Dr. Brook Taylor's Principles of Linear Perspective, Or, The ...

Brook Taylor

DR. BROOK TAYLOR'S

PRINCIPLES OF

LINEAR PERSPECTIVE.

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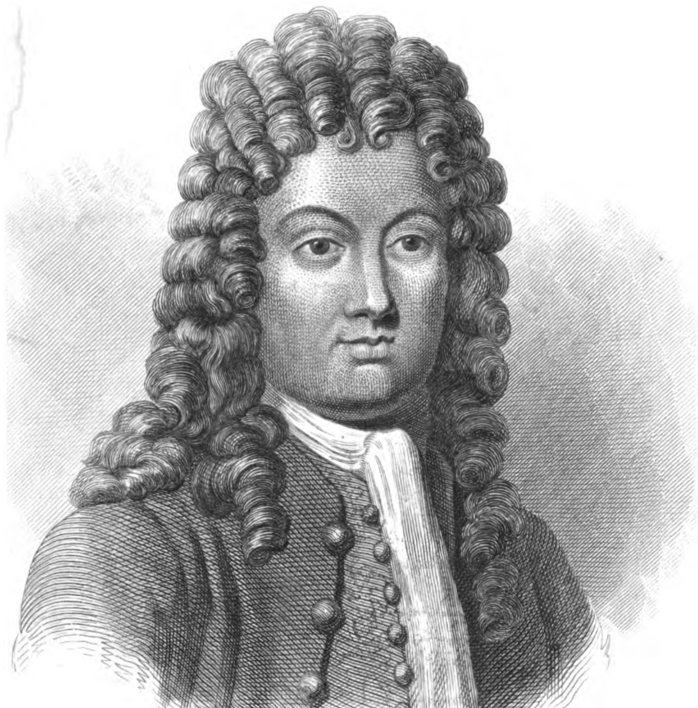

Brook Taylor LL.D. & R.S.S.

London, M Taylor, 6 Barnard's Inn, Holborn, 1835.

Dr BROOK TAY[illegible]

PRINC[illegible]

LINEAR PERSP[illegible]

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THE ART OF DESIGNING [illegible]

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[illegible]RESENTATION OF ALL SORTS OF [illegible]

AS THEY APP[illegible] TO THE EYE.

A NEW EDITION

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JOSEPH JOP[illegible]

[illegible]

LONDON

[illegible]

[illegible] SUCCESSOR T[illegible]

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M.DCC[illegible]

DR. BROOK TAYLOR'S

PRINCIPLES OF

LINEAR PERSPECTIVE;

OR,

THE ART OF DESIGNING UPON A PLANE

THE

REPRESENTATION OF ALL SORTS OF OBJECTS,

AS THEY APPEAR TO THE EYE.

A NEW EDITION:

WITH ADDITIONS, INTENDED TO FACILITATE THE STUDY OF THIS MUCH ESTEEMED WORK,

BY

JOSEPH JOPLING, ARCHITECT,

AUTHOR OF THE "PRACTICE OF ISOMETRICAL PERSPECTIVE," &c. &c.

LONDON:

PUBLISHED BY M. TAYLOR

(NEPHEW AND SUCCESSOR TO THE LATE JOSIAH TAYLOR),

6 BARNARD'S INN, HOLBORN.

M.DCCC.XXXV.

MEMOIRS

OF THE

LIFE OF THE AUTHOR.

Brook Taylor was born at Edmonton, August 18th, 1685. He was the son of John Taylor, Esq. of Bifron's House, in Kent; by Olivia, daughter of Sir Nicholas Tempest, of Durham, Baronet.

The father of the subject of this memoir is represented as one, who with the principles which distinguished the Puritans of the preceding generation had retained a full measure of the austere temper and ungracious manners which too often characterised that sect. His numerous children, and all who depended on him, were habituated to the strictest forms of reverence and submission. Domestic endearments unbended to no relaxation from ceremony throughout the day. The patriarch was always upon his throne. The father sought not a display of his affections, but of his

authority. From these circumstances it is no subject of surprise, that in after life little harmony should have subsisted between the father and the son.

The fine arts were, however, no strangers to this family; music was cultivated with assiduity and success, and particularly by Brook Taylor. He was also accomplished in the arts of design; and, according to the relation of his descendant, the drawings and paintings executed by him which have been preserved, require not those allowances for errors and imperfections which are given to the performances of dilettanti; but will bear the test of scrutiny and criticism from artists of the greatest abilities.

Brook Taylor was educated at home, under the tuition of the Rev. Mr. Sacchette, and being deemed qualified for the university at the early age of fifteen, he was entered a fellow-commoner at St. John's College, Cambridge, in 1701. At that period mathematics engaged more particularly the attention of the University, and roused the emulation of every youth possessed of talents and application. We may presume that Brook Taylor immediately followed the course of study, which a Machin, a Keil, and above all a Newton, had opened to the mind of man, as we find him ho-

noured with early notice and kind attention from these eminent persons.

In 1708 he wrote his treatise "On the Centre of Oscillation," as appears in a letter to professor Keil, though that essay did not appear in the Philosophical Transactions until some years afterwards. In the year 1709, he took his degree of Bachelor of Laws. In 1712 he was chosen a Fellow of the Royal Society. Having, during this period, exercised his skill and acquirements in private correspondence and discussion, he now ventured forth as a competitor for scientific fame in agitation of those points of knowledge which engaged most of the genius and learning then flourishing in Europe. June 25, 1712, he presented his first paper to the Royal Society, "On the Ascent of Water between two Glass Planes;" his second production, "On the Centre of Oscillation," was read in July; his third, "On the Motion of a stretched String," was read in September of the same year. By a letter to Kiel, dated July 13, 1713, it appears that Brook Taylor had presented to the Royal Society a paper of that date, on his favourite subject of music, which, however, is not preserved in their Transactions.

January 13, 1714, he was elected to the office of secretary to the Royal Society, and in the

same year he took his degree at Cambridge of Doctor of Laws. About this time he transmitted, by a letter to Sir Hans Sloane, "An Account of some curious experiments relative to Magnetism." Early in the year 1715, he published in Latin his *Methodus Incrementorum;* in May he presented a very curious essay entitled, "An Account of an Experiment for the Discovery of the Laws of Magnetic Attraction," and in the same year he published his learned and able treatise, "On the Principles of Linear Perspective." He was further engaged in a controversial correspondence with the Count Raymond de Montmort on the subject of the tenets of Malbranche, which occasioned him to be particularly noticed in the eulogium of the French Academy, pronounced on the decease of that most fanciful philosopher.

The correspondence which Brook Taylor had for some time carried on with learned foreigners had given him a general reputation abroad. The mathematicians of Paris for the most part espoused the cause of Newton, and sought a nearer acquaintance with its able supporter, the accomplished Secretary of the Royal Society. In the year 1716 he accepted the pressing invitation of the Count Raymond de Montmort, the Abbé Conti, and others, to visit them in Paris.

Brook Taylor joined with his scientific knowledge, the attainments of a general scholar, an elegant person, and the accomplishments of a finished gentleman. Thus endued, he could not fail to meet a distinguished reception in the gay and enlightened metropolis of France. Among the friends whom he acquired at that time may be named, in contra-distinction to mathematicians, Lord Bolingbroke, the Count de Caylus, and Bossuet; and of the ladies who honoured him with a particular regard, Marcilly de Villette and Miss Brunton, the beautiful and accomplished niece of Isaac Newton.

In February 1717, Brook Taylor returned to London, and in that year he composed, and presented to the Royal Society, three treatises. The first entitled, " An Attempt towards an Improvement of the Method of Approximating in the Extractions of Roots of Equations in Numbers." The second, " A Solution of Demoivre's Fifteenth Problem, with the Assistance of Combinations and Infinite Series." The third, " A Solution of the Problem of G. G. Leibnitz, proposed to the English."

Intense application to study having impaired his health, he was enjoined by his physicians to repair to Aix la Chapelle, which was probably the reason which induced him to resign his laborious

office of secretary to the Royal Society. Returning from Aix la Chapelle early in the year 1719, he applied himself to recomposing in a new and improved form his "Treatise on Linear Perspective." It was in consequence of this work that a dispute arose between the author and John Bernouilli, which terminated in a most inveterate quarrel between those learned men. In a treatise published in the "Acts of Leipsic," Bernouilli had taken occasion to speak of this work of Brook Taylor, as abstruse to all, and unintelligible to artists for whom it was more especially written. In reply he published, "An Apology against J. Bernouilli's Objections," which, with some letters preserved by Sir W. Young, shew the nature of this dispute, and the asperity with which it was conducted.

Towards the end of the year 1720, Brook Taylor accepted an invitation from Lord Bolingbroke, to pass some time at his country seat of La Source, near Orleans. He returned to England in 1721, and in that year published the last paper appearing with his name in the Philosophical Transactions; it is entitled, "An Experiment made to ascertain the Proportion of Expansion of Liquor in the Thermometer, with regard to the Degree of Heat."

In the year 1721, he married Miss Brydges, of Wallington, in Surrey, a young lady of good family, but of small fortune, which latter circumstance occasioned a rupture with his father, whose consent had never been obtained. Early in the year 1723, his wife died in child-bed, together with her infant son. This melancholy event softened the heart of his father, who, after a full reconciliation, received Brook Taylor at his house of Bifrons. At this place he continued during the two succeeding years, where the musical parties so agreeable to his taste and early proficiency, and the affectionate attention of a numerous family welcoming an amiable brother so long estranged by parental resentments, not only soothed his sorrows, but ultimately engaged him to a scene of country retirement, and domesticated and fixed his habits of life.

Having formed an attachment to an agreeable young lady, in the year 1725, with the full approbation of his father and family, he married his second wife Sabetta, daughter of John Sawbridge, Esq. of Olantigh, in Kent. In 1729, John Taylor died, and his son Brook succeeded to the family estate of Bifrons. But he was soon doomed to experience a second wreck of his domestic happiness, for in the following year he lost his wife

Sabetta, who died in child-bed. The infant daughter lived, and was married to the father of Sir William Young, from whose life of his maternal grandfather this memoir is abridged.

From the period of his first marriage, Brook Taylor appears to have suffered his attention to be engaged with the ordinary duties and employments of domestic life, to the exclusion of his scientific pursuits, with the sole exception of the composition of a "Treatise of Logarithms," placed in the hands of his friend Lord Paisley (afterwards Abercorn), to prepare for the press, but which seems never to have been printed.

After the loss of his second wife, the term of his life was short, and his days appear to have been passed in sorrow. An essay entitled, "*Contemplatio Philosophica*," was written about this time, and probably with a view to abstract his mind from painful recollections and regrets. His friends offered every comfort and consolation; but having survived his second wife little more than a year, Brook Taylor died of a decline, on December 29, 1731, in the forty-sixth year of his age.

THE

AUTHOR'S PREFACE.

CONSIDERING how few and how simple the principles are upon which the whole art of PERSPECTIVE depends, and withal how useful, nay, how absolutely necessary this art is to all sorts of designing; I have often wondered, that it has still been left in so low a degree of perfection, as it is found to be, in the books that have been hitherto wrote upon it. Some of these books indeed are very voluminous: but then they are made so, only by long and tedious discourses, explaining of common things; or, by a great number of examples, which indeed do make some of these books valuable, by the great variety of curious cuts that are in them; but do not at all instruct the reader, by any improvements made in the art itself. For it seems that those who have hitherto treated of this subject, have been more conversant in the practice of designing, than in the principles of geometry; and therefore when, in their practice, the occasions that have offered have put them upon inventing particular expedients, they have thought

them to be worth communicating to the public as improvements in this art; but they have not been able to produce any real improvements in it, for want of a sufficient fund of geometry, that might have enabled them to render the principles of it more universal, and more convenient for practice. In this book I have endeavoured to do this; and have done my utmost to render the principles of the art as general, and as universal as may be, and to devise such constructions as might be the most simple and useful in practice.

In order to this, I found it absolutely necessary to consider this subject entirely anew, as if it had never been treated of before; the principles of the old Perspective being so narrow and so confined, that they could be of no use in my design: and I was forced to invent new terms of art, those already in use being so peculiarly adapted to the imperfect notions that have hitherto been had of this art, that I could make no use of them in explaining those general principles I intended to establish. The term of horizontal line, for instance, is apt to confine the notions of a learner to the plane of the horizon, and to make him imagine, that that plane enjoys some particular privileges, which make the figures in it more easy and more convenient to be described, by the means

of that horizontal line, than the figures in any other plane; as if all other planes might not as conveniently be handled, by finding other lines of the same nature belonging to them. But in this book I make no difference between the plane of the horizon, and any other plane whatsoever; for since planes, as planes, are alike in geometry, it is most proper to consider them as so, and to explain their properties in general, leaving the artist himself to apply them in particular cases, as occasion requires.

My design in this book is not to trouble the reader with a multitude of examples, nor to descend to a perfect explanation of any particular cases; but to explain the general principles of Perspective: which, if I have been so happy as to have done, in such a manner as may be intelligible to the reader, I do not doubt but he will easily be inclined to pardon my shortness: for he will find much pleasure in observing how extensive these principles are, by applying them to particular cases which he himself shall devise, while he exercises himself in this art, than he would do in reading the tedious explanations of examples devised by another.

I find that many people object to the first edition that I gave of these principles, in the little

book entitled, "Linear Perspective," &c. because they see no examples in it, no curious descriptions of figures which other books of Perspective are commonly so full of; and seeing nothing in it but simple geometrical schemes, they apprehend it to be dry and unentertaining, and so are loath to give themselves the trouble to read it. To satisfy these nice persons in some measure, I have made the schemes in this book something more ornamental, that they may have some visible proofs of the vast advantages these principles have over the common rules of Perspective, by seeing what simple constructions, and how few lines, are necessary to describe several subjects, which in the common method would require an infinite labour, and a vast confusion of lines. It would have been easy to have multiplied examples, and to have enlarged upon several things that I have only given hints of, which may easily be pursued by those who have made themselves masters of these principles. Perhaps some people would have been better pleased with my book if I had done this: but I must take the freedom to tell them, that though it might have amused their fancy something more by this means, it would not have been more instructive to them; *for the true and best way of learning any art, is not to see a great many examples done*

by another person, but to possess one's self first of the principles of it, and then to make them familiar by exercising one's self in the practice. For it is practice alone that makes a man perfect in any thing.

The reader who understands nothing of the elements of geometry can hardly hope to be much the better for this book, if he reads it without the assistance of a master; but I have endeavoured to make every thing so plain, that a very little skill in geometry may be sufficient to enable one to read this book by himself. And upon this occasion, I would advise all my readers who desire to make themselves masters of this subject not to be contented with the schemes they find here, but upon every occasion to draw new ones of their own, in all the variety of circumstances they can think of. This will take up a little more time at first; but in a little while they will find the vast benefit of it, by the extensive notions it will give them of the nature of these principles.

