L. and Ranon R., “VEX-CMS: A tool to design virtual exhibitions and walkthroughs that

integrates automatic camera control capabilities,” Proceedings of SG 2010: 10th International Symposium on Smart Graphics, Lecture Notes in Computer Science 6133, Springer Verlag, Berlin Heidelberg, June 2010, pp. 103-114.

There is also the possibility of trying the software by downloading it at <http://hclilab.uniud.it/vex/>

Our second experimental application is another modeling tool for the production of viewpoint paths. In this case, we focus on situations where the modeler wants to “film” some 3D scenes, possibly with animations, and produce an edited movie. The application takes a manually produced “script” (not much different from scripts used in cinema) and proposes, for each “event” in the script, different viewpoints that semantically and cinematically represent distinct choices for visualizing the current narrative. In computing viewpoints, our system considers established cinema conventions of continuity and composition. The result is a novel workflow based on interactive collaboration of human creativity with automated intelligence that enables efficient exploration of a wide range of cinematographic possibilities, and rapid production of computer-generated animated movies. More details on this work, which involved the collaboration with researchers from INRIA France and Millsaps College, USA, can be found in the paper: Lino C., Christie M., Ranon R. and Bares W., “*The Director’s Lens: An Intelligent Assistant for Virtual Cinematography*,” Proceedings of MM 2011: 19th ACM International Conference on Multimedia, ACM Press, New York, December 2011, pp. 323–332.

The results of the second area of work are presented in this book, in the chapter titled “*A Mobile Learning Application for Descriptive Geometry*”.

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Natural Interfaces for the Shape Genesis and Improvement in Artefact Design

Agostino De Rosa, coordinator

A “natural” interface is one that meets the normal cognitive mode by which a designer performs complex modeling tasks, regardless of their significance in project planning. To consider both sides of an interface, our research has been divided into two distinct approaches: an *object-interface* approach, which views software architecture and hardware procedures in relation to the properties of the geometric model under construction, and a *subject-interface* approach, which focuses on the ways in which software and hardware show operations, sample directories, and handle all aspects of feedback in the construction and testing of the model.

The subject interface is based on projective design and executed with descriptive-geometry representation methods. Today subject interface is a part of object interface, which with digital representation separates visual (screen) output from gestural input. The geometric intuition of a 3D modeler is still exercised between an optical surrogate of the geometric model (on the screen) and model parameters stored in a data base. Designers can create more complex geometric shapes by exploiting the opportunities offered by NURBS geometry or powerful topological processing techniques—like the geometric hybridization of morphing, warping, folding—and they can easily make projective transformations, like 3D perspective or axonometry. A better progression of subject interfaces for designing and testing, as well as for describing geometric forms, would lead to greater awareness and control of their properties' significance in the object modeled, where such properties take on physical, plastic and iconic meanings. It therefore seems appropriate to reformulate certain traditional geometric problems with a view to the problems generally typical of design—formal research that responds to certain optical properties and to mechanical, tactile, iconic and plastic parameters in nature.

In this connection, a further possible context for studying configurative relations in abstract mathematical space, where proprieties and relations of and among forms develop, other than the one allowed by the virtual space of digital modeling, is presently offered by the new ways of producing art installations, characterized by an immersive and multi-sensorial appeal. The prominent projective quality of these installations, achieved by means of a high-technological employment of light and sound, creates the possibility to verify the intelligibility of certain descriptive geometry constructions in the actual space of sensory experience, through direct action by the user, who is, now, physically located within the “demonstration” space.

The working hypothesis proposed by the IUAV University of Venice research unit foresees an operational test concerning the interplay between interface and user, immersivity and multisensoriality, as well as a comparison between the specific semiotics of the arts and the human body and the semiotics of representations, to be carried out by developing an installation at IUAV University of Venice's historical headquarters.

The subject-interface research of this unit involves four points of view reflected in four initially distinct tasks, and a fifth step focused on verification:

1. A case study of state-of-the-interface modeling and visualization processes, conducted on the basis of ergonomic criteria (cognitive);
2. A perspective on organizing and simplifying generative processes in geometric modeling software;
3. A general theory of the technical representation of artifacts;
4. An analysis of the technologies presently employed in producing user-friendly immersive spaces, and a development of planning strategies by means of which the foreshadowing of projective relations can be perceived in the space of the phenomenological experience.
5. Verification of the analysis performed, to be achieved by planning an installation at the IUAV University of Venice's historical headquarters in Venice's Tolentini neighborhood (contingent upon further funding).

The program of the Venice work group was extended to the broader national context by soliciting collective and individual contributions from outside scholars. The former refer to points 5 and 2 of the General Research Objectives, which have to do, respectively, with proposing a standard and defining repertory of problems; the latter refer to point 3, concerning applications. Regarding Point 5, defining a standard for computer-modeling software interfaces, the Venice unit has teamed up with the units in Udine and Rome to create the database and design the data-entry masks.

Analysis has focused on the following software applications, classified here by type of representation primarily implemented and main goals, for which records have been compiled:

- Mathematical representation: Solidthinking;
- Numerical or polygonal representation: 3D Studio Max;
- Hybrids: AutoCAD;
- Specialist: ArchiCAD.

Ultimately, it was this unit's job to contribute to the formulation of a proposal concerning the standard, and particularly its specific contributions:

- Compiling a repertory of problems and their respective solutions in an innovative key. This unit will specifically analyze problems concerning the planar sections of surfaces and their developments, as they are collected in the reference repertory, in the second part of Chapter IV (problems 503 to 591). This task, as explained in the national project, will produce commentaries on classic and innovative solutions, in their historical context, as well as models of the solutions using the mathematical representation method.
- As regards compiling a repertory of descriptive geometry applications employing digital representation, the Venice unit will work on the topics summarized in the abstract, set in a current conception of the notion of interface, as is explained in detail below.