The art of Perspective is necessary to all arts, where there is any occasion for designing; as Architecture, Fortification, Carving, and, generally, all the mechanical arts; but it is more particularly necessary to the art of Painting, which can do nothing without it. A figure in a picture, which

is not drawn according to the rules of Perspective, does not represent what is intended, but something else. So that it seems to me, that a picture which is faulty in this particular is as blameable, or more so, than any composition in writing which is faulty in point of orthography or grammar. It is generally thought very ridiculous to pretend to write an heroic poem, or a fine discourse upon any subject, without understanding the propriety of the language wrote in; and to me it seems no less ridiculous for one to pretend to make a good picture without understanding Perspective: yet how many pictures are there to be seen, that are highly valuable in other respects, and yet are entirely faulty in this point! Indeed this fault is so very general, that I cannot remember that I ever have seen a picture that has been entirely without it; and, what is the more to be lamented, the greatest masters have been the most guilty of it. Those examples made it to be the less regarded; but the fault is not the less, but the more to be lamented, and deserves the more care in preventing it for the future. The great occasion of this fault is, certainly, the wrong method that generally is used in the education of persons to this art. For the young people are generally put immediately to drawing; and, when they have acquired a facility

in that, they are put to colouring. And these things they learn by rote and by practice only, but are not at all instructed in any rules of art; by which means when they come to make any designs of their own, though they are very expert at drawing out, and colouring every thing that offers itself to their fancy, yet, for want of being instructed in the strict rules of art, they do not know how to govern their inventions with judgment, and become guilty of so many gross mistakes, which prevent themselves as well as others from finding that satisfaction they otherwise would do in their performances. To correct this for the future, I would recommend it to the masters of the art of Painting, to consider if it would not be necessary to establish a better method for the education of their scholars, and to begin their instructions with the technical parts of Painting, before they let them loose to follow the inventions of their own uncultivated imaginations.

The art of Painting, taken in its full extent, consists of two parts; the inventive, and the executive. The inventive part is common with poetry, and belongs more properly and immediately to the original design (which it invents and disposes in the most proper and agreeable manner) than to the picture, which is only a copy of that design

already formed in the imagination of the artist. The perfection of this art of Painting depends upon the thorough knowledge the artist has of all the parts of his subject; and the beauty of it consists in the happy choice and disposition that he makes of it; and it is in this that the genius of the artist discovers and shews itself, while he indulges and humours his fancy, which here is not confined. But the other, the executive part of Painting is wholly confined, and strictly tied to the rules of art, which cannot be dispensed with upon any account; and, therefore, in this the artist ought to govern himself entirely by the rules of art—not to take any liberties whatsoever. For any thing that is not truly drawn according to the rules of Perspective, or not truly coloured, or truly shaded, does not appear to be what the artist intended, but something else. Wherefore, if at any time the artist happens to imagine that his picture would look the better if he should swerve a little from these rules, he may assure himself that the fault belongs to his original design, and not to the strictness of the rules; for what is perfectly agreeable and just in the original objects themselves, can never appear defective in a picture where those objects are exactly copied.

Therefore, to offer a short hint of the thoughts

I have sometime had upon the method which ought to be followed in instructing a scholar in the executive part of Painting, I would first have him learn the most common effections of practical geometry, and the first elements of plain geometry, and common arithmetic. When he is sufficiently perfect in these, I would have him learn Perspective. And when he has made some progress in this, so as to have prepared his judgment with the right notions of the alterations that figures must undergo when they come to be drawn on a flat, he may then be put to drawing by view, and be exercised in this along with Perspective, till he comes to be sufficiently perfect in both. Nothing ought to be more familiar to a painter than Perspective; for it is the only thing that can make the judgment correct, and will help the fancy to invent with ten times the ease that it could do without it. For the colouring: before the young artist is employed in copying of pictures, where there are great variety of colours to be imitated, it would be well that he should be instructed in the theory of the colours; that he should learn to know their particular properties, their different relations, and the various effects that are produced by their mixture; and that he should be made well acquainted with the nature

of the several material colours that are used in Painting. These things ought to be learned in a regular method; and the artist ought not to depend entirely upon the several indigested observations that may occur to him in practice. And to this last purpose of colouring, I cannot help thinking that the theory I have endeavoured to explain in the Appendix, from Sir Isaac Newton, may be of very great use to learners. There may be regular methods, also, invented for teaching the doctrine of light and shadow;* and other particulars relating to the practical part of Painting may be improved and digested into proper methods for instructing the young artist. But I only hint at these things, recommending them to the masters to reflect and improve them.

The book itself is so short that I need not detain the reader any longer in the Preface, by giving him a more particular account of what he may expect to find in it.

* See a work entitled, "Sciography; or, Examples of Shadows, with Rules for their Projection, &c." by Joseph Gwilt, F. S. A.

EDITOR'S PREFACE.

Dr. Brook Taylor's " Principles of Perspective" are admitted to have been the foundation, for, now, more than one hundred years, of every subsequent treatise of note on that subject in this country. T. Malton, indeed, in the preface to his large work, says, " I do not pretend to have found out new principles, nor do I think there can be need of any other; those given by Brook Taylor being sufficient for every purpose whatever: and the principles on which he has founded his system are the most simple and perfect that can possibly be conceived."

These testimonials alone would fully justify, as the work is now out of print, a new Edition of Brook Taylor, without *any* alterations or additions.

But, as it must be admitted that learners have found much difficulty in comprehending his brief explanations, or getting so impressed on their minds those important principles as to be able to apply them with facility to any case which may occur in practice, and, as will be seen in the Memoirs of the Author, the work has been characterised " as abstruse to all, and unintelligible to artists, for whom it was more especially written,"

it appeared desirable, if possible, to render more obvious the principles of the Author. The Editor, therefore, has had to consider in what way he could add to render it easier to learners, at the same time to leave it substantially and manifestly, "Brook Taylor's Linear Perspective."

In order to do this, his attention has been directed to the Definitions, and to the explanations of the use of terms, including those by which Perspective is distinguished from other kinds of projection, but with which it is most intimately connected, and to the diagrams explanatory of these. In the last Edition, about nine pages only, the greater part in large print, was the extent of the definitions; in the present Edition, these occupy thirty-three pages, including seventeen new illustrative diagrams. In the Theorems, several notes have been added, and the Problems are prefaced by an enumeration of the various circumstances required to be given or determined, before any sort of projection can be made.

Fig. 24, which illustrates the first Problem, has been separated from a figure in the former Edition, which also illustrated other problems, and in further illustration of the principles of this figure, figs. 25 and 26 have been added. To the first Problem, have also been added figs. 27, 28, 29, 30, 31, and 32; all of which evidently arise out of

the principles of Brook Taylor, although none of them were separately explained by him.

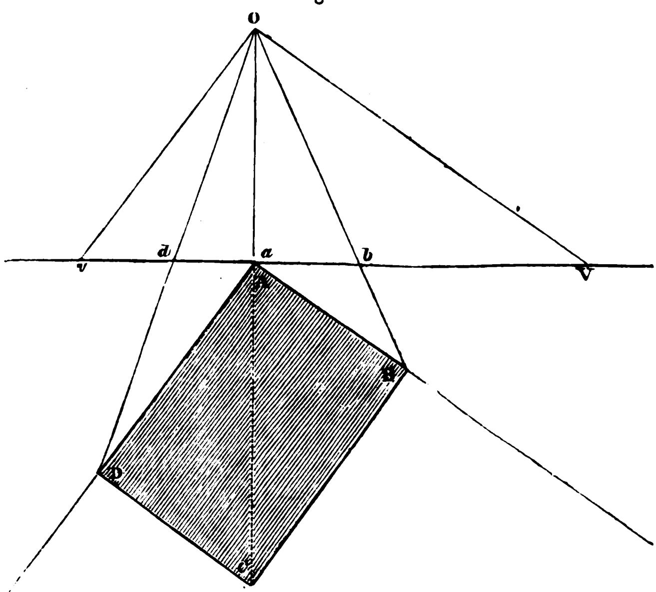
In a similar way in other Problems, although not to a great extent, diagrams have been separated, to render them more distinct, and notes and diagrams have been added, to make them better understood, and more easily recollected.

In order to make students familiar with, and to impress on their minds the great importance of trying, extreme cases, in order to have a comprehensive idea of Projection, the following figures are introduced here, which shew the method of projecting, and the appearance of a finite and infinite cube, when three sides are seen at the same angle, the adjacent corner of the cube being the centre of the picture, and the point of sight at only one inch and a quarter distant from it.

Let ABCD, fig. 59, be the diagonal plane of a cube whose sides are one inch square, with the diagonal AC perpendicular to the plane of projection, the section of which is shewn by the line *v*V. Then let O be the point of sight at $1\frac{1}{4}$ inch from *a*, the centre of the plane of projection, and also from the angle A of the cube in the line of the diagonal CA produced; then O*v* and OV being drawn parallel to AD and AB, the points *v*V are the vanishing points of those lines.

The projection of the lines AB or AD, infinitely produced, would be equal to *a V* and *a v*; *a* V, *a* V, and *a* V, in fig. 60, would therefore be the

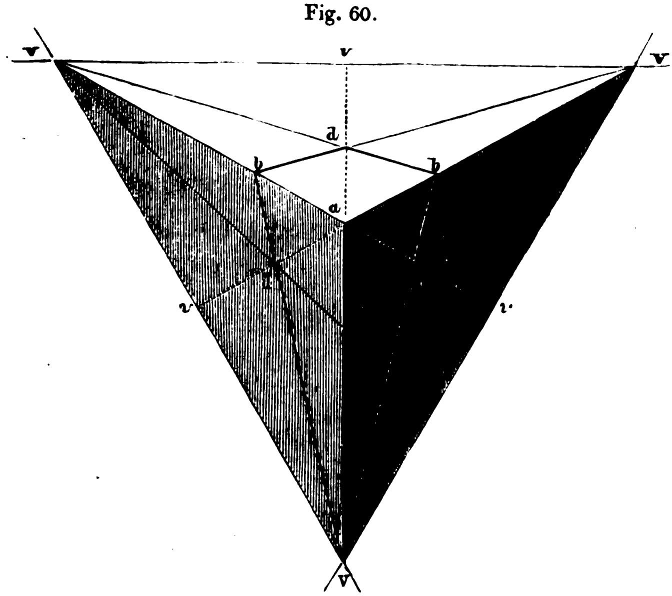
Fig. 59.



projection of the three lines forming the solid angle of an infinite cube, viewed at 1¼ inch from the point *a*. The projection of the finite line AB, fig. 56, being *a b*, fig. 60; the projection of the line AD, fig. 59, infinitely produced, would be *a v*, fig. 60; the projection of the diagonal of a face of an infinite cube, viewed at 1¼ inch from its adjacent angle *a*, while the projection of AD, the diagonal of the face of a cube, whose sides are one inch square, would be *a d*.

The vanishing lines V *v* V may be continued each way from the centre *v*, infinitely; on these

Fig. 60.



lines, as drawn, and thus produced, would be the vanishing points of every line that could be drawn on each face of the cube as seen. The points V being each in common, the vanishing points of all lines on any two faces having a common angle; and the points *v* being the vanishing lines of all lines parallel to any diagonal *a d;* and any lines parallel to a diagonal *b b* would be parallel, because the vanishing point is at an infinite distance. Here, then, in fig. 60, are the representations of a cube, each face of which is $1\frac{1}{4}$ square, and also of

a cube, each face of which is infinite; the adjacent angle of both touching the plane of projection, and the point of sight being at the distance of $1\frac{1}{4}$ inch from it. When fig. 60 is viewed from a point at $1\frac{1}{4}$ inch from *a*, the finite cube will appear to be the exact representation of a real cube of the dimensions given, viewed at the same angle and at the same distance; and were it possible to tint the projection of the representation of the infinite cube exactly as such an object would appear, the appearance of this small figure viewed from that point would be infinite. As it is tinted in the wood-cut, if viewed steadily from the proper point of sight, this figure becomes greatly magnified.

A solid angle, less than a cube, thus viewed, would have the vanishing lines nearer; if greater than a cube, they would be more distant.

Except this cube be viewed from the true point of sight, it appears much more distorted than the smaller cubes represented in figs. 5 and 8, which are in direct or isometrical projection; thus the student is presented with two extreme cases, between which are all the appearances of any cube when each face is seen equally and at a greater distance than one and a quarter, of the length of one of its boundary lines.

LINEAR PERSPECTIVE.

DEFINITION I.

LINEAR PERSPECTIVE is the art of describing exactly, on paper, canvass, or any surface, the representations of any given objects as they would appear from any given point.

DEFINITION II.

A Plane is either a real or imaginary surface, on which, if right lines are placed in every possible direction, they would perfectly coincide with it in every part, as far as they extend. A plane has no thickness. The section of a plane is represented by a right line. The boundary of a plane may be a figure of any form; and every plane may be supposed to be produced or extended in any or every direction, as conveniency requires. Each face of the cubes, for example, represented in figs. 4, 8, and 12, are planes.

N. B.—As it is almost impossible so to arrange a work, that each figure shall be immediately connected with the letter-press, in all places where it may be referred to, persons who are desirous of learning perspective are recommended to make copies of each diagram; and if they are all arranged on one sheet, they will find it greatly to facilitate the acquisition of a perfect knowledge of the art.

DEFINITION III.

Plane of Projection is the face of a plane picture, or any plane surface, on which the representation of any object is to be described. It is to drawing the representation of objects on such a surface that this work principally refers. And with respect to any point, line, or a plane of any object to be represented, the plane of projection may either be parallel or inclined to it at any angle, and either touch it or be at any distance from it.

DEFINITION IV.

A Ray is any right line which is drawn, or imagined to be drawn or proceed from any point of an original object to the plane of projection; or from the point of sight, to or through its corresponding point on the plane or surface of projection, to any original point.

A *a*, fig. 4, is a *direct* ray proceeding from the point A of the cube to the point *a* on the plane of projection.

E *e*, fig. 8, is an *oblique* ray proceeding from the point E of the cube to the point *e* on the plane of projection.

O A, fig. 12, is a *visual* or *diverging* ray proceeding from the eye at O, through the point *a* on the plane of projection, to the corner A of the original cube.

DEFINITION V.

A System of Rays is all the lines taken together which produce the projection of any object, as the whole of the rays in figs. 4, 8, and 12.

DEFINITION VI.

A Plane of Rays is all the lines taken together which produce the projection, or may be supposed to proceed from every point of an original right line to every point in its representation; or from the eye, through every point of the representation of a right line, to the original line. The lines forming a plane of rays may be either parallel or diverging.

The rays A a and B b in fig. 4, and A a and E e in fig. 8,	and all intermediate rays forming the planes A a b B and A a e E, are parallel.
The rays O D and O E, in fig. 12,	and all intermediate rays forming the visual plane O D E, are diverging.

DEFINITION VII.

A Surface of Rays is all the lines taken together which produce the projection of any line, whether a right or curved line. Rays proceeding from the eye, or point of sight, to a circular line, form a conical surface of rays. A surface of rays is thus obviously determined by the form of the line to or from which the rays proceed.

DEFINITION VIII.

A Body of Rays may be imagined to fill up the whole of the space between any original objects and their projections, or between the original objects and the point of sight.

DEFINITION IX.