During the first year the research unit was engaged in four main tasks:

- 1a. To contribute, in parallel to the central unit, to the main, traditional morphological repertoire of architecture (stereotomic and plastic) by reading typical surfaces and geometry, checking their genesis as a function of the tools of representation used, with a view to unifying procedures and systems of representation in 3D modelers in the future.
- 2a. To maintain records of state-of-the-art hardware and software interfaces, in terms both of methods and of tools.
- 3a. To formalize and publish a general theoretical outline, putting side by side a semiotics of technical representation and a rhetoric of artifact forms.
- 4a. To analyzing and devise systems of communication and exploitation of digital data, through processes on the borderline between science and art—the latter understood in its contemporary performative and spatial meaning—with the aim of outlining new approaches, including multi-sensorial ones, to the projective visualization, interpretation and conception of forms.

The creation of a repertory of geometric surfaces in present and past architecture, checking in each instance their generative algorithms in software, has allowed us to standardize representative language in order to obtain a mathematic correspondence between the institution designed and the rule that governs it. We have considered not only sliding surfaces (developed and undeveloped), but also algebraic surfaces, of difficult encoding, and have traced how they are described.

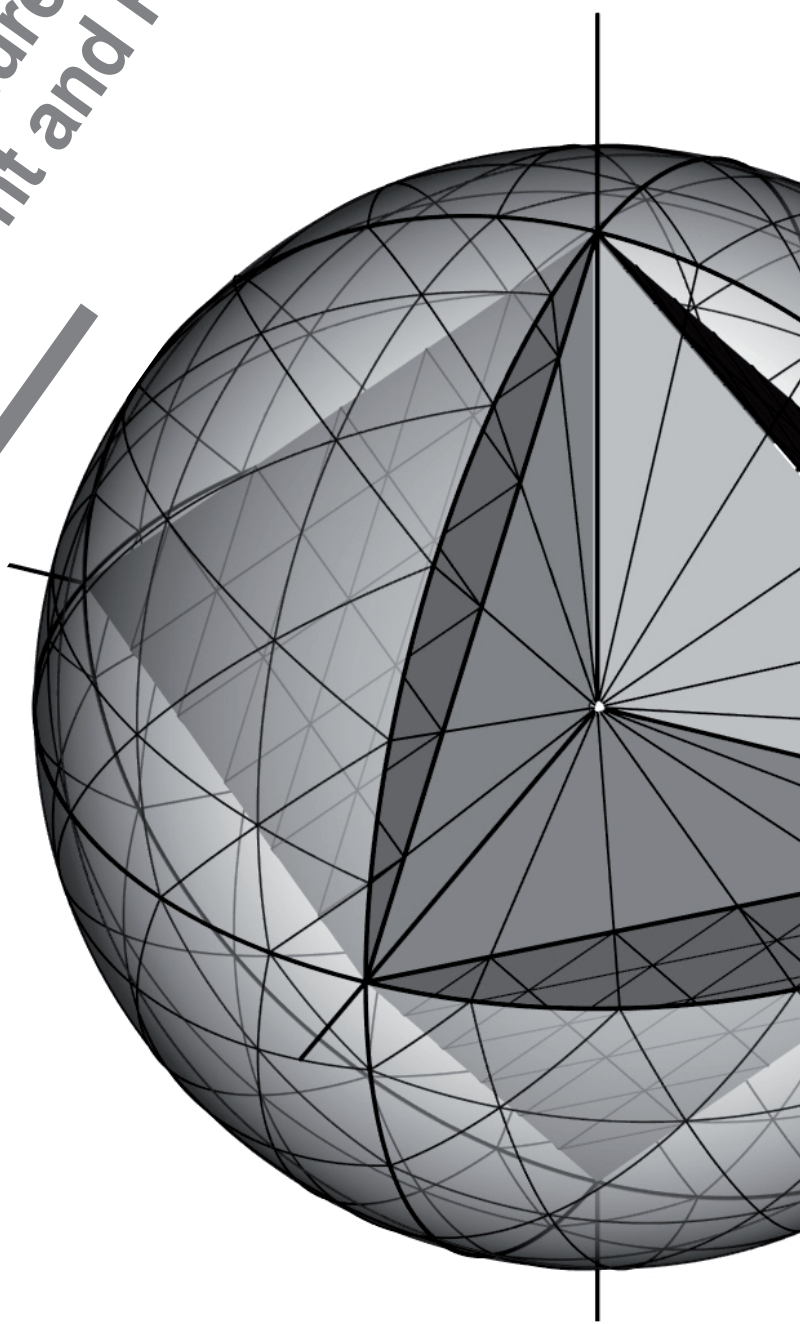
At the same time, the definition of a taxonomy of current hardware and software interfaces has allowed us to classify the methods and tools to identify and, in relation to the first objective of this phase, the points of continuity and discontinuity in relation to the overall objectives of the research.

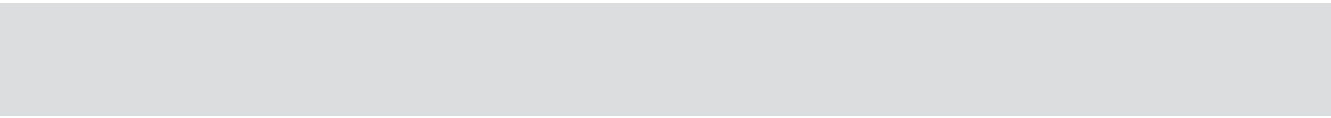
The third task of this phase, which is design-oriented, aspired to identify a total image that will classify formal manipulation techniques in design and, particularly, to distinguish among different types of morphing, from geometrical morphing to analogical morphing (which is based on the exchange of iconic and plastic attributes, among other things), by methodology used.

The next stage, which took place in the second year of research, was responsible for:

- 1b. Identifying the means of general theoretical organization and simplification of processes generating geometric and formal control in software modeling;
- 2b. Implementing some, with open-source modeling software.

! The Relation Shape/Color. Digital Procedures for Color Management and Representation !