Direct Radial Projection of any object is made when the system of rays which produce the representations are all parallel to each other and perpendicular to the plane of projection. In fig. 1, A B C D is the plan of the base of a square pyramid, with the boundary lines of the sides, from the apex, directly projected thereon; and one side A B, and the vertex E, is again, by direct rays, projected to *a*, *e*, and *b*, on another plane of projection which is parallel to A B. The line *a e b* is the section of the other plane of projection, which plane is to be supposed to be perpendicular to the plane on which the plan is drawn. In fig. 2 is shewn the direct projection from four points on the plan of the base, and one from the seat of the vertex, neither side of the pyramid being parallel to the plane of projection. In fig. 3 is shewn the direct projection of the height of the elevation of the pyramid, E being the apex, and A B *e* C the section of the plane of the base. The seat of a point or line is always determined by direct projection. The Roman capital letters mark the original points, and the small italics of the same letters, the corresponding projections.

Fig. 1.

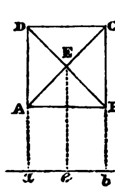


Fig. 2.

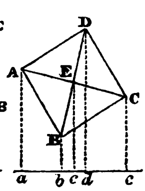


Fig. 3.

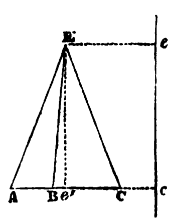
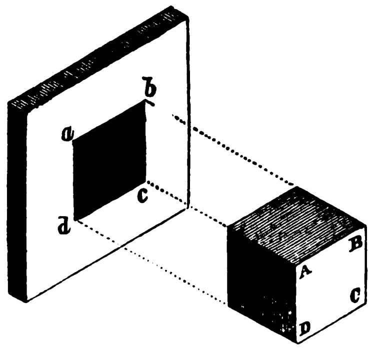


Fig. 4.



Isometrical Perspective, which for practical purposes is so useful, the pyramids just described, and the plain plans, elevations, and sections of a building, are of this projection: also, the shadows of any objects on any plane on which the sun or moon shines direct, as the rays of these (to all sense) are parallel to each other. The sun is supposed to shine directly on the face of the cube ABCD, fig. 4, and also directly on the plane *a b c d,* on which its shadow is by the direct rays projected: or, if *a b c d* be horizontal, it will be the plan of the face of the cube ABCD; or, if it be vertical, it will be the elevation of that cube. If the plane on which any direct projection is made be neither a vertical nor horizontal plane, it may, as to these or some other plane, either given or supposed, be an inclined plane.

DEFINITION X.

Oblique Radial Projection of any object is made when the plane of projection is only perpendicular in one direction to the parallel system of rays. In figs. 5 and 6 is shewn the oblique rays from the plan of a square pyramid in two

positions; and fig. 7 shews oblique rays from the elevation of the same. In fig. 8, the ray *e*E, and all the rays parallel to it, are in a plane perpendicular to the plane of projection in the direction *ea*, but oblique to a perpendicular plane in the direction *ed*; or, in other words, a plane passing through or lying on the rays *a*A, *b*B, and *e*E, would be perpendicular to *af*; but a plane on the rays *e*E and *d*D would be inclined to *ea*, therefore oblique.

Oblique projection is not recommended to be used for the representation of objects, and more especially a great degree of obliquity in the rays should be avoided.

Fig. 5.

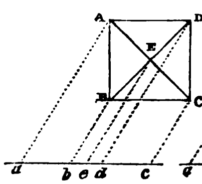


Fig. 6.

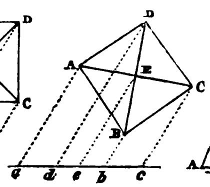


Fig. 7.

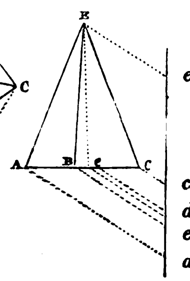
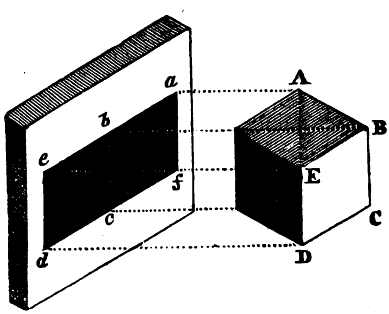


Fig. 8.



The shadows of any objects on any plane on which the sun or moon does not shine direct, are of this projec-

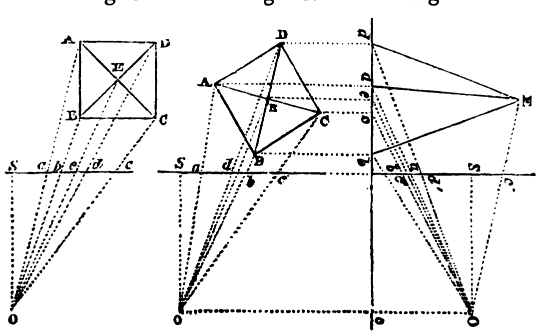
tion. But the shadow of an object is not the appearance of an object. Any cube, for example, would, if seen from an infinite distance, or from the sun or moon, appear the same, on whatever plane its shadow might be projected. That is, its appearance would be always as in the direct projection of it. Two objects very unequal in their proportions may have their shadows of the same dimensions. If one object be long and another short, but in other respects the same, by a greater obliquity in the rays, the projection of the shorter may be made as long or in any degree longer than the other—therefore to represent objects by oblique projection, is to give them a false appearance. Oblique sections of any object, as the sections of any moulding or cornice, made at any other angle than a right angle, are the same as this projection. In fig. 8 the sun is supposed to shine on each of the two faces of the cube which are under the lines A B and B E, at an angle of 45°, and its shadow is projected on the plane *aedf*, with which the rays, in one direction, also make an angle of 45°; the length A E being increased to *ae*, while the length B C is the same as *bc*. Thus, the dimensions of the shadows of any objects made by the rays of the sun shining obliquely on a plane or in the oblique projection of it, are always the same as the dimensions of the original object in one direction; and the greater the obliquity of the rays, the more the length of the shadow or projection exceeds the dimensions of the object in the other direction.

DEFINITION XI.

Visual Radial Projection of an object is made when the system of rays which proceed from the several parts

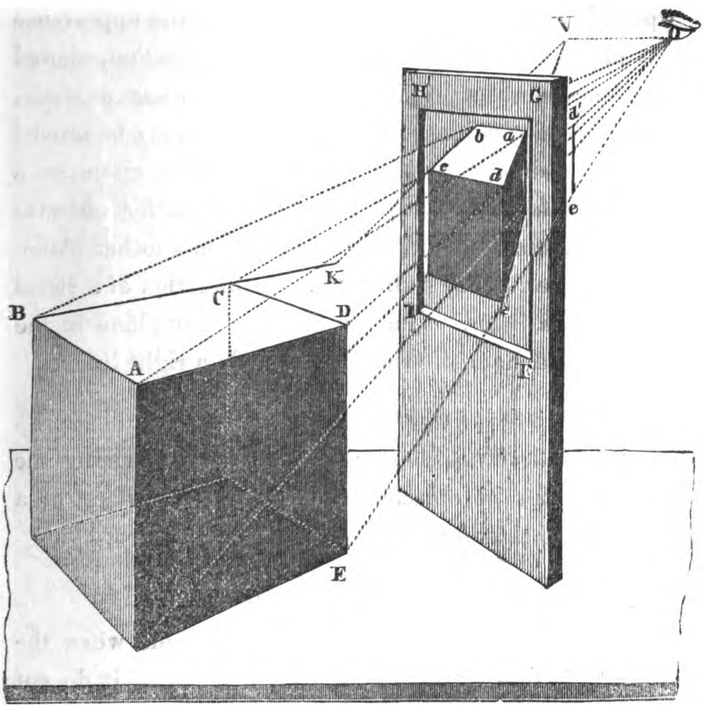
of it, and produce the representation of it on the plane of projection, all tend to the same point; or, when the system of rays proceeding from a point to, or through, and past the several parts of any object, fall on the plane of projection. In figs. 9 and 10 is shewn the plan of a square pyramid, and the seats of the visual rays in two positions on planes perpendicular to the plane of projection. Fig. 11 shews the elevation of the pyramid, and also the visual rays projected by direct rays from fig. 10, on a plane perpendicular to the plane of projection. Whether on the plan or elevation of the visual rays, those only are represented of the actual length, between the point of sight *o* and any point on the original object, or on the picture, that are parallel to the plane on which they are projected. In this case, the only rays thus parallel are those from the point of sight *o*, to S the centre of the picture in each figure.

Fig. 9. *Fig.* 10. *Fig.* 11.



In fig. 12, the diverging or visual rays, which proceed from the eye of the spectator at *o*, to the angle of the cube A B C D E, pass through the plane of projection F G H I, at the angles *a b c d e* of the picture representing the cube.

Fig. 12.



The representations of any other object, as it appears to the eye fixed at any given point, on any plane of projection, is of this projection; also the shadows of figures falling on a plane, when the light is a candle or lamp, and considered only as a point, and the reflections of objects in a plane mirror, or from one plane mirror to another. When the representations of the shadows of objects proceeding from a candle falling upon a plane are to be made as they appear from any other point, this projection, as far as the shadows are concerned, must be repeated. The shadows of a plane circle on a plane of

projection may, in this way, be either a circle, ellipse, hyperbola, parabola, or a right line: but the appearance of a plane circle, beyond the plane of projection, viewed from any point of sight, can only be either a circle, ellipse, or a right line. These statements may be proved by ocular demonstration, by placing a plane circle on a plane, which may be adjusted to any angle for observation, or for the projection of its shadow on another plane. The projection of any right line in the direction of a visual ray is only a point. The projection of any plane in the direction of a plane of visual rays is only a right line.

DEFINITION XII.

Panoramic Projection is the method of forming the representations of objects on the concave surface of a cylinder, the rays of projection proceeding from a point.

DEFINITION XIII.

Irregular Projection of an object is made when the rays which proceed from the several parts of it do not all fall upon the same plane, but upon an *irregular surface* on which the representation is projected. The representation of an object on an irregular surface may, of course, be produced by rays either parallel to each other or diverging.

Having thus defined several kinds of projection, that of "*Visual Radial Projection*," which is the generally understood, but confined meaning attached to the term "*Linear Perspective*," which it is the chief object of this work to explain, will, it is hoped, be better understood. Indeed, it is absolutely necessary that both *direct, and oblique radial projection,* should be known, for they are

of essential and constant use, and by them visual radial projection is frequently very greatly facilitated; and, indeed, without one or the other, any sort of projection, or even practical geometry, cannot be performed.

DEFINITION XIV.

The Art of Painting consists of three parts, viz. Drawing, Light and Shadow, and Colouring; and the principles of all these parts, especially Drawing, or Linear Perspective, are to be obtained from the consideration that the rays of light ought to come from the several parts of the picture to the spectator's eye with all the same circumstances of direction, strength of light and shadow, and colour, as they would do from the corresponding parts of the real objects seen in their proper places. A picture actually so drawn and placed in the right position, and in a proper light between the eye and the real objects, could not be distinguished from them.

DEFINITION XV.

Point of Sight is that point before a picture, or the real objects represented, where the spectator's eye ought to be placed, in order that the several parts of each may have exactly the same appearance. From the point of sight, rays in right lines proceed to every point in the picture, and continued, if the picture be before the figure in the same right lines, to the corresponding points of the real objects represented. If the point of sight be supposed to be the centre of a sphere or globe, with visual rays extending in every direction, the plane of projection and any other plane will be a tangent plane, or a plane parallel to a plane of tangents, to some points of the sphere. The

point of sight is marked by the letter *o* in an eye, in figs. 13, 14, and 15, which are on Plates I. and II.

When the representation of an object is not viewed from its proper point of sight, it appears more or less distorted, according as the point of sight and the vanishing points are at a less or greater distance. The points of sight and the vanishing points for the representations of the prism in figs. 16, 17, and 18, are at a very little

Fig. 16.

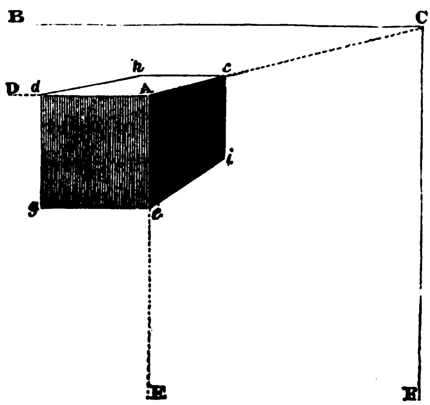


Fig. 17.

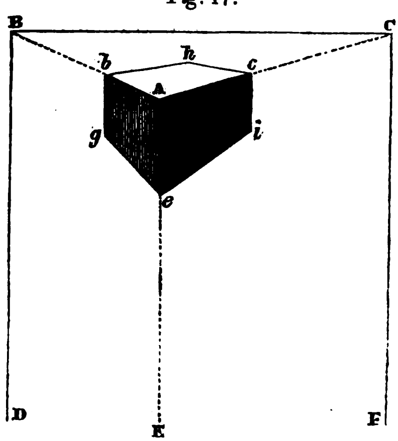
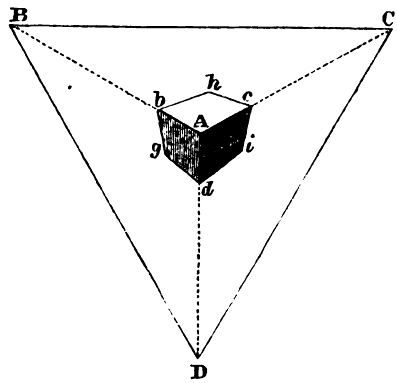


Fig. 18.



distance, in order that these figs. may occupy but little space in the work, and consequently they will appear, when viewed at the ordinary distance of holding the book to read its contents, to be very much distorted. However, if viewed from the proper point of sight, in whatever direction they may be before the observer, they will appear correct representations of prisms. This will be the more obvious if the same figures are drawn to a much larger scale, so that the point of sight may be proportionably farther removed from the plane of projection. It may be worthy of remark here, that, for example, the prism, fig. 16, appears more distorted when viewed out of its proper point of sight if the boundary line from A to C, tending to the finite vanishing point, be towards the observer, than when either of the other boundary lines A E, or A D, from the angle A, is directed to the person. The former position, which is the same as representations of buildings known by the name of American Perspective, has received the appellation of "*Perspective run mad.*" But the latter, although seen at as great a distance from

the true point of sight, is seldom considered to give an inaccurate representation.

DEFINITION XVI.

Centre of a Picture, is that point on the plane of projection from which, by a direct or perpendicular ray is the shortest distance to the point of sight. Or it may be defined as the tangent-point where the plane of projection would touch a sphere whose radius is equal to the distance between the plane of projection and the point of sight. A line drawn from the centre of a sphere to the centre of a plane of tangents, or from the point of sight to the centre of the picture, is perpendicular to the plane of projection. The letter S is the centre of the picture in figs. 13 and 14.

DEFINITION XVII.

Original Point, line, figure, surface, or solid, is the real object placed in the situation where it is intended to be represented to be in the picture.