1. The Geometry of Colour. Shape as a Measuring Tool

Michela Rossi

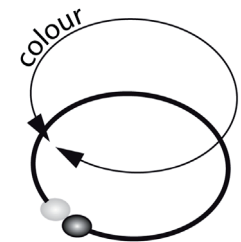
1.1 Introduction. Sign and Color

Color has always been one of the least defined elements in design works, an attribute to be assigned to the finished work, as if it were an element foreign to projects. This lack of attention reflects the prejudices of Classical thought, which gives drawing the task of expressing knowledge of the concept of form, and painting the job of recounting the experience of the senses, belonging to the world of phenomena. The result is the association *design/substance* and *color/appearance* affirmed by Classical philosophers, for whom painting describes the appearance, whereas drawing goes into the substance, which Aristotle identifies with shape. For sighted people, color is in fact the first element in the recognition of forms, and therefore it involves the relationship between *subjectivity and objectivity*. Ancient philosophy places it in the category of phenomena and regards it as a secondary element in relation to the concepts of *matter, form and geometry*. So Western culture attributes to drawing the transcription of a form (its size, geometry and structure) by the sign of its outline, whereas color, referred to the appearance of things and not to their substance, is part of painting.

G. C. Argan explains that painting represents reality using descriptive images that privilege color as a distinctive feature, unlike drawing, in which the mark expresses concepts that imply a deeper knowledge.¹ The conception of color as a secondary attribute linked to appearance is confirmed by Renaissance treatises, which refer to color in relation to painting, saying that architectural design must describe the form, and not insist on ornament and other accessory elements, among which is color. Vitruvius recognizes its importance for mural decorations in domestic interiors. As a consequence, projects and their representation incline to neglect chromatic aspects, thought to be a factor of secondary importance that can be delegated to the construction.²

The digital technology, which has focused its attention on the 3D representation, uses the color as an important element of the visualization of shapes on the monitor. The new tools of computer representation, facilitating the color reproduction even in the technical design, have radically changed the graphic standards of the project.

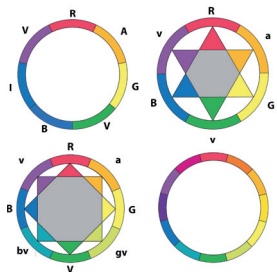
In virtual models through which is developed an important part of the design process, the shape is made recognizable by the color as in painting and not the outline, which does not exist in reality. Digital technology, which has focused its attention on 3D representation, uses color as an important element of the visualization of shapes on the monitor. New tools of computer representation, by facilitating color reproduction even in technical design, have radically changed the graphic standards of projects. In the virtual models through which an important part of the design process develops, shape is made recognizable by *color* as in painting and not by *outline*, which does not exist in reality. Computer representation, which dominates contemporary project management, shifts the focus to the physical nature of color, highlighting its dual nature, as representation and realization, due to the mixing of pigments. In digital projects, control of chromatic combinations and blending of pigments goes through the additive synthesis of the lights, which generates the virtual image, before its subtractive synthesis on paper. The shift of attention from the planar image of the drawing to the three-dimensional model requires that one redefines the role of color in the project and in its representation, where it is forced to mediate between the physics of the digital representation and the chemistry of the construction. The impact of the tools of representation on the management and representation of projects demands that one investigate the operative methods of showing color, to which an important role is acknowledged in the characterization of urban space, and so, in design research.





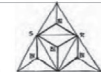









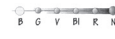






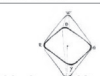
















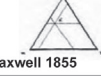













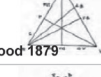




















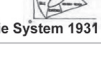

(figure 1) The first color pattern organised the hues in a line between black and white, the dark and the light that generates the phenomenon.



(figure 2) In XVII century, Fludd combined the white and the black in a 7 colors wheel, stressing the continuity of colours.






































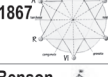























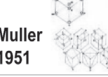




(figure 3) Newton detected 7 color in the white light and in his wheel he combined red and violet that are opposite in the spectrum. Goethe suggested the a symmetrical 6 color pattern, with complementary colors on the diameter extremes. With 4 primary instead of 3, change the couples of complementary, then the harmony, not the pattern.

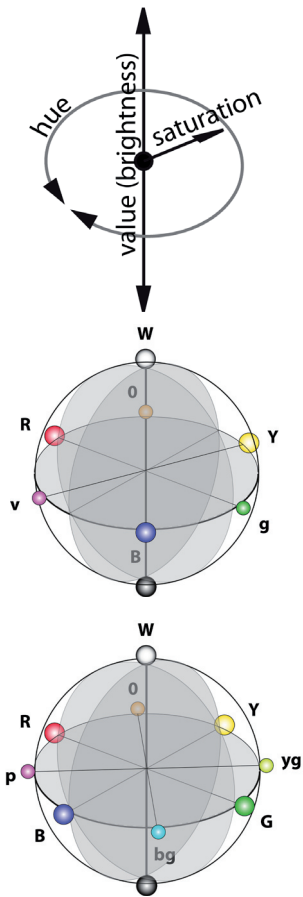
Lines, arcs, tables	Cones	Wheels	Globes	Triangles	Polyhedron	Complex surfaces
 Pythagoras		 Fludd 1607		 Plato	 Plato	
 Aristotle		 Newton 1672		 Alberti 1435		
 Alberto Magno	 Grosseteste 1230	 Boutet 1708		 Waller 1686	 Benson 1868	
 Leonardo		 Castel 1740		 Lambei	 Hoeffler 1883	
 Aguilonius 1613		 Harris 1766		 Mayer 1758	 Hebbinghaus 1902	
 Kircher 1646		 Schiffemuller 1772		 Lacombe 1792	 Becke 1924	 Luterg Nieberg 1927
 Glisson 1650	 Von Bezold 1874	 Goethe 1810	 Forsius 1641	 Sowerby 1809	 Boring 1929	 Pope 1929
 Waller 1670	 Wundt 1893	 Hayter 1826	 Runge 1810	 Maxwell 1855	 Muller 1951	 Johnson 1936
 Le Blon 1741	 Ostwald 1916	 Field 1854	 Chevreul 1839	 Helmholz 1860	 Hickethier 1952	 Mc Adam 1944
 Young 1801	 NCS 1953	 Blanc 1867	 Wundt 1874	 Rood 1879	 OSA system 1960	 Cie Stile 1946
	 Coloroid 1962	 Hering 1872	 Munsell 1905	 Lacouture 1890	 Cie lab system 1976	 Hesselgren 1953
	 Garritsen 1975	 Rood 1879	 ICS - NBS 1955	 Fick 1892	 RGB - 1981	 Din - System 1953
	 ACC System 1978	 Jacobs 1925		 Winter 1910	 CMN - 1986	
	 HLS System 1991	 Birren 1923		 Cie System 1931	 Albert Vanel 1990	

(figure 4) Chronological table of color models. Shapes with similar patterns are gathered in the same family.