In figs. 13 and 14, G and F are original points,
G F is an original line,
G F H is an original surface or plane.

In fig. 12, the cube A B C D E, is an original solid, containing points, lines, and planes.

When a plane of any original plane-figure is not perpendicular to the plane of projection, the angle which it makes therewith is to be set off on a plane which is perpendicular both to the original plane and also to the plane of projection.

DEFINITION XVIII.

Distance of a Picture, is the length of the direct visual

ray or line between the point of sight and the centre of the picture, as O S, in figs. 13 and 14.

DEFINITION XIX.

Directing Plane, is an imaginary plane of visual rays parallel to the plane of projection, at the distance of the picture from it, passing through the point of sight, as is shewn in figs. 13 and 14.

DEFINITION XX.

Directing Line, is the line where any original plane, produced if necessary, intersects a visual ray on the plane of rays called the directing plane, as the line thus called in figs. 13 and 14.

DEFINITION XXI.

Directiny Point, is the point where any original line, produced if necessary, meets a visual ray on the directing plane, as the point D in figs. 13 and 14.

DEFINITION XXII.

Director Ray, is that visual ray on the directing plane between the directing point and the point of sight. This visual ray being always parallel to the projection of the original line on the plane of projection, its direction is determined by it. (See figs. 13 and 14.) This visual ray is perpendicular to the original line produced, from the point D.

DEFINITION XXIII.

Centre of Directing Line, is the point where the direct visual ray meets it, and is the same as the centre of any original plane produced which is perpendicular to the plane of projection. (See Centre of a Plane.)

DEFINITION XXIV.

Centre of a Plane, is that point of any plane, produced if necessary, which is nearest to the point of sight, or from which, if a direct ray pass through the point of sight, it will be perpendicular to that plane: all other visual rays proceeding from the point of sight to any plane are diverging. In figs. 13 and 14, S is the centre of the plane of projection; E is the centre of the original plane. If this point on any original plane be ascertained, and a line be drawn from it to the centre of any circle on that original plane, the visual projection of that line on the picture will be the direction of the minor axis of the elliptical representation of the original circle. And if from the centre of any orginal plane be drawn the two seats of the visual rays which are tangents to any circle thereon, the projection of a line or chord drawn through the circle between the tangent points, will be the major axis of the elliptical representation.

DEFINITION XXV.

Centre Line, is the direct visual ray or line drawn from the point of sight to the centre of any line or plane to which it is perpendicular.

Figs. 13 and 14, O S is the centre line to the plane of projection, also the centre line to the vanishing line.

O E is the centre line to the original plane, and also to the directing line.

O *b* is the centre line to the intersection line.

O D is the centre line to the original line.

DEFINITION XXVI.

Intersection Point, is the point where any original line, produced if necessary, cuts the picture or plane of projection. On figs. 13 and 14, B is the intersection point of the line FG. An intersection point may occur in the centre, or on any part of the picture.

DEFINITION XXVII.

Intersection Line, is the line where any original plane, produced if necessary, cuts the plane of projection. (See figs. 13 and 14.) An intersection line may occur on any part of the picture, and in any direction. When an original plane is not perpendicular to the plane of projection, two intersection lines are necessary, then called First and Second Intersection Lines.

From the first the angle of the original plane is set off; and the second, which is the intersection of the original plane with the plane of projection, is always perpendicular thereto, at the point B, for example, figs. 13 and 14, of the intersection of the angle.

DEFINITION XXVIII.

Parallel of an original Line, is a visual ray, drawn parallel to any original line, through the point of sight, and which, if not parallel to the plane of projection, cuts it at the vanishing point. (See figs. 13 and 14.)

DEFINITION XXIX.

Parallel of an original Plane, is a plane of visual rays parallel to any original plane, passing through the point of sight, and cutting the plane of projection if not parallel

thereto. (See figs. 13 and 14.) This plane, and every plane passing through the point of sight, may also be called the vanishing plane, as every plane that would, if produced, pass through the eye of the observer, cannot be seen, and therefore such a plane is represented on the plane of projection by a right line, which right line is the vanishing line of all planes parallel thereto.

DEFINITION XXX.

Vanishing Point, is the point where the visual ray which is parallel to any original line cuts the plane of projection. (See V figs. 13 and 14.) When an original line is perpendicular to the plane of projection, its vanishing point is in the centre of the picture. When an original line is parallel to the plane of projection it has no vanishing point, for the parallel of the original line passing through the point of sight would not intersect the plane of projection, if infinitely produced. In any representation of an object, all lines that are parallel to any line in any original object have the same vanishing point. Vanishing points may occur on any point of the plane of projection, however far it may be produced. Three of the boundary lines in the definite prism represented by fig. 16 have one vanishing point C at a finite distance, and for the other six boundary lines no vanishing points. Three lines being parallel to the vanishing line B C, and three to the vanishing line C F. Six of the boundary lines in the prism represented by fig. 17 have two vanishing points B and C at a finite distance, the other three lines being parallel to the vanishing lines B D and C F, which are parallel. And the nine boundary

lines of a prism, as represented by fig. 18, have three vanishing points A B C at a finite distance. The portions of two infinite cubes represented by figs. 19 and 20, and the infinite cube represented by fig. 21, have, the first, one point C; the second, two points B and C; and the third, three vanishing points B, C, and D, at a finite distance.

Fig. 19.

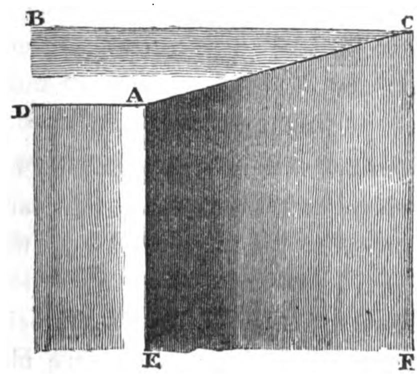


Fig. 20.

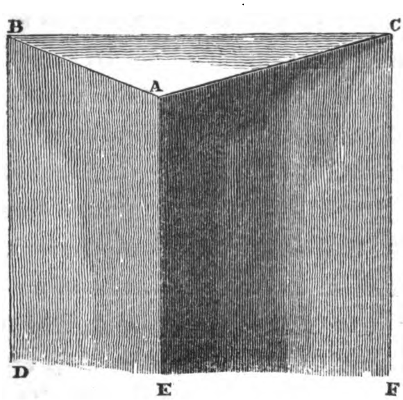
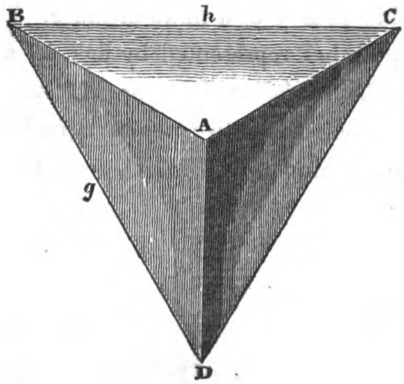


Fig. 21.



DEFINITION XXXI.

Vanishing Line, is a right line parallel to the intersection line where the plane of visual rays parallel to any original plane, or to the plane passing through any original line and its seat cuts the plane of projection: all lines upon the same original plane have their vanishing points in the same vanishing line on the plane of projection, the position of the vanishing point on the vanishing line varying with the angle which different lines make with the plane of projection. A prism, as in fig. 16, may have portions of two vanishing lines B C and C F at a finite distance; or, as in fig. 17, one finite vanishing line B C and portions of two other vanishing lines B D and C F at a finite distance; or, as in fig. 18, three finite vanishing lines B C, C D, and D B, at a finite distance. Thus, as in fig. 19, one boundary line A C meeting other two A D and A E at the adjacent angle of an infinite cube is represented in a finite space; in fig. 20 two boundary lines A B and A C meeting another A E at the adjacent angle A, and one face A B C of an infinite cube, are re-

presented in a finite space. And in fig. 21, the three boundary lines A B, A C, and A D, meeting at the adjacent angle A, and the three faces A B C, A C D, and A D B of an infinite cube, are represented in a finite space. When three faces of a finite cube or any prism are seen, its representation has nine boundary lines; but the representation of an infinite cube appears to have only six boundary lines; the angle formed by the boundary lines B *h*, and *c h*, for example, in fig. 21, at the infinite distance, having vanished. The sum of the four angles of the representation of any face of a finite cube or prism is 360°, but the sum of the angles forming the representation of a face of an infinite cube or prism is only 180°.

The vanishing line of a plane perpendicular to the plane of projection always passes through the centre of the picture. A plane parallel to the plane of projection has no vanishing line. The position of the vanishing lines of planes varies with the angle which they make with the plane of projection, from passing through the centre of the picture, until the planes become parallel, when they have no vanishing line.

DEFINITION XXXII.

Centre of Vanishing Line, is the point in that line to which a visual ray or right line drawn from the point of sight is perpendicular. See figs. 13 and 14. When an original line is perpendicular to the plane of projection, its vanishing point, then at the least distance, is the centre of the vanishing line; the centre of the picture and the centre of the vanishing line in figs. 13 and 14 are in the same point S.

DEFINITION XXXIII.

Distance of the Vanishing Line, is simply the length of the visual ray between the centre of the vanishing line and the point of sight. As O S, figs. 13 and 14.

DEFINITION XXXIV.

The Seat of a Point, Line, or Surface, is the direct radial projection of a point, line, or surface. If from the original point, line, or surface, a direct ray, a plane of direct rays, or a body of direct rays be drawn perpendicular to the plane of projection it or they will pass through, or to, and determine their repective seats. When a plane, revolving on an original right line, becomes perpendicular to the plane of projection, in the intersection of the two planes in those positions is the seat of the right line. In fig. 1, the seat of the right line A B is the right line *a b*, the extremities *a* and *b* being the seats of the points A and B; in fig. 4, the shadow *a b c d,* is the seat of the surface or face of the cube A B C D, and in that projection also of the cube itself. The seats of the original right line and of the points G and F are *g'* and *f'* on the intersection line; the seats of the visual rays are marked on the original plane, and the seat of the point of sight on the original plane produced, is at E, in figs. 13 and 14.

The direct ray proceeding from any original point to its seat on the plane of projection, is on a plane perpendicular thereto. When an original line is not perpendicular to the plane of projection, the plane on which its angle therewith is set off is perpendicular thereto—that is, a plane passing through any original line and its seat is perpendicular to the plane of projection.

DEFINITION XXXV.

Centre of Intersection Line, is a point in that line, produced if necessary, to which, if a visual ray or right line be drawn from the point of sight or from the centre of the picture, it will be perpendicular thereto. This point is marked by the letter b, in figs. 13 and 14, the ray $o\,b$ being perpendicular to $b\,f'$.

DEFINITION XXXVI.

Vertical Line, or Vertical Plane, is a line or plane that is perpendicular to the horizon.

The representations of buildings in visual projection are generally supposed to be drawn on a vertical plane. Thus drawn, the picture may be looked upon from points considerably deviating from the true point of sight without appearing much distorted.

DEFINITION XXXVII.

Horizontal Line, or Plane, is a line or plane that is parallel to the horizon.

When the representations of buildings are drawn in visual projection on a horizontal plane, except they are viewed from the true point of sight, they appear much distorted. Drawings were made some years ago by an American in this way, and such projections are known by the term American perspective, already referred to at page 13.

DEFINITION XXXVIII.

Ground Plane, is an original plane that is level with the surface of the earth, or the foundations on which an object stands.

DEFINITION XXXIX.

Ground Line, is the line where the ground plane intersects the plane of projection.

DEFINITION XL.

Station Point, is the centre or seat of the point of sight on the ground plane.

The 37th and 38th definitions are only relative terms, and, with the definitions 39 and 40, are not at all essentially connected with the principles of perspective; but when a vanishing line is a vertical, and an horizontal line passes through the vanishing points, these terms may assist the comprehension of the others.

EXPLANATION OF THE CONSTRUCTION OF, AND OBSERVATIONS ON, FIGS. 13 AND 14.

These figures have already been referred to in various parts of the definitions; but as they contain the most important principles in the practice of perspective, and are designed to explain six different methods of finding the visual projection of a point and a right line, whatever may be their position with respect to the point of sight and the plane of projection, this connected explanation of the construction of them, and these observations which they are calculated to illustrate, are given with the hope of facilitating a clear comprehension of the subject of this work.

In fig. 13, the eye of the observer, at the point of sight O, is supposed to be looking at G and F, called original points, or at the right line G F, an original line,

situated in space, but whose distances can be measured. It is required to draw the representation of these points, and that right line, on a plane If'A, called the plane of projection, whose position has been taken at pleasure, between the point of sight O and the original objects G and F. The plane of projection may be supposed to be transparent as a plate of glass or talc.

Having determined the position of the plane of projection, find the direct distance, or the visual ray, from the point of sight O to the point S, which is the centre of the picture or the plane of projection. The plane of projection may be supposed to extend in every direction equally from the point S, which is the nearest point on it to the point of sight O. The direct distances from the original points G and F to the points g' and f', which are the seats of the points G and F, on the plane of projection, must be ascertained; or the angle which the original line G F makes with the plane of projection, and the distance from the point B, where the line F G produced would intersect the plane of projection, must be measured. Now the original plane, passing through the original points G and F, the intersecting point B, and the point g' and f', is perpendicular to the plane of projection, and, being produced, determines the *intersection line*.

Parallel to the *plane of projection,* passing through the point of sight, is the *directing plane;* and that meeting the original plane, further produced, gives the *directing line*, which is at the same distance from the point of sight O as the *intersection line* is from S, the centre of the picture.

The original line F G being produced, till it crosses

the directing line, determines the directing point D, and a line from D to O is the *director ray*. Parallel to the *director ray*, from the intersecting point B, draw the line B V, on the plane of projection: and parallel to the original line G F, draw the line O V, and where these lines meet in V, is the vanishing point on the plane of projection. Parallel to the intersection line, through the point V, draw the *vanishing line*, which, in this instance, the original plane being perpendicular to the plane of projection, also passes through the point S, the centre of the picture.