Design Application, the Color

Philosophy, linguistics	Art and design	Physics and astronomy	Medicine and nat. sciences	Industry and chemical	Physiology, psychology	Colorimetry
Plato 	Leonardo 1430 	Forsius 1611 	Grosseteste 1230 	Waller 1680 		
Phythagora 	Leon Battista Alberti 1435 	Aguilonius 1613 	Fludd 1637 	Waller 1688 		
Aristotle 	Boutet 1708 	Newton 1672 	Kircker 1646 	Le Blon 1741 		
Alberto Magno 	Castel 1740 	Mayer 1758 	Lambert 1758 	Sowerby 1809 		
	Goethe 1810 		Harris 1766 	Chevreul 1839 	Young 1801 	
	Runge 1810 		Schiffenmuller 1772 	Maxwell 1855 	Wundt 1854 	
	Hayter 1826 	Field 1854 	Lacombe 1792 	Lacoutre 1890 	Wundt 1874 	
	Von Bezold 1860 	Helmholz 1860 		Becke 1924 	Hoefler 1883 	
	Blanc 1867 	Rood 1879 	Hering 1872 	Johansson 1936 	Hebblinghaus 1902 	Winter 1910 
	Benson 1868 	Fick 1892 		Hickthier 1952 	Ostwald 1916 	Luter Nieberg 1927 
	Winter 1910 		Munsell 1905 	Hessलगren 1953 	Pope 1929 	Cie Mc Adam 1944 
	Faber Birren 1923-25 			OSA-System 1960 	Boring 1829 	Cie Sile 1946 
	Jakobs 1925 			NCS 1968 	DIN-System 1953 	
ICS-NBS 1955 	Muller 1951 			HLS 1975 	CMN 1986 	

(figure 5) Chronological table of main models applications.



(figure 6-7-8) Three-dimensional shapes allow a simple representation of the color with polar coordinates, no matter the shape and the number of primary colours. The spheres show Runge's (1810) and Wundt's (1874) models.

1.2 Color and Geometry

Theory leads color to *geometry*, which has always had a close relationship with the design and the project. Geometry arises from the need to *measure the earth*. The Greek philosophers developed arithmetic as the result of the measure of the sizes of the forms, identifying *number* as a modular element able to express the distances between visible elements. The changeover from *shapes* to *numbers* opened the way to systems of computing physical lengths and the measurement of phenomena. Number and mathematical calculation become the tools for quantifying phenomena of any kind and today allow their digital transposition.

So geometry is the privileged tool for the representation of cognitive models that can express different elements, even abstract ones. It displays concepts in the relationship between shape and number, which manifests the association between the arts of space (and time), which share the concept of harmony and proportion with mathematics. This double bond also ties the description of color to the forms of geometry, the first reference to the explanation and measure of phenomena. The use of geometric models dates back to the ancient world: historical models refer to the geometry of elementary figures to give simple explanations for complex facts.³ Theories associate a geometric rule to the relationship between the qualities of color, now called hue, brightness and saturation. The geometric model rationalizes the rule that organizes the system, and shows the gradual transition from one color to another along favorite lines.

The association of color to geometry is justified by the role that the Western world has given to mathematics in an attempt to explain reality, consisting of elements expressed by *shape* and verifiable by *measurement*—number. The diversity of models increases in relation to the development of practical applications, exploding in the last two centuries. The continuity of nuances requires ordered systems of basic geometric shapes, able to express themselves in a small number of factors and to measure *quality* in terms of numerical *quantities*. Theoretical models show in a single construction the correspondence of the characteristic elements of form with the *primary components*, the *variables* and *harmonies* of colors. Geometry indeed finds its complement in the harmony of proportions and symmetries of regular figures, which display harmonies of color through the concept of proportion.

The objective elements of geometry enable one to overcome the difficulties in the measurement of perception due to the physiology of the eye and the physical nature of color, generated when light strikes opaque bodies. Abstraction, which transposes the color in a shape, expresses the choice of the primary colors and the measure of their relative amounts in the derivative colors, in relation to the quantity of light (brightness) and the color intensity (saturation). The theoretical model allows practical applications in which the shape determines the mixing of pigments in the production of color gradations. The difference between the chemical and the physical nature of the phenomenon is seen in the proposal of alternative systems in primary colors, but not necessarily in the form, which translates the quantitative and qualitative data into coordinates. The geometric model expresses the qualities of color distances and angles, and the analogical reference facilitates visualization and storage.

The immediacy of association with theoretical concepts justifies the success of geometry as a tool for measuring color, explaining the number of models offered in support of the many theories. The wheel and the sphere are the best known forms in a varied panorama. They are rooted in the aesthetics of Western culture thanks to persistent teaching and experimental application by the Bauhaus 4 and Itten, who used the dual model proposed by Goethe and Runge in the previous century. These two artists had the merit of having popularized the reference to geometry for the explanation of color, even if they reworked previous models. The success of these forms is the result of their effectiveness as an explanatory support to the harmonies of color, which is associated with the search for a link between *painting* and *sculpture/architecture*, and between *color* and *shape*.

The study of geometric models of color is not unknown. The existing literature provides a significant overview of the theories, schools and purposes, highlighting the potential applications of the different systems. The comparison of models and theories explains the complexity of the components of the problem and underscores the interest of different scientific domains accompanying the persistence of similar shapes and the coexistence of different assumptions. However it can't be said to be conclusive, although it is a necessary step for the study of this phenomenon in relation to the project, with the development of digital applications for color surveying in urban areas and the control of matching and chromatic harmonies in interior design and printed textiles.

The research started by reordering the results of previous studies⁵, to investigate similarities and differences on the basis of a reorganization of formal models in families, and to verify the potential of geometric models in relation to the representation of color in the graphic tools of projects, historically conditioned by the objective limits of their tools of representation.

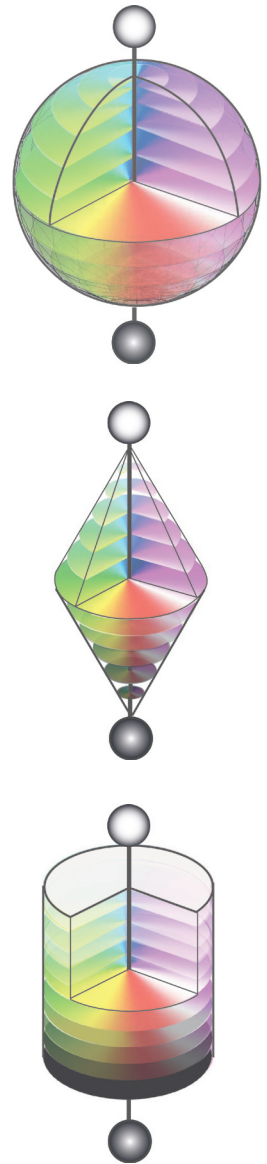
Shape is not the only method of cataloging, but it is the most obvious manifestation of the specific features of theory and thus becomes the first comparison criterion in an attempt to reorganize the many interpretations developed. Other possible classifications are history, scientific fields and application purposes, geographic and cultural areas. The classification of geometric models produces thematic maps to help one understand the complexity and multiple relations of the aspects the theme involves and can become a basis for the development of effective tools for the management of various information, capable of interweaving formal, aesthetic, linguistic, perceptual and cultural references.

Models for the representation of color have attracted continuous interest, which has been rendered all the more intense by the development of applications that often go back to painting, but are also useful in the applied arts and in industry. The number and diversification of forms demonstrate the effectiveness of geometry in displaying concepts and the impossibility of a unitary, ultimate solution, because of the dual nature of color.⁶ The comparison of models in relation to form, cultural areas or theoretical and practical purposes emphasizes the relevance of concepts based on different assumptions for the number and nature of the primary colors, along with the persistence of deep-rooted symbolic references.

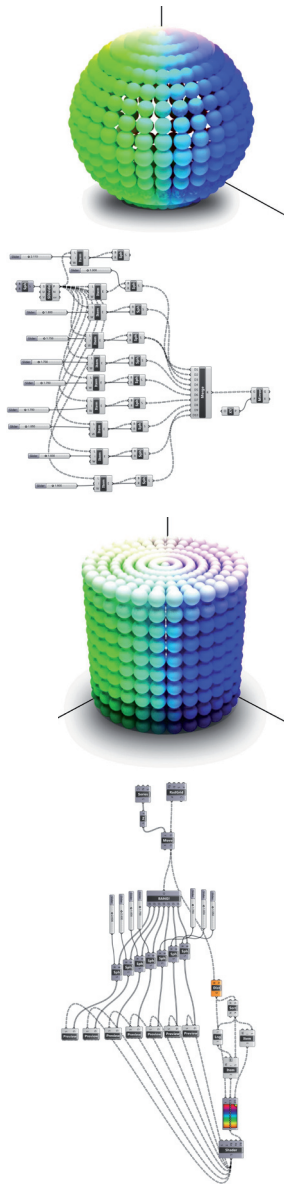
The shapes of geometric models provide ideas for the development of new solutions for the control and visualization of color in projects with contemporary tools of representation. These identify color as an element complementary to sign, and apply calculation algorithms for which the mathematical reference finds in geometry an intuitive and accessible interface. The specificity of the different solutions suggested we test the potential of geometry in the construction of digital models using software capable of reproducing, in a parametric way, color change as a result of the process of construction of the model itself.

1.3 Theories, Models and Practical Applications

The first attempt to theorize color harks back to ancient Greece. Empedocles recognizes three basic colors, with the red between white and black to separate light colors from dark ones in the alignment that connects the two poles of the globe. Plato, for whom all reality is made up of two types of triangles and the elements that constitute it have the form of the five regular solids, in the *Timaeus* uses the tetrahedron. At its vertices, there are four basic colors: white, black, red and bright. From the mixture of white, red and bright comes yellow; white, black and bright create blue; white, black and red generate purple, with no brightness. The shape corresponds to the element, fire, and it is interesting to note that this first conceptual model links the manifestation of color to light. The linear structure remains in the explanation put forth by Pythagoras and is confirmed by Aristotle: seven colors associated with the planets link the harmony of colors back to cosmic harmony. In the Middle Ages the Classical heritage flows into the



(figure 9-10-11) The main models with ruled surface keep the white at the top of the polar axis and the black in the bottom, such as *albedo* and *nigredo* in Grosseteste's double cone.



(figure 12) Generative color modeling on ruled surfaces with Grasshopper. The software, which has a color algorithm, allows immediately the automathical generation of two variables (hue and value). Colors hue are associated to solid elements with decreasing/increasing dimensions and ou can easily modify a solid in a different one, changing the shape algorithm. The rotational shapes don't allow a regular filling but on their surface. (G. Buratti)

Arab world, which refines optical knowledge with innovations later detailed by the English Franciscans. Grosseteste, Bacon's teacher, explains color with a double cone with white/light and black/dark at the two vertices and pure colors on the base in between.

The first theoretical models link color to the light/shadow antithesis and insert it in the gradual transition between black and white. From the 1400s a more practical interest develops in the applied arts, particularly in fabrics. When the Italian Renaissance rediscovers ancient knowledge, the practical interest of painters is aimed at the theoretical models of the Classical world. The regularity of the elementary figures suggests their application to the measurement of mixtures of pigments.