Any plane passing through the point of sight, and being parallel to any original plane which makes any angle with the plane of projection, determines the vanishing line. The parallel of the original plane, in this example, is perpendicular to the plane of projection, because the original plane is so. The parallel of any original plane may be called the vanishing plane. From the point of sight any plane being in the parallel of any original plane cannot be seen—this plane may therefore be said to vanish, and where it intersects the plane of projection is the vanishing line of all other planes parallel to it. If an original plane made, for example, an angle of 45° with the plane of projection, the parallel of the original plane, passing through the point of sight, would also make an angle of 45° with the plane of projection. In that case, the vanishing line on the plane of projection would be at the same distance from the centre of the picture as the centre of the picture is from the point of sight. All lines on the parallel of any original plane, passing through the point of sight, may be said to

vanish. They can only be represented by points on the plane of projection, and each point is the vanishing point of all lines parallel to the visual ray passing through it. Any line on the ray O V can only be represented by the point V on the plane of projection, which is the vanishing point not only of G F, as has been already stated, but of all lines, however situated, that are parallel to G F. Any line in the ray O S can only be represented by the point S, which is its vanishing point, and the vanishing point of the line f' F, and all lines parallel thereto. If an original plane passed through the point of sight and the line G F, the line B V would be its intersection and also its vanishing line on the plane of projection, and the director ray would be its directing line. It will be seen by this figure, that if three points, as the points O, V, S, for example, are determined, the position of a plane passing through these points is determined. If an original plane is parallel to the plane of projection, the directing plane is its parallel, which being also parallel to the plane of projection, if both were infinitely produced they would never meet; therefore such original planes can have no intersection line, directing line, or vanishing line; nor can any lines on such original planes have any intersection point, directing point, or vanishing point. It is most important to be clearly understood, that where the parallel of any original line, passing through the point of sight, intersects the plane of the picture, produced if necessary, is its vanishing point; and that where the parallel of any original plane, passing through the point of sight, intersects the picture, is the vanishing line. The points D and E on the direct-

ing line cannot, of course, be projected on the plane of projection; but any portion, for example, of the line D B, if the plane of projection were produced, might be projected thereon, and would be a continuation of the line V B. If a circle, for example, be drawn between the directing and intersection lines, on the original plane produced, the whole of it might be projected, as seen from the point of sight O, on the plane of projection produced. If, however, any part of a circle, or any other original figure, either touch or is cut off by the directing plane, the whole in the first case, and only a portion in the latter, could be represented, even if the plane of projection were infinitely produced. If the object be a circle, in the first case the projection would be a parabola, in the second case an hyperbola.

The point E, which is the centre of the original plane produced, and the nearest point on that plane to the point of sight O, is also the seat of the point of sight O, on the original plane. The visual rays are represented as they proceed from the point of sight O to the points G and F, the original objects. The seats of these visual rays E G and E F are represented on the original plane, which is perpendicular to the plane of projection. These seats of the visual rays determine the points g'' and f'', on the intersection line. The seat of the direct visual ray O S is the line E b; and the length of the seat is the same as the length of the direct visual ray,—that is, the seat of the point of sight is at the same distance from the point b, on the intersection line, that the point of sight is from S, the centre of the plane of projection. The distance between the directing point D and the inter-

secting point B, is the same as between the point of sight O and the vanishing point V; and the distance between the directing point D and the point of sight O, and the distance between the intersection point B and the vanishing point V, are equal. The points O, V, D, B, G, F, are in the same plane. The points O, S, E, *b*, are in another plane. The lines g' S and f' S are drawn from the seats of the original points to the centre of the picture. The lines $g''g$ and $f''f$ on the plane of projection are perpendicular to the intersection line from the points where the seats of the visual rays cross it.

The point S is the centre of the vanishing line;
b is the centre of the intersection line;
E is the centre of the directing line; and
O *b* is the direct visual ray to the intersection lines.

The points *g* and *f*, and the line *g f*, are the visual projections of the original points G and F and the line GF.

The point *g*, for example, the visual projection of the point G may be found—

1st. By the intersection of the visual ray OG with a line drawn from its seat *g* to S, the centre of the picture.

2d. By the intersection of the same visual ray O G with a perpendicular to the intersection line from the point g'', the seat of that ray.

3d. By the intersection of the same visual ray OG with a line from the intersecting point B, to the vanishing point V.

4th. By the intersection of the line *g* S with the perpendicular $g'' g$.

5th. By the intersection of the line B V with the perpendicular $g'' g$.

6th. By the intersection of the line f' S, with the line B V.

In a similar way, by either of these methods, the visual projection of the point F and of the line G F is determined.

The point of sight O, and the lines B V and B F, being in the same plane, if the line B F be supposed to be infinitely produced, the line B V, when viewed from the point of sight O, would cover its infinite extent; for O V being parallel to B F, these lines, if infinitely produced, would never meet; therefore B V is the definite projection of the line B F, infinitely produced; but while an infinite line is thus represented by a finite one, the finite line B D could not all be represented on the same plane of projection, however far it might be produced.

If the point of sight O, the line O S, and the plane of projection as represented in this diagram, be supposed to be fixed, and all the other planes and lines be supposed to revolve on the direct visual ray O S, then visual projections of original points, or an original line similarly situated with respect to the points of sight and the centre of the picture, may be made at the same distance from the centre of the picture, in any direction on the plane of projection.—Or in this diagram the plane of projection may be supposed to be vertical, horizontal, or inclined at any angle to some vertical or horizontal plane; and the original plane may be supposed to be either below, above, to the right, or to the left; and in all or any of these

positions the projections would be correct. Again, suppose the original plane to be a horizontal plane intersecting the plane of projection below the centre of the picture, and if another original plane parallel to this were as much above the centre of the picture as this is now supposed to be below it, the visual projection of the points and line similarly situated would be as much above the centre of the picture as in the diagram it now appears below it. Further, suppose this original plane to be to the right of the centre of any picture, and another similar original plane, with the points and right line as much to the left, the plane of projection being extended accordingly, the visual projections of these points and that line would be as much to the left as these are now supposed to be to the right of the centre of the picture. By viewing this diagram with different sides directed to the person, a more complete notion of the universality of these principles will be obtained: but by means of a model with threads or wires for the visual rays, &c. they would, with more certainty, be permanently impressed on the mind of the student.

Suppose the angle f' B F to be the angle which another original plane makes with the plane of projection, then the intersection line of that other original plane will be a line on the plane of projection perpendicular to the line Bf' from the point B. The line Bf', in that case, would be the intersection line of the plane on which the angle is set off, or it may be called the seat of the angle.

From what has already been said, it is hoped it will be perceived that the intersection lines of original planes

and the seats of their angles may occur on any part of the plane of projection. While it is necessary to have a clear comprehension of this in order to know how to make a judicious selection of intersection lines, in practice it is desirable to have as few of them as possible. The way to avoid many of them is first to make direct projections of any original object or objects on planes, either known or assumed to be, in any convenient situation, perpendicular to the plane of projection. Sometimes one plane, and occasionally two planes, are given, as being perpendicular to the plane of projection, as for example, those on which the plan and elevation of a building are drawn.

The very particular explanation just given of fig. 13 will, it is hoped, enable the student more readily to understand how the principles including the several planes, lines, and points, are to be applied in practice on one plane, as in fig. 14. If the planes in fig. 13 are supposed to turn, as on hinges, on the intersection line, the vanishing line, the line passing through the point of sight where the parallel of original plane and directing plane meet, and on the directing line; and if by that means the plane of projection, the parallel of original plane, and the directing plane, be folded down on the original plane, the four planes will thus be brought together, and all the lines on them may now be represented on one plane. The action of folding down the three planes is represented by fig. 15. The several lines and points on fig. 13 being described by the same names and letters as in fig. 14, the student will, it is hoped, find no difficulty in tracing the corresponding points and lines of the first to those on the last. In prac-

tice, in order to avoid having too many lines upon the drawing, it is convenient to have a separate paper on which to draw the direct projection of any original object, the intersection line, the directing line, and either the visual rays from points of the original object to the point of sight, or the seats of the visual rays to the seat of the point of sight: also for the purpose of finding the vanishing lines and vanishing points. From this paper to the drawing, is to be transferred only the intersection line with the points thereon, and the vanishing line and vanishing points by which means the visual projection of any object may be drawn. It may be proper to observe here, that when the distance of the centre of the picture from any intersection line is equal to the distance of the point of sight from the centre of the picture, both the directing line and the vanishing line are in the same place when folded down on one plane. When an intersection line is near the centre of the picture, the directing line is near the point of sight. When an intersection line passes through the centre of the picture, the directing line passes through the point of sight. The directing line is always at the same distance from the intersection line that the point of sight is from the vanishing line. Students are recommended to try extreme and intermediate cases in those things that will admit of them: as, for example, the projections of lines perpendicular, parallel, and making an angle of 45°, with the plane of projection; the representation of an object to appear near the centre, and also of the same object to appear at the extremity of a picture; the plane of projection being near the object, and the point of sight at a considerable distance; the point of

sight near the plane of projection, and the object at a considerable distance from it.

AXIOM I.

The common intersection of two planes is a right line.

AXIOM II.

If two right lines make any angle with each other, or meet in a point; or, if they are parallel to each other, a plane may be so placed as to coincide with every point of each line.

AXIOM III.

If three right lines cut each other, but not in a common point; or, if two of the lines are parallel to each other, and they are both cut by the third, all the three lines will be in the same plane; or, if a plane coincide with any two of the lines, it will coincide with the third also.

AXIOM IV.

Every point in any right line is in the same plane as that right line.

AXIOM V.

Two parallel right lines, or two parallel planes, cannot meet or intersect each other.

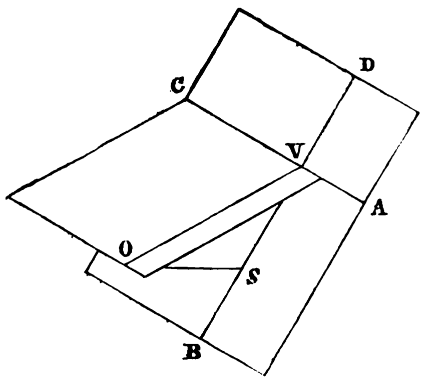
LEMMA I.

Fig. 22. It is assumed that if C V O and A B C D be two planes cutting each other in the line A V C, and if, from any point O on one of the planes, the line O V be drawn to cut the line A C, at right angles in V, and another line O S, be drawn perpendicular to the plane A B C D, then a right line drawn from S to V, will be perpendicular to A V C.

This follows from the 11th Proposition, 11th book of

Euclid, where it is demonstrated how to draw a straight line perpendicular to a plane from a given poiut above it.

Fig. 22.



THEOREM I.

It is considered to be an acknowledged truth, that a line drawn from the centre of the picture to the centre of a vanishing line is perpendicular to that vanishing line.

DEMONSTRATION.

Fig. 22. Suppose ABCD to be the plane of projection, or the picture; O to be the point of sight, and OVC to be a plane, parallel to some original plane, which, by its meeting or cutting the plane of projection, produces the vanishing line AVC; and let V be the centre of that vanishing line, and S the centre of the picture.

Then having drawn OV, OS, and SV: OV is perpendicular to the vanishing line AC, as stated in Definition 32; and OS is perpendicular to the plane of projection ABCD, as stated in Definition 16. Therefore, SV is perpendicular to AVC (Lem. 1), which was to be proved.

Corol. 1.—The distance OV of any vanishing line

AVC, is the hypothenuse of a right angled triangle, whose legs are OS, the distance of the picture, and SV, the distance between the centre of the picture and the vanishing line.

Corol. 2.—Thus a plane, passing through the point of sight, the centre of the picture, and the vanishing point (when the vanishing point is not in the centre of the picture), is perpendicular both to the picture and to the parallel of any original plane.

Or if, from any point in the right line, formed by the intersection of two planes, making any angle with each other, a right line perpendicular to that line be drawn upon each plane, a plane passing through both these latter right lines will be perpendicular to both planes. The intersection of this third mentioned plane with one of the other planes, is the seat of the angle of the other plane. Also, whatever angle two planes which intersect may make with each other—right lines may be drawn upon each that shall form any angle with each other, from 0° to 180°: but, if a line upon one plane be perpendicular to the line of intersection, no line drawn upon the other plane (if the angle which the planes make with each other be *less* than a right angle) will exceed an angle of 90°. And no line drawn upon the other plane (if the angle which the planes make with each other *exceed* a right angle) will be at a less angle than 90°.

Corol. 3.—Also that any plane on which the angle of any original plane is to be set off, is perpendicular, both to the original plane and also to the plane of projection.

THEOREM II.

The Perspective representation of any object is the

same as the visual or diverging radial projection of it on the plane of the picture, the point of sight being the point of intersection of all the rays.

DEMONSTRATION.

Fig. 12. In the definition of the principles of the art of painting it has been stated, that the light and colour from any point *a* of the projection of any object must come to the spectator's eye at O, in the same direction as it would do from the corresponding point A of the original object. Indeed, it is evident, that the rays *a* O and AO are in one and the same straight line. Hence, it is evident, that the projection *a* is the intersection of the ray AO by the plane of the picture, and that the whole projection *abcde* is the diverging radial projection of the original cube ABCDE, made by the body of rays which proceed from the point of sight O, which was to be proved.

Corol. 1.—The projection of a right line is a right line. For the triangular surface ODE, which may easily be imagined to be covered with a system of rays which terminate in O, and produce the projection *de*, which is the representation of the line DE, all the rays going to O, from the several points of the line DE, being in the same plane as the lines DO and EO. Therefore *de* is the intersection or representation of the line DE, formed by the intersection of the triangular surface of rays with the plane of projection, and consequently is a right line, which is considered to be self-evident.

Corol. 2.—The original of a projection may be any object that will produce the same system of rays. Thus

the original of the projection *de* on the picture may be any line *d′e′* between the point of sight and the picture, which it is evident produces a similar triangular surface of rays, as the line DE does on the other side of the picture.

This being the case, that a small object near the spectator's eye, on one side of the picture or plane of projection, may have the same representation as a larger object of the same form at a greater distance, either on the same side or the other side of it,*—how, therefore, it may reasonably be asked, is it, that any figures drawn upon a picture appear to be the objects they are designed to represent?

The reason is, that the mind by habit knows that the same objects that are at a greater or less distance vary in colour, light, and shade; and by a judgment thus formed it determines their situation. An imitation of all these circumstances is necessary to render a picture complete; although a simple drawing in lines only is sometimes nearly sufficient to convey a correct idea of an object, on account of the relation the several parts have to each other. In a drawing of a pavement, for example, the stones may all appear to be square, of equal sizes, and, of course, some at a less or greater distance than others, although the figures by which the squares are represented are very irregular figures, any one of

* From this circumstance the exact size of the original of a projection cannot be obtained, unless a point or line of intersection of some part be given on the picture; if that be known, as is shewn in this work, the original may be found: but without a line or point of intersection be given, the proportions only of the several parts of the original can be ascertained from a correct representation.

which would not appear to be the representation of a square, if there were no other objects to bias the judgment by their relation to it.

THEOREM III.

The projection of a right line, not parallel to the plane of projection produced if necessary, passes through both its intersection and vanishing points.

DEMONSTRATION.

Fig. 13. The explanation which has been given in the Definitions of the several parts of this figure, and afterwards of its construction being understood, it will be remembered *fg* is the projection of the original line FG; that is, OF and OG are the visual rays which produce the projections of the points F and G, at the points *f* and *g*. The line *fg* on the picture being the intersection of the triangular plane of rays OFG with the plane of projection; all the line FGD is also in the same plane produced, and, consequently, the intersection point B; therefore *fg*, on the plane of projection being produced, must pass through the point B, the point of the intersection of the original line, which was first to be proved.

O V being parallel to F G is in the same plane as the triangle O F G. Therefore *fg* continued will pass through V, which was also to be proved.

This theorem being the principal foundation of all perspective, the student should make himself familiar with it. To help him a little further in his reflections upon it, the sense may also be explained by fig. 12, where the projection *bc* meets the original line BC in its intersection K, and passes also through its vanishing

point V, which is produced by O V being parallel to B C; the point K being on the plane of the picture produced.

N.B.—When on any original plane any original right line produced passes through its vanishing point* on the lane of the picture, the whole projection of it is in that point; in that case the right line may be said to vanish. This is one reason for using the term vanishing point. Another reason is, the further any object is off, upon any right line, the smaller is its projection, and at the same time, the nearer it appears to the vanishing point; and when it comes into that point its magnitude vanishes, because the original is at an infinite distance. This may be easily conceived, by imagining a man to be going from you in a long walk, who appears smaller and smaller the further he goes. The reason of this diminution will appear from the following Corollaries.

Corol. 1.—The projection of all original right lines that are parallel to each other, but not to the plane of the picture pass through the same vanishing point: for they have but one parallel common to them all, and consequently but one vanishing point.

This may be seen represented in fig. 12, where the projections *da* and *cb* of the parallel lines DA and CB meet in their common vanishing point V.

Corol. 2.—The centre of the plane of projection is the vanishing point of all lines perpendicular to the picture: for where the parallel of any such line passing through the point of sight cuts the picture is the vanishing point.

* Such an original line, if further produced, would also pass through the point of sight.

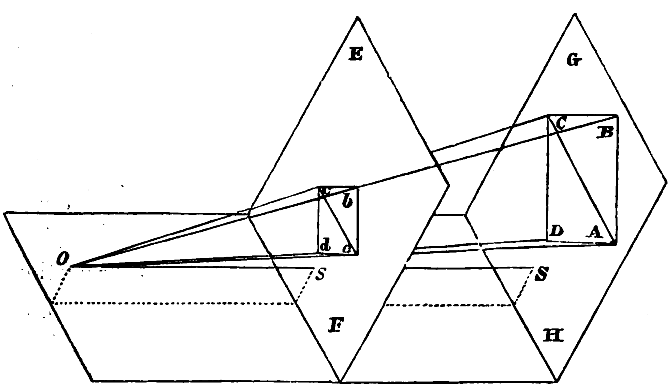
THEOREM IV.

The projection of a line parallel to the picture is parallel to the original.

DEMONSTRATION.

Fig. 23. Let the plane E F be the picture, and G H an original plane parallel to it, then *a b* will be the projection of A B, if O be the point of sight. The visual rays proceeding from the point of sight form the triangular plane O A B, and the line *a b* is the intersection of the plane of rays with the plane of projection. Therefore the plane H G being parallel to the plane E F, *ab* is parallel to A B; for they are both on the triangular plane O A B, and would not meet if produced.

Fig. 23.



N.B.—This fact has, however, puzzled several; for many good draughtsmen are neither mathematicians nor geometricians; and they have asked, Will not the two horizontal and parallel right lines, for example, forming

the top and base of a long wall, appear, to a person who may stand in front of the middle of it, to get nearer and nearer to each other the further they extend both to the right and to the left of the centre? Have not such lines, therefore, the appearance of curves? and ought not their representation, when drawn on a plane, as thus viewed, to be curved lines, as there is no appearance of any angle in the middle?

The answer to these questions is, that straight parallel lines on the plane of projection, when viewed from the point of sight, will appear to approach each other; and also appear as much curved, in proportion to their length, as the long lines of the top and base of the wall would so appear from the point, at a proportionable distance, from which they were viewed.

Corol. 1. Any number of original lines being parallel to each other, and to the picture, the projections of the whole of them are also parallel to each other; thus AB and CD are parallel to each other, and so are their projections *ab* and *cd*.

N. B.—This is the reason why the perpendicular lines of a building, for example, projected on a vertical plane, are all drawn parallel to each other, and if the plane of one front be also parallel to the plane of projection, the representations of all the vertical and all the horizontal lines of that front will be parallel to each other. When on any given or original plane figure there are no right lines parallel to the plane of projection, that is, to the intersection line, such lines may be drawn either at pleasure or from any given points, to facilitate the visual projection of the other lines.

Corol. 2. The projection of *abcd* of any plane figure ABCD which is parallel to the picture, is similar to its original: for, having drawn the diagonal AC and its corresponding projection *ac*, the sides of the triangle *ab*, *bc*, and *ac* are parallel to their corresponding originals AB, BC, and AC: therefore the angles *a*, *b*, and *c* are equal to their corresponding angles A, B, and C; and, consequently, the triangle *abc* is similar to the triangle ABC. For the same reason *acd* is similar to ACD; and, consequently, the figure *abcd* is similar to ABCD.

Corol. 3. The length of the line *ab* in the projection is to the length of the original line AB as the distance of the picture is to the distance between the point of sight and the plane of the original figure. Let O*s*S be perpendicular to both planes, cutting them in *s* and S, then will *ab* : AB : : O*a* : OA, or O*s* : OS. By the 17th Proposition, 11th book of Euclid, it is demonstrated, "that if two straight lines be cut by parallel planes, they are cut in the same ratio."

The centre of the picture is *s*, and O*s* is the distance of the picture. The line OS is the distance between the point of sight O, and the original plane ABCD.

Therefore *ab* is to AB as the distance of the picture is to the distance between the point of sight and the plane of the original figure.

THEOREM V.

The projection of a right line is parallel to its director.

DEMONSTRATION.

Fig. 13. It has already been explained, that in this figure the lines OF, OG, OD, *fg*, are all in the same

plane; and here it may be stated, that as the directing plane O D E must be parallel to the plane of the picture A B I C, therefore the director O D is parallel to the projection *fg*, by the 16th Prop. 11th Book of Euclid, where it is demonstrated, that "If two parallel planes be cut by another plane, their common section with it are parallels."

Corol. 1.—The projections of lines that have the same director are parallel to each other.

Corol. 2.—When the original line is parallel to the picture, its director is parallel to it, and, consequently, is in the parallel of any given plane passing through that original line, and, therefore, the vanishing line of that plane and the projection of the line are parallel to each other.

THEOREM VI.

The vanishing, intersection, and directing lines of any original plane are parallel to each other.

DEMONSTRATION.

Fig. 13. The explanations before given being understood, and the parallel plane O V C being parallel to the original plane D F H, and also the directing plane O D E being parallel to the plane of the picture C A B; therefore the vanishing line C V, the intersection line I B, and the directing line E D, are parallel to each other, by the proposition in Euclid, quoted in last Theorem, which was to be proved.

Corol.—The distance of V between the projection of the point *f* and the vanishing point V, is to the distance B V between the intersection B and the same vanishing point V, as the distance O V of that vanishing point V from

the point of sight O is to the distance D F, between the director D and the original point F. For O V B D is a parallelogram; therefore B V is equal to D O, and the triangles fO V and O F D are similar, because their sides Vf and O D are parallel: therefore

$$fV : VO :: DO\ (= BV) : DF.$$

THEOREM VII.

The vanishing points of all right lines on any original plane, are in the vanishing line of that plane on the plane of projection.

DEMONSTRATION.

For as the parallels of all original right lines on the same plane pass through the point of sight, all these parallels of the original right lines will be in the plane which is parallel to the original plane: by the 15th Proposition, 11th book of Euclid, where it is demonstrated, that "If two straight lines meeting one another, be parallel to two other straight lines that meet one another, but which are not in the same plane with the first two: the plane which passes through the first two straight lines is parallel to the plane passing through the other two straight lines."

Corol. 1.—Original planes that are parallel, have the same vanishing line.

Corol. 2.—The vanishing point of the common intersection of two original planes, is the intersections of their vanishing lines.

Corol. 3.—The vanishing line of a plane, perpendicular to the picture, passes through the centre of the picture.

THEOREM VIII.

The intersection of all right lines in the same original plane, are on the intersection of that plane.

This needs no demonstration.

***Corol.* 1. The intersection of the common intersection of two original planes, is the intersection of their intersection.**

***Corol.* 2.—Planes, whose common intersection is parallel to the picture, have parallel intersections, and also parallel vanishing lines.**

In order to assist students in comprehending the data in the following problems, those circumstances which are required either to be given or determined in order to make upon a plane either *direct*, *oblique*, or *visual* projection of points, lines, or planes, are enumerated as follows:—

***Case* 1.—Before the *direct* projection of a point can be ascertained, the position of the plane of projection must be determined. In fig. 2, for example, the position of the line *ab*, which is the section of the plane of projection, must be determined before the direct projection of the point A to the point *a* can be ascertained.**

***Case* 2.—Before the *oblique* projection of a point can be determined, the position of the plane of projection and the angle of the ray with the plane of projection must be given. In fig. 7, for example, the position of the line *e′a*, the section of the plane of projection, and the obliquity of the angle *e′a*A must be determined before the projection of the point A to *a* can be ascertained.**

Case 3.—Before the *visual* projection of a point can be made, the position of the plane of projection and the point of sight, or its seat on a plane perpendicular to the plane of projection, must be determined; these being done, the direct distance of the plane of projection from the point of sight, that is, to the centre of the picture, may be measured; then the direct distance from the original point to the plane of projection, that is, to the seat of the point, and also the distance from the seat to the centre of the picture. In fig. 11, for example, *o* represents the position of the point of sight, the line *e′c′* the section of the plane of projection; and on fig. 10, *o* is the seat of the point of sight on a plane perpendicular to the plane of projection, and the line *sc* is the section of the plane of projection. These being determined, suppose the pyramid to stand vertically, the visual ray in fig. 11 from *a* to *o* crossing the plane of projection at *a′* shews how much the projection of that point is below *s* the centre of the picture, and in fig. 10 how much the projection of the point A is to the right of *s*, also the centre of the picture.

Case 4.—Before the *direct* projection of a right line can be found its length must be given, and the angle the line makes with the plane of projection must be known. After the explanation of the 1st Case, by a reference to the line AB, fig. 2, further explanation of this will be unnecessary.

Case 5.—Before the *oblique* projection of a right line can be obtained, its length must be given, the angles which the line makes with the plane of projection, and the angle which the oblique ray is to make with that

plane, must be determined. After the explanation of the 2d Case, all that can be necesssary is to refer to the line AB, fig. 6, which is projected to *a b*.

Case 6.—Before the *visual* projection of a right line can be obtained, its length must be given, and the position of the plane of projection with respect to it and the point of sight or its seat, the distance and the angle which the line makes with the plane of projection and its seat must be determined. After the explanation of the 3d Case, a reference to AB, fig. 10, will enable the student to understand this.

Case 7.—Before the *direct* projection of a plane surface can be made, its dimensions, or a correct drawing of it, must be given; and also the angle it makes with the plane of projection must be known. See 1st and 4th Cases.

Case 8.—Before the *oblique* projection of a plane surface can be made, its dimensions, or a correct drawing of it, must be given; also the angle that it and the oblique rays make with the plane of projection must be known. See 2d and 5th cases.

Case 9.—Before the *visual* projection of an original plane surface can be determined, its dimensions, or a correct drawing of it, must be given; the position of the plane of projection, the point of sight or its seat, the centre of the picture, the angles and its seat which the original plane makes with the plane of projection, must be known. See 3d and 6th Cases.

Case 10.—Before any projections of objects of any other forms are obtained, it is necessary to have given and determined such of those things which are required to find the projection of points, lines, or plane surfaces

which are in them, or may be marked or formed on them as have been already described for the three kinds of projection.

N.B.—The plane of the seat or the plane of the angle of any original line is always perpendicular to the plane of projection. When an original plane is perpendicular to the plane of projection, it is in the plane of its seat.

PROBLEM I.

To find the visual projection of a Point, having given,

Fig. 24.—A, its original position at its given distance from

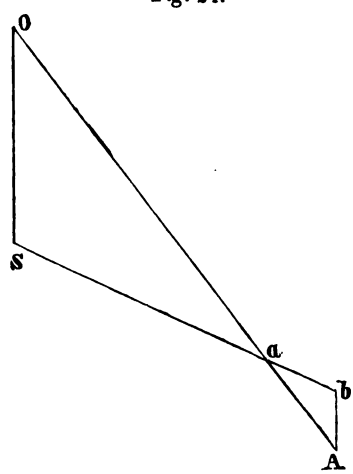
b, its seat on the plane of projection;

S, the centre of the picture, and

O, the point of sight at its distance from the centre of the picture.

The line shewing the distance of the picture S O, must

Fig. 24.



be parallel to the direct ray A *b*, between the original

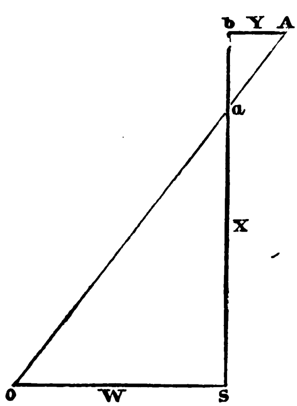
position of the point and its seat, for they are both perpendicular to the plane of projection. These four points, O, S, *b*, A, and the two lines O S and *b* A, being described on your paper, draw the diverging ray A O (that is, the line from the original point to the point of sight) and the line *b* S (that is, the line from the seat of the point on the plane of projection to the centre of the picture), and where these two lines cross each other at *a*, is the projection of the original point A.

This method of finding the projection of a point is the *first* described at page 29.

DEMONSTRATION.

Suppose the angle O S *b* had been a right angle, and the angle A *b a* in consequence also a right angle; then suppose the line S *b* to be the section of the plane of the picture, as in fig. 25, the original point A being on one

Fig. 25.



side perpendicular thereto, on the direct ray from *b*, and the point O, the point of sight, on the other side being

perpendicular to the point S, the centre of the picture, the diverging ray, proceeding from the original point A to the point of sight O, passes through the line S *b*, the section of the plane of the picture, at *a*. But the point *a* is the same, whether the angle O S *b* be a right angle or not; because the triangles O S *a* and A *b a* are similar, and therefore

$$Sa : ab :: SO : bA,$$

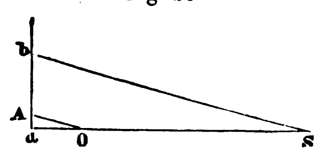
which ratio is not altered by the angles O S *b* and *a b* A deviating from a right angle, so long as they are similar.

Therefore, in all cases, the point *a* thus found is the visual projection sought of a point, whose seat on the picture is *b*, and its distance from its seat is *b* A, the point of sight O being in the situation described.

Corol. 1.—Having drawn S *b*, the point *a* may be found by a scale and compasses, dividing the line S *b* in *a*, so that S *a* may be to *b a*, as the distance of the picture S O is to the distance *b* A of the original point from its seat, as in fig. 26, where

$$aS : ab :: OS : Ab.$$

Fig. 26.

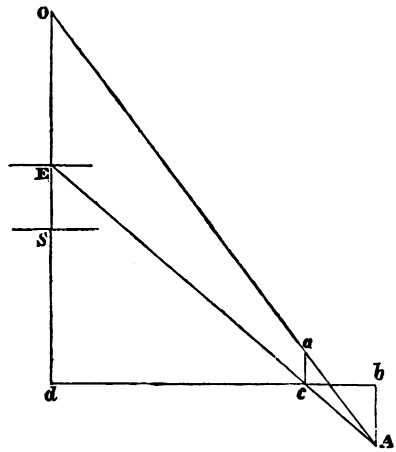


Corol. 2.—By this proposition the visual projection of any *line* may be found, by finding the projection of two points in it, and then drawing a line between these projections of the two points.