As science relaxes its ties with doctrine, the invention of the printing press accelerates the diffusion of scientific and technical knowledge above all in France, England and Germany, where the first industrial applications are tested. From the seventeenth century the need to explain physical phenomena in a rational and measurable way leads to the development of new models, both flat and 3D. They are aimed at the control of mixtures of light or pigment, which provide a formal continuity with those of medieval philosophers and Renaissance artists.⁵ Mixtures of pigments are known, but the development of proto-industrial techniques produces new applications in dyeing textiles and printing (Le Blon, Chevreul). They stimulate study of the effects of perception of the interactions between colors, eventually leading to the classification of chromatic harmonies. The best known classification is based on the opposition of primary and complementary colors, popularized by the work of Goethe and the Bauhaus, with important consequences for the culture of color in contemporary arts.⁶ Goethe's work is not very original, but credit is due to the poet for recognizing the importance of perceptual interferences of colors. His analysis of the perceptual effects of the combination of colors provides a first reference for the studies of psychologists, who associate it with complex surfaces models that give an objective measure to stimuli and physiological reactions.

The quantity and quality of the primary colors in the system are displayed by the symmetries of the model, which binds the coordinates of the color to a finished surface, in a space in two or three dimensions. The constants that emerge from comparison highlight the significance of certain elements related to the symbolism of astrological and alchemical knowledge, pointing out the derivation of chemistry from alchemy and the symbolic power of color. An obvious example is the vertical axis of the achromatic line (*albedo/nigredo*), almost always arranged with the light at the top and the dark at the bottom, but also the number of primary colors in the different systems (three, four, seven). They emerge in the seven-color wheels proposed by Fludd and Newton, despite the obvious contradiction to the theory of the light spectrum.

Just as similar situations may give rise to different patterns, so different situations may be associated with a single shape. Therefore the choice of the shape is arbitrary and doesn't depend on the concept. The shape is a tool for explaining the concept, not the concept itself. Indeed, some theories refer to different shapes. The most distinctive elements of the concept are the number and/or the choice of the primary colors and their place in the model. There is a great difference between physical and chemical synthesis, but in both of them one finds very similar geometric references.

The formal concept of the theory shows the analogical relationship between the shape and the coordinates that measure its color, depending on the organization of the primary colors on the surface and the form of the surface, but it does not create the colors. It expresses a measuring tool, but it is not the measurement, which is always the quantity of each primary in the mixture.

The number of primaries changes the mixtures and therefore the colors of the system. In the 1600s and 1700s interest expands, involving physics, medicine and the natural sciences, chemistry and industry, in light of the practical applications of scientific discoveries. The first interesting applications concern printing and the textile industry. In the nineteenth and twentieth centuries colorimetry answers to the need to measure color scientifically, and interest is addressed also to physiology and psychology. During the first half of the twentieth century, new models attempt to represent the perceptual

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relationships in a scientific way, through the use of complex mathematical surfaces. The return to the simpler forms of the cube and tetrahedron underpins color research in television and information technologies.

The chronology identifies three phases characterized by:

- Looking for patterns capable of arranging colors in a brightness range between white and black, which expresses the symbolic antithesis of light and shadow;
- Recourse to symmetries for the control of mixtures of primary pigments (or lights), from which it is possible to obtain all derivative colors, with practical applications in art and industry;
- An effort to make the factors of perception measurable and objective, focusing mainly on the physiological nature of vision receptors.

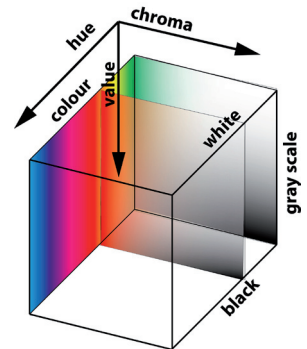
The models are linked to figures and elementary solids grouped into families in two and three dimensions, with specific symmetries associated with the explanation of the harmonies and mixtures of the primary colors:

- Circles, spheres and "roses";
- Polyhedra (tetrahedron, cube, octahedron);
- Triangles, pyramids and "modified triangles" (with knots and spirals);
- Cones, double cones and cylinders;
- Complex surfaces;
- Algebraic structures and various mathematical models.

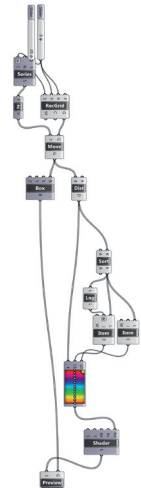
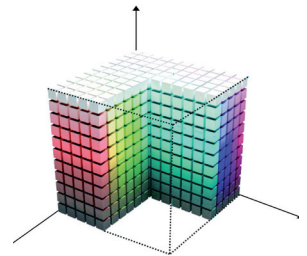
The last group includes mathematical models in which numerical calculation shows complex and subjective aspects of the phenomenon. This family, which differs from the others because of the difficulty of visualization of the models, brings together efforts to integrate in a single system the objective parameters of perception, influenced by physiological factors. It opens the way to the use of multi-dimensional mathematical models, such as the matrices produced in relation to the development of artificial intelligence systems and speech applications, whereas the simpler forms have practical applications. The recent ones call attention to the conceptual validity of many theories of the past.

After Plato, Grosseteste and then Lambert pointed out the inadequacy of planar models to express the three qualities of color. Solid models allow one to discretize on their flat sections the changes of two variables leaving the third constant, to display different situations depending on the chosen section, to define discretized shades in the continuous variation of color. To families originating from the closure of the linear models in a flat form must be added tables, used, from the end of the seventeenth century to represent and measure the mixing of pigments and showing their possible combinations in the structure of Cartesian space.⁷ The first, proposed by Waller in 1686 for use by craftsmen and naturalists, defines over 100 colors by mixing seven pure pigments (white, several blues and reds) on the first line, with a range of yellows and reds arranged on the left column with white on top and darker red below. The table represents the first attempt to standardize colors and documents the recognition of red and blue as basic primitives, to which yellow is added in the formulation of the square model, with the primary colors in the middle of the sides.

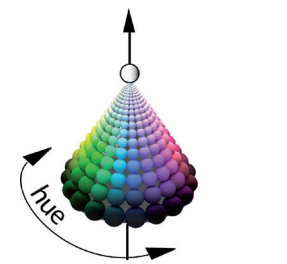
Formal similarities emphasize the topologically equivalent value of models with convex surfaces, and the sphere is the transition between one form and the other. In curved solids a different space is left for the various colors: the polar conception shows how, on approaching non-color, one loses the recognition of colors. In the cone and sphere the amplitude of shades is reduced toward the poles, in the first graduation is linear, and in the second increases progressively, accentuating the visibility of the phenomenon. The symmetries of vertices and edges in the polyhedra show the relationship between the primary elements of chromatic structure. They set out the basic colors in the main symmetries, on the top or center of the polygon faces or edges. The models with continuous or discontinuous surface represent the graduation, or discretization, of color. In the language of digital modeling, these models could be created with NURBS or mesh.



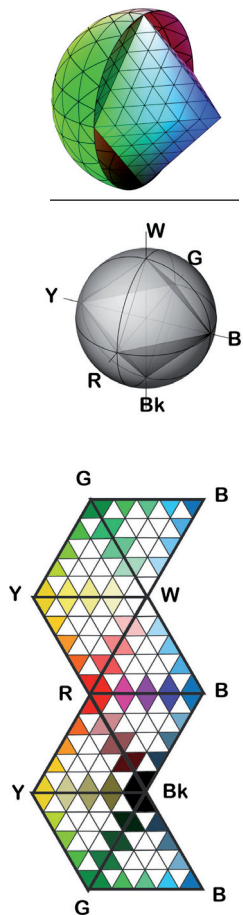
(figure 13) The cubic model represent the color by Cartesian coordinates.



(figure 14) Generative tessellation of color in the cube; this tessellation allows the modular filling of the space between the discrete elements of each color (G. Buratti).



(figure 15) Generative tessellation of color on cone surface. Double cone should have the full colors on the main diameter. (G. Buratti)



(figure 16) The color-octahedron (Höfner, Hebbinghaus, Boring). The geodesic projection on the spherical surface produces a symmetrical distribution with respect to the 6 pure colors (white, black, red, yellow, green, blue); on diameters there are the mixtures of two colors; on the faces those of three.

1.4 Conclusion. The Parametric Representation of Color

The digital space of computerized representation, adopting the cubic model of the RGB system to display color on the screen, can adjust the multidimensional structures of graphs to the interwoven relationships between the influencing factors of color. It resolves, in virtual space, a complexity that cannot be resolved and displayed in Euclidean space. Therefore research is justified in structuring a "smart" program for data collection and for the management of general information related to color as a tool of projects, appropriate both to design and to architecture, which are the two faces of the contemporary artificial microcosm of living.⁸

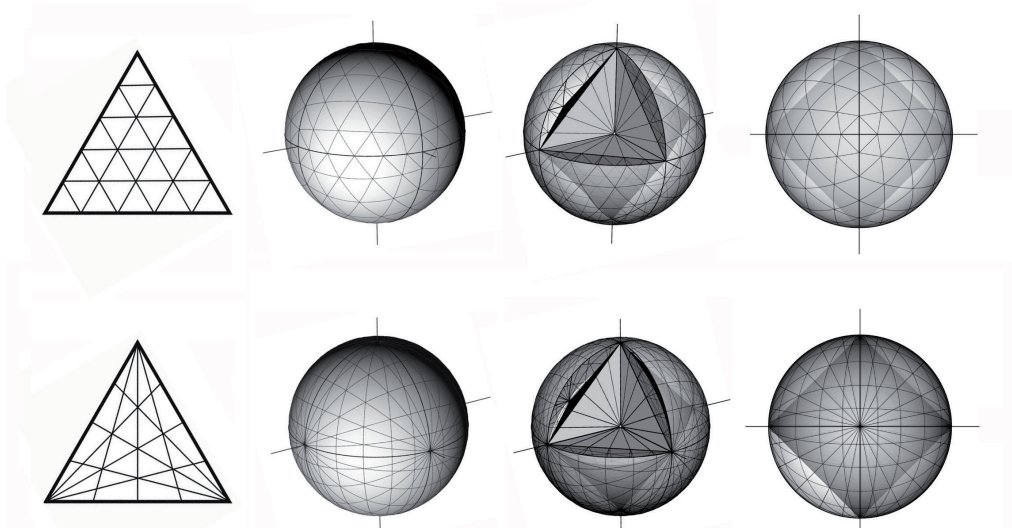
During the research that investigated different aspects of color, the feasibility of different computerized applications has been verified. They allow the development of auxiliary tools of various kinds for the color control in the project. Hypotheses and experiments conducted led finally to the definition of a project for a color model that offers alternative combinations of equivalent colors in the design of textiles, wall papers and polychrome finishes. References and possible areas of application are urban renewal, the survey of color, the color restoration and color representation in the project, and especially the design value of the color combinations. This is one of the aspects for which the geometry of the color has the greatest interest, because the theories of color harmony applied to the formal arts, particularly painting, refer to the symmetries of geometric figures. Digital modeling of a parametric color-solid with a software that enables the subsequent transformation of its Euclidean form, simulates the transition between topologically equivalent forms and demonstrates that the essence of the model is expressed not by the form but by the reference system of its coordinates.

The shape is an abstraction aimed at visualization. Digital modeling, with technicians capable of generating successive and gradual transformations from a basic element through reiterated algorithms, makes it possible to simulate the automatic discretization of the chromatic scale, applying the parametric processes to *color* rather than *form*. The experimental models describe color changes in connection to the system coordinates and preserve the possibility to transform the primitives. The transition from one form to another, the gradualism between adjacent tones and the definition of the position of the primary colors on the model can be controlled with a few simple operations, even later on, by changing the geometric primitive or the color gradient.