Fig. 27, shews the application of the *second* method of finding the visual projection of a point, described at

page 29, which is, by the intersection of the visual ray

Fig. 27.

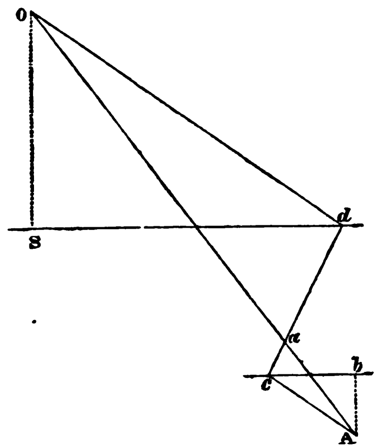


with a perpendicular from the intersection point of the seat of the visual ray.

Perpendicular to the direct ray A *b*, draw *d b*, which is the intersection line of an original plane, perpendicular to the plane of projection. The seat of the point of sight, on an original plane produced, which is perpendicular to the plane of projection, is always at the same distance from its intersection line, as the point of sight is from the centre of the picture. Therefore make *d* E equal to S O, and E, which is on the directing line, is the seat of the point of sight O. From the original point A, draw the visual ray A O, and from the same point draw the line A E, which is the seat of the visual ray. From the point *c*, where the seat crosses the intersection line *d b*, draw the perpendicular *c a*, and the point *a*, where the perpendicular meets the visual ray, is the projection of the original point A.

Fig. 28, shews the application of the *third* method of finding the visual projection of a point, described at page 29, which is, by the intersection of the visual ray from the given point with the definite projection of any infinite right line passing through the given point, on an original plane, perpendicular to the plane of projection.

Fig. 28.

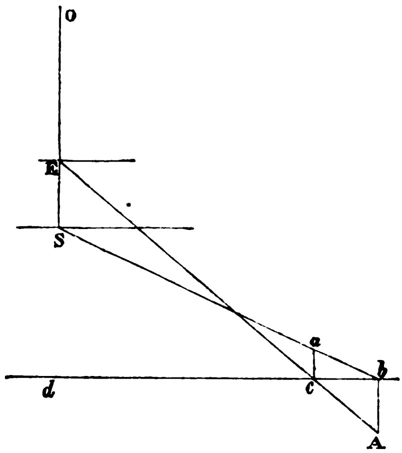


Draw a portion of any infinite right line A *c*, through the original point A, crossing the intersection line at *c*, the intersection point. Then, parallel to A*c*, draw O *d*, crossing the vanishing line S *d*, at *d*, the vanishing point. Next, draw *c d*, the definite projection of *c* A infinitely produced, and the point *a*, where it crosses the visual ray O A, is the projection of the original point A.

Fig. 29, shews the application of the *fourth* method of finding the visual projection of a point, described at page 29, which is, by the intersection of a right line, drawn from the seat of the original point to the centre

of the picture, with a perpendicular drawn from the intersection point of the seat of a visual ray.

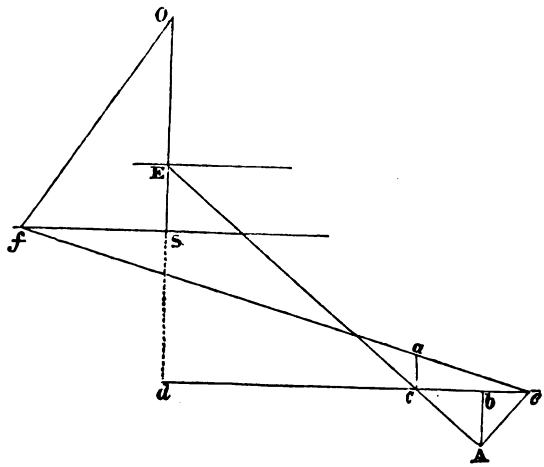
Fig. 29.



Find the seat E of the point of sight O, A E the seat of the visual ray, and the intersection point *c;* then draw the perpendicular *c a,* as described in fig. 27, and draw the right line *b* S, crossing the perpendicular *c a,* at *a,* which is the projection of the point A. It may be observed of this method, that it may be applied to great advantage if the point of sight O is off the picture, and if the seat E of the point of sight O can be got on the plane of the picture. The distance E O by this method not being required.

Fig. 30, shews the application of the *fifth* method of finding the visual projection of a point, described at page 30, which is, by the intersection of a perpendicular drawn from the point of the intersection of the seat of the visual ray with the definite projection of an infinite right line.

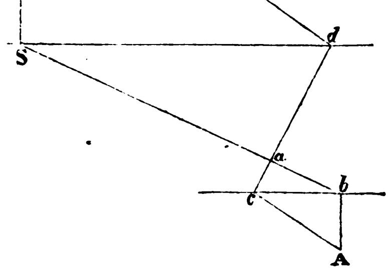
Fig. 30.



A E is the seat of the visual ray;
ca is the perpendicular from the intersection point *c*;
ef is the definite projection of *e* A infinitely produced;
and *a* is the projection sought;
O*f* is parallel to *e* A.

Fig. 31, shews the application of the *sixth* method of

Fig. 31.



finding the visual projection of a point, described at page 30, which is, by the intersection of the definite projection of any infinite line passing through the point with a line drawn from the seat of the original point to the centre of the picture.

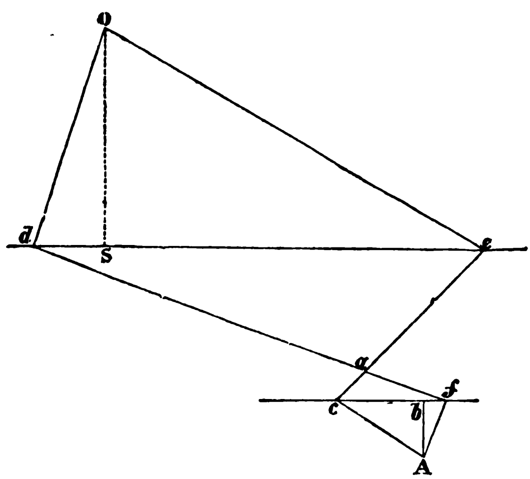
c d is the definite projection of the line *c* A infinitely produced;

b S is the right line from the seat of the original point to the centre of the picture;

and *a* is the projection sought. O *d* being parallel to *c* A.

Fig. 32, shews the application of the *seventh* method of finding the visual projection of a point, which is, by

Fig. 32.



the intersection of the definite projections of any two infinite lines passing through the given point.

The line *d f* is the definite projection of the line *f* A infinitely produced; and *e c* is the definite projection of

the line *c* A infinitely produced, and *a* is the projection of the original point A.

The line *d o* is parallel to *f* A, and *o e* is parallel to *c* A.

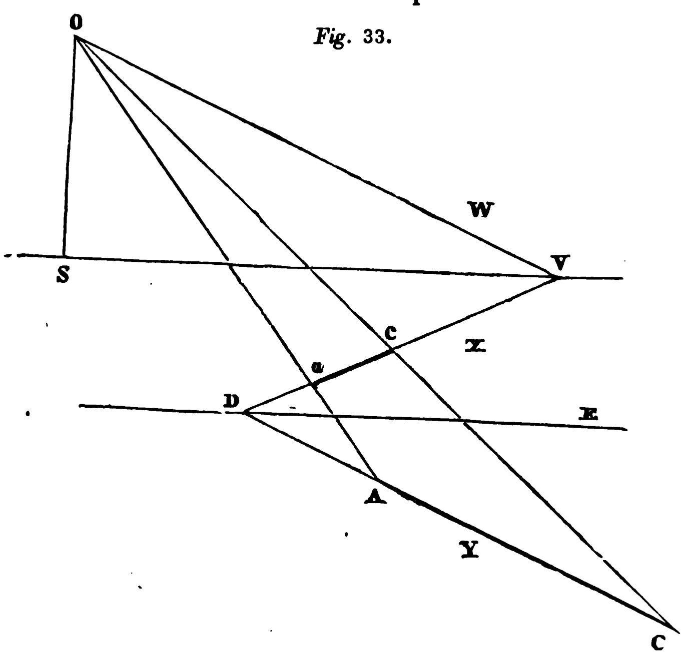
The vanishing point of the line *f d* being *d*, and the vanishing point of the line *c e* being *e*.

PROBLEM II.

To find the visual projection of a line, by means of the intersection point, the vanishing point, and the visual rays, having given

Fig. 33. A C, the original line, the angle C D E, which it makes with its seat on D E, the intersection line; the point S, the centre of the picture, and the point O, the distance of the point of sight from the centre of the picture.

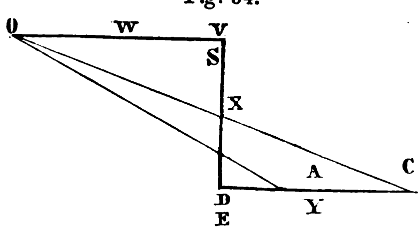
Fig. 33.



On your paper draw A C, produced to D, at the given angle with the intersection line D E. Also let the position of S, the centre of the picture, be marked, and then perpendicular to D E (the intersection line) draw S O, and make it equal to the distance of the picture, O being the point of sight. These being done, all things given in this problem are marked on your paper.

Now, parallel to the intersection line D E draw the vanishing line S V through the given point S, the centre of the picture, and parallel to the given or original line A C draw the parallel ray O V through the given point of sight O, and where the lines O V and S V cross each other, is the vanishing point, and O V is its distance from the point of sight. Next, draw D V, which is the definite projection of the original line A C, supposed to be infinitely produced in the direction A C, from D; for, however long the line A C, produced from D, may be, its appearance from O can never be longer than the line D V. From the point O draw the diverging rays O A and O C, and these cutting the line D V, in *a* and *c*, *a c* is the projection of the definite original line A C.

Fig. 34.

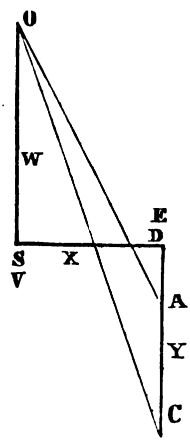


DEMONSTRATION.

It has been shewn how the planes, represented in

fig. 13, are to be folded down on one plane; now imagine how the three parts, W, X, and Y, of this may be bent or turned on the intersection line D E and the vanishing line S V. First, suppose the space X between these lines to be turned up on the line D E, at right angles to the plane C D E, on the space Y, and then the plane O S V, or space W, turned down, so as to be at right angles to the plane S V E D. The student is recommended to bend his paper on which his drawing is, in this manner, which

Fig. 35.



is represented by fig. 34. He will, however, observe that when so bent, the projection of the original line will appear to be on the back of the picture when viewed from the point of sight O; which has before been explained. If he bend his paper, as in fig. 35, the projection of the line will appear on the face of the picture, while the original line, when viewed from the point of sight O, will appear to be at the back of the original plane.

N. B.—The letters W, X and Y, are put on several of the figures, in order to shew, if the plane of projection was folded at the intersection and vanishing lines, where the original plane, the plane of projection, and the parallel of the original plane would appear.

The student is recommended to write the several names of the lines and points on his drawing, when it will be evident that O being the point of sight, and A C being the original line, and the ray O V being parallel to it, V is its vanishing point, and the line D V being cut by the diverging rays, consequently *a c* is the projection of the original line A C.

Corol. 1.—The line D A C being thus conceived to be the original line laid on the picture, by turning the plane C D E round the line E D, the projection of any other point of the line A C may be found upon the line D V, by drawing diverging rays across it from any two points to O the point of sight. For the points *a* and *c* depend only on the parallelism of the ray O V and line D C, and their proportion:—

$$aV : aD :: VO : DA,$$

and $cV : cD :: VO : DC$, upon account of the similar triangles *a* V O, and *a* D A and *c* V O and *c* D C.

The better to understand this, should any thing more be necessary, the student is recommended to compare figs. 13 and 14, with figs. 33, 34, and 35, where the points O, V, *f*, *g*, B, G, F, in the first figs. are analogous to the points O, V, *a*, *c*, and D, A, C, in the latter figures respectively.

Corol. 2.—Having found D V, the projection c of the point C may be found by a scale and compasses, as

$$cV : cD :: OV : CD.$$

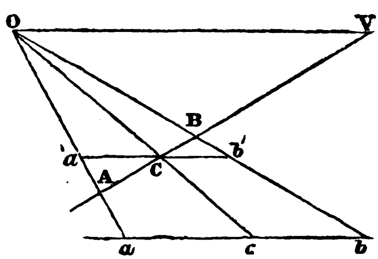
N. B.—The projection of the points A and C may be found by any of the methods described in the first problem.

PROBLEM III.

Having given the projection of a line, and its vanishing point; to find the projection of the point that divides the original line in any given proportion.

Let A B (fig. 36) be the given projection of the line to be divided, and V its vanishing point. Draw at pleasure O V and $a\,b$ parallel to it, and through any point O, of the line O V, draw O A and O B, cutting $a\,b$ in a and b. Divide $a\,b$ in c, in the proportion given, and draw O c, cutting A B in C. Then will C be the projection sought, the original of B C being to the original of C A, as $b\,c$ is to $c\,a$.

Fig. 36.



DEMONSTRATION.

O V being parallel to $a\,b$, $a\,b$ may be considered as the original line, and O V as its parallel, consequently O, as the point of sight, and O a, O b, O c, as visual rays, projecting the points, A, B, C.

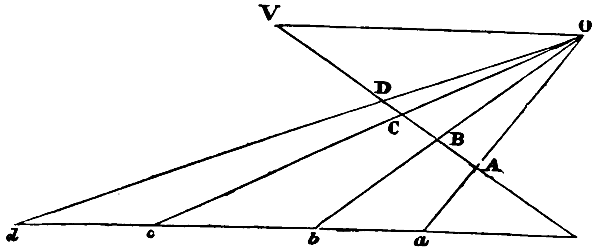
Corol.—The mathematical reader will easily find, that $AC \times BV : CB \times AV :: ac : cb$. Whence the point C may be found by a scale and compasses, making $AC : CB :: AV \times ac : BV \times cb$.

PROBLEM IV.

Having given the projection of a line, and its vanishing point; from a given point in that projection, to cut off a segment, that shall be the projection of a given part of the original of the projection given.

Let A B (fig. 37) be the projection given, V its vanishing point, and C the point from whence is to be cut off the segment. Draw at pleasure V O and *a b c* parallel to it, and from any point O, in V O, draw O A, O B, O C, cutting *a b* in *a*, *b*, *c*. Make *c d* to *a b*, as the given part is to the original of A B, and draw O *d*, cutting A B in D. Then will C D be the segment sought.

Fig. 37.



DEMONSTRATION.