Giorgio Buratti collaborated with the study of a digital application for displaying color combinations in interior architecture (furniture and finishes). This allowed the evaluation of: the available interface, the problems connected with the digital representation of color, the experimenting with generative software (Grasshopper by Rhinoceros) in the parametric construction of geometric models of color. The verification concerned the recurring forms of elementary solids, giving priority to those who had useful applications in the arts and industry. The models of different shapes reveal a common understanding linked to the polar representation of the 3 dimensions of color in a solid with ruled surface. The testing concerned the parametric modeling of simplified models based on the elementary geometry of the cone, the cylinder and the sphere. The control of two independent variables is immediate, while the third variable requires the use of more complex algorithms than those available, but theoretically the program supports the automatic generation of color from the default parameters as desired.

The problems highlighted by the modelling of simple solids (sphere, cone, cylinder and cube) have emphasized the limits of tessellations of these three-dimensional shapes in relation to the symmetries on which the chromatic harmonies are built. These bind the important experiments promoted under the Bauhaus to the empirical observations of Goethe, expressing an aesthetic conception of the color that has become a classic and is still recognized as a point of reference. The project proposal concerns the realization of a solid of color that allows the immediate visualization of equivalent chromatic combinations (i.e. with the same symmetry with respect to the chromatic center of

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gravity), through the rotation in the space of a chosen number of radial elements whose direction is bound to the choice of the starting series.

The Descriptive Geometry suggests a solution to the problem facilitated by the application of the graphics capabilities of the generative software, from where you can get easily the tessellation of any surface.

The first obligated reference can be found in the researches of Buckminster Fuller and Maurits Cornelius Escher, differently applied the geometry of the sphere and the symmetries of regular solids inscribed in it in search of innovative solutions. They produced a personal reworking of the "natural" geometry of the regular solids that Plato used to explain the Nature, in which the geometry of the shape is the element that provides the solution to the design problem. Fuller's geodesic domes are one of the well known examples of the applications of Descriptive Geometry, and it continue to be the object of attention from continuously new points of view, as evidenced by the scientific work of the schools of Florence and Rome.

The design of a symmetrical "color pattern" requires a regular solid to which apply the colors and then proceed with a geodesic tessellation with a regular deformations of the modules. The octahedron meets the requirements: its 6 vertices allow you to place in a prominent position the 4 psychological primary (red, yellow, blue, green) and the two achromatic colors (black and white) and to geodetically project them on the sphere, inserting all the gradations in between. The starting setting is similar to that of the double cone NCS and remind in the form that defines the position of the fundamental colors the twentieth-century models of Hoeffler, Hebbinghaus and Boring, but also the organization of the colors in the Wundt's sphere.

The choice of the geodetic lattice of the tessellation with the repeated application of the same generative algorithm is conditioned by the need to minimize the deformations of the tiles in which the shades of color must be inserted. They would be inferior using a solid with a higher number of faces, but among the regular solids, this best suits the geometry of color. It also allows to dispose on the surface both the RGB primary of the pc screens and the primaries of the mixing of pigments and the achromatic colors, so the tints and shades of color, while the saturation can be controlled on the same surface through the transparency filters in almost all software of design and digital modelling.

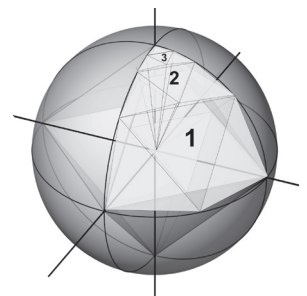
The graphical structure of the model allows to display, on the continuous surface of the sphere, the colors obtained by mixing two components (along the diameter of the stereographic projection of the edges of the tetrahedron) and the nuances of mixture of three colors in the tiles of the projection of the tessellation of the faces, the centre of which, as in the Plato's solid, you find the intermediate tint between the colors of the

(figure 17) The geodesic geometry teaches that the mode of tessellation of the faces allow to limit the deformations of the tiles in the next periodic projection on spherical triangles.

The colors have symmetrical layout with respect to the center of the sphere. As Fuller teaches, the deformations are minimum with repeated projection of triangles built on the midpoint of the faces, which are equilateral only at the center of the 4 main face, regardless of the increasing number of projections.

The different shape of the colored pieces can immediately see the primary (quadrilaterals) mixtures of pure colors (diamonds) and the shades of three colors (triangular facets).

In the center of the 6 main spherical triangles (equilateral triangle) there are the 6 different combinations of balanced triads of contiguous primary (inscribed cube).



(figure 18) Repetitive tessellation of the sphere with continuous geodesic projections.

three sides. The generative tessellation lead to the establishment from time to time the number of the tiles (thus of colors) depending on the difference that you want to have between contiguous colors. In this way the arrangement of colors on the surface of the sphere, which is symmetric about its centre, allows to "measure" equivalent color-combinations by the rotation of a regular figure into the sphere.

The Descriptive Geometry confirms itself as a measuring and representation tool.

Notes

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- [2] Vitruvius, *De architectura libri X*, Book VII, Coatings.
- [3] N. Silvestrini, 1988; A. Marotta, 1999; Rolf G. Kuehni, 2005; Sarah Lowengard, 2006. A. Marotta offers a wide historical analysis on colour theories and with Falzone's work introduced a new approach in Italian studies about colour in Architecture.
- [4] R. Scheper, 2005.
- [5] The team started its research on colour in design from the geometric

model, crossing the reference of Silvestrini's and Marotta's studies. On the research collaborate: M. Bisson, R. de Paolis, G. Amoroso, G. Mele, G. Buratti, C. Boeri, V. Vezzani, D. Sigona, E. Alberti, D. Bontempi.

- [6] R. de Paolis, 2011
- [7] Giampiero Mele, 2011.
- [8] J. Albers, *'Interaction of colours'*, Yale, 1973.

[9] M. Rossi, Geometry, shape and colour in design. Research notes from historic colour theory, in *Colour and Light in Architecture*, edited by P. Zennaro, Nemesi, Venezia, 2010.

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