If V O be conceived as the vanishing line of any plane passing through the original of A B V; *a b c d*, being parallel to it, may be conceived as the projection of a line parallel to the picture (by *Cor.* 2. *Theor.* 5), and therefore its parts *a b* and *c d* will be in the same proportion

to one another as their originals (by *Theor.* 4). But because of the vanishing point O, the originals of O a, O b, O c, O d are parallel (by *Cor.* 1. *Theor.* 3). Wherefore the original of C D is to the original of A B, as $c\,d$ is to $a\,b$ (by *Prop.* 2. *Lib.* 6. *Elem.*) Which was to be proved.

N. B.—This proposition might have been demonstrated as the foregoing, and the foregoing may be considered as a particular case of this, viz. when the point C of this proposition coincides with one of the points A or B.

Corol.—The point D may be found by a scale and compasses, making

$$DC : DV :: dc \times AB \times CV : ab \times AV \times BV.$$

This problem is applicable; if, for example, the breadth of the visual projection of a window, or other opening or space, in the front of a building be given; and if, from any other point also given, it is required to find the breadth of the projection of a window of the same or other opening to be set off from that point. Suppose the actual breadth of a window is four feet, whose visual projection is A B, and that from another point C, it is required to cut off the projection of an opening, known to be five feet—then make $a\,b$ a scale of four feet, and one foot being added to it, it will be a scale of five feet, which is the distance from c to d—the projection of which is C D.

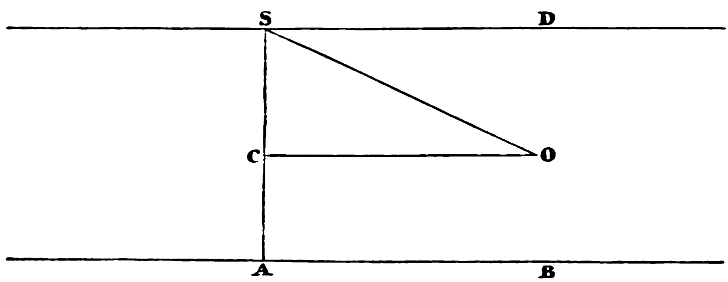
PROBLEM V.

Having given the centre and distance of the picture; to find the vanishing line (with its centre and distance) of a plane, whose intersection is given, with the angle of its inclination to the picture.

Let A B (fig. 38) be the given intersection of the

plane, and C the centre of the picture. Draw C O parallel to A B, and equal to the distance of the picture, and draw C A perpendicular to A B. Draw O S, cutting A C in S, so that the angle O S C may be equal to the inclination of the original plane to the picture. Draw S D parallel to A B. Then will S D be the vanishing line sought, S its centre, and O S its distance.

Fig. 38.



DEMONSTRATION.

Imagine the triangle O S C to be raised up on the picture, so that O C may be perpendicular to the picture. In that case O will be the point of sight, and S D, being parallel to A B, a plane passing through the line S D, and the point O will be the parallel of any original plane passing through A B, and inclined to the picture in the angle O S C. Wherefore S D is the vanishing line sought. And O S, in that supposition, being perpendicular to S D, S is the centre, and S O the distance of the vanishing line S D. Which was to be proved.

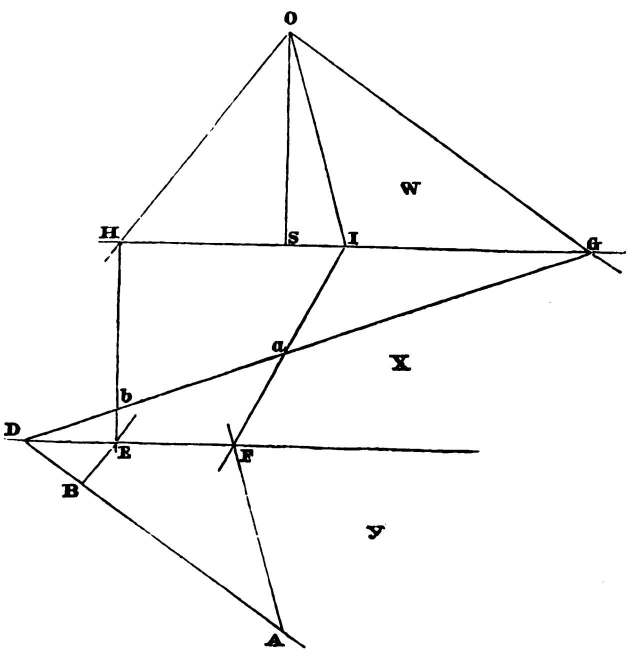
N. B.—Taking O C for radius, C S is the cotangent, and O S the co-secant of the inclination of the original plane to the picture.

PROBLEM VI.

Having given the intersection of an original plane, with its vanishing line, and the centre and distance of that vanishing line; to find the projection of any line in the original plane, having the original figures drawn out in their just proportions.

Let D F (fig. 39) be the intersection given, H G the vanishing line, and S its centre. Draw S O perpendicular to G H, and equal to the distance of the vanishing line G H, and let the space Y be the original plane, seen

Fig. 39.



on the reverse, as objects appear in a looking glass; the space W being the parallel plane in the same manner

folded down on the picture; and let A B be the original line, whose projection is sought. Let A B cut the intersection in D, and draw O G parallel to A B, cutting the vanishing line in G. Draw D G, which will be the definite projection of the right line D A, infinitely produced. Through A and B draw at pleasure A F and B E, and in the same manner find the definite projections of both A F and B E infinitely produced, which are F I and E H, cutting D G in *a* and *b*. Then will *a b* be the determinate projection of A B, *a* being the projection of the extremity A, and *b* the projection of the extremity B.

N.B.—The projection of the line A B, in fig. 39, may be found by visual rays, drawn from the points A and B, across the definite projection of the line D A, infinitely produced, as in fig. 33. In fig. 33, the original line A B was treated as being on an original plane, perpendicular to the plane of projection—that is, on a plane passing through the original line and its seat. In that case, the visual ray or distance O S was both the direct distance of the point of sight from the centre of the picture, and the direct distance of it from the centre of the vanishing line; for the centre of the picture and the centre of the vanishing line were at the same point. But in fig. 39, the line A B is treated as being on a plane, making any angle with the plane of projection. In this case, the length of the visual ray, or distance from the point of sight O to S, is the distance to the centre of the vanishing line; which distance, if the original plane is not perpendicular to the plane of projection, will be greater than the distance from the centre of the picture. For example, O S, in fig. 38, which is the distance from the point of sight O to S, the

centre of the vanishing line is longer than O C, which is the distance from the point of sight O to C, the centre of the picture. The distance O S varies with the angle which any original plane makes with the plane of projection; and the more an original plane is from being perpendicular to the plane of projection, the more will S be from C; and the greater will be the difference between O C and O S. By thus finding the vanishing line, as in fig. 33, and setting off from it the distance of the point of sight, as in fig. 39, the visual projection of any line upon a plane, making any angle with the plane of projection, is determined.

As this is the most important of all the author's new principles, a further explanation may not perhaps be unacceptable to the student. If the distance O S was equal to the distance between the point of sight and the centre of the picture, then the original plane Y, on which the line A B is placed, would, if folded up to its proper situation, be perpendicular to the plane X; but, if the distance or ray O S from the point of sight to the centre of the vanishing line be greater than the distance of the point of sight from the centre of the picture, then the plane Y, when in its proper position, would make a less angle than a right angle with the plane of projection X; and when the distance O S, from the point of sight to the centre of the vanishing line, is infinitely long, then the original plane Y, when in its proper situation, is parallel to X, the plane of projection. In all these cases the plane W, when folded up to its proper place, is parallel to the plane Y.

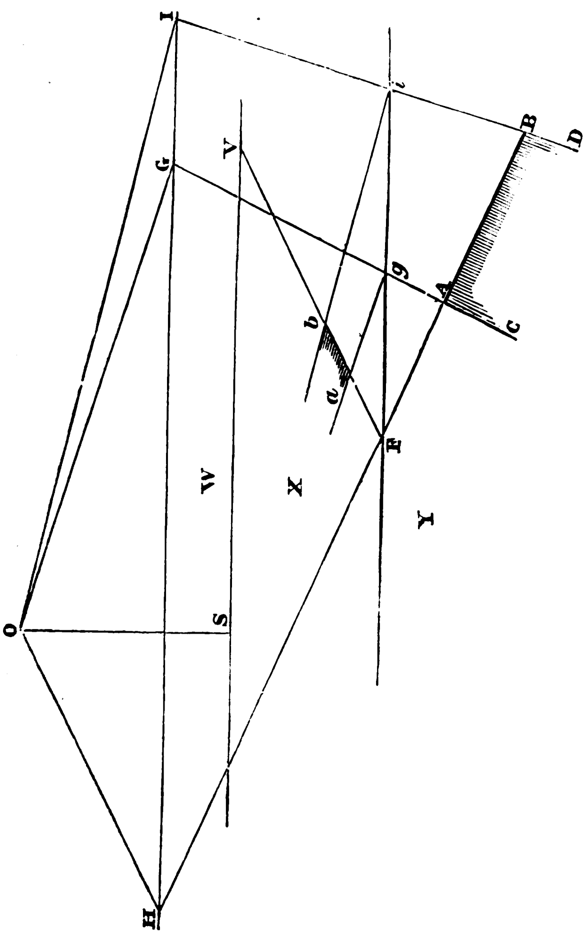
Otherwise, by the Directors.

Let F *i*, fig. 40, be the given intersection line. And let the original plane, &c. be folded on the plane of the picture; the line H I, being the directing line, and the line S V, the vanishing line; and the distance between the lines F *i* and S V being equal to the distance of O, the point of sight, from the centre of the intersection line H I. Now, to find the projection of any original line A B, continue it until it cuts the intersection line F *i* in F, and the directing line H I in H. Then H O is the director of the line A B, and if from F the line F V be drawn parallel to H O, then F V is the definite projection of F B infinitely produced. Find, in the same manner, the director O G of the line C *g*, and the director O I of the line D *i*; then the lines *g a* and *i b* being drawn from the points *g* and *i*, parallel to their respective directors, will cut off *a b*, on the line F V, which is the length of the visual projection of the original line A B.

N. B.—The figure in the former editions of this work, which contains the illustration of this, and many other problems, shews the vanishing and directing lines in the same place when folded down. As this can only be the case when the distance O S, for example, fig. 13, is equal to the distance O E, it has been thought better to vary the diagram in fig. 40, in order to shew the vanishing and directing lines when folded down in different positions. The intersection line, the director, and the angle, which the original plane makes with the plane of projection, and the position of the point of sight being known, both

the directing and vanishing lines may be determined. If the original plane produced through its intersection line pass through the point of sight, both the directing line and vanishing line would be represented by the intersection line on the plane of the picture. It has been shewn,

Fig. 40.



that the directing line and the vanishing line, when folded down, may occur in the same place; and also that the vanishing line may be between the intersection and directing line; or, as in fig. 14, the vanishing line may be further than the vanishing line from the intersection line. It may be further stated, that according to the position of the point of sight, with respect to the position and direction of an original plane, the intersection line may be between the vanishing and directing lines.

DEMONSTRATION.

Imagine the figs. 39 and 40 to be folded up, till the original plane W, the parallel of the original plane Y, the directing plane with the point of sight (which may be supposed to lie under W), and the plane of projection X, come into their proper places. Then you will find that D, in fig. 39, and F, in fig. 40, are the intersection points of their respective original lines A B; and that H, in fig. 40, is the directing point. Then O G, in fig. 39, will be found still parallel to A B; therefore G is its vanishing point, and D G the definite projection of D A infinitely produced. In *Theor*. 3, it is demonstrated that the visual projection of a right line, not parallel to the plane of projection, produced, if necessary, passes through both its intersection and vanishing points. In fig. 40, F V is still parallel to H O, the director of the original line A B; therefore *a b* is the visual projection of the line A B. In *Theor*. 5, it is demonstrated that the visual projection of a right line is parallel to its director. The points *a* and *b*, fig. 39, found by the intersection of the lines F I and E H with the line D G; and the points *a* and *b*,

in fig. 40, found by the intersection of the lines *g a* and *i b* with F V, are obvious. If the visual rays are drawn from A and B to O, the construction would be the same as fig. 33, *Prob.* 2.

N.B. 1.—The method of folding down the several planes, shewn by fig. 15, will also shew how they may be folded up on the intersection, vanishing, and directing lines, in figs. 39 and 40.

N.B. 2.—If, in fig. 40, the line D B produced, had passed through G, the line *b i* would have been parallel to *a g;* for O G in that case would have been the director to both lines, according to *Corol.* 1. *Theor.* 5. where it is stated, that the projections of lines that have the same director are parallel to each other.

PROBLEM VII.

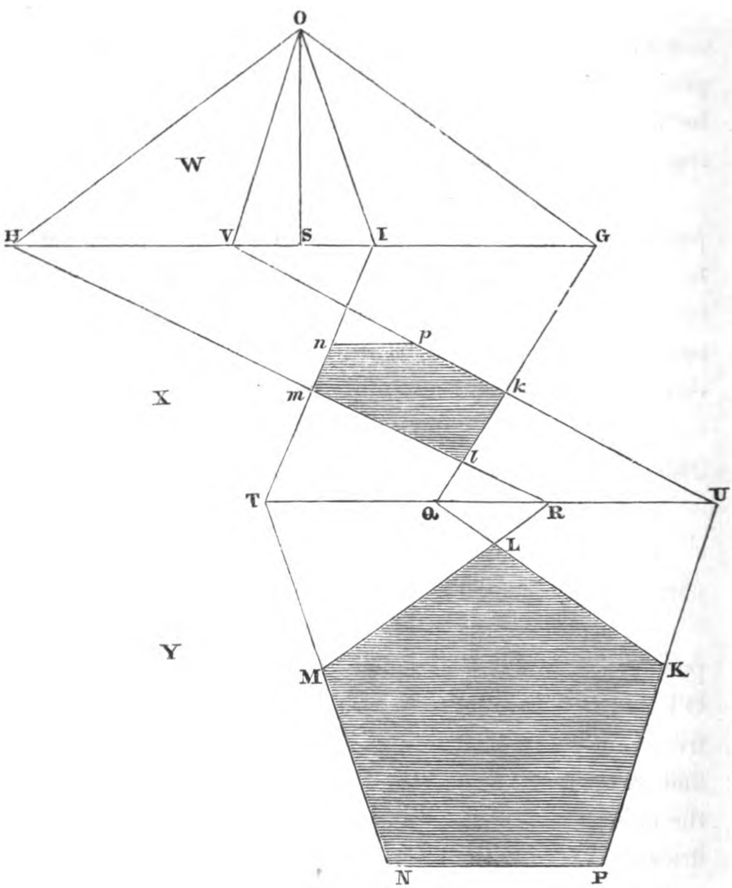
Having given the same things as in the foregoing problem; to find the projection of any figure in the original plane.

This is done by finding the projections of the several parts of the figure given, by the foregoing problem.

For example, the projection *k l m n p* (fig. 41) of the pentagon K L M N P is found thus. Drawing O G, O H, O I, and O V parallel to K L, L M, M N, and K P respectively, the points G, H, I, and V are their vanishing points; and K L, L M, M N, and F K being continued to cut the intersection in their intersections, Q, R, T, U; whence, drawing Q G, R H, T I, and U V, are got the projections *l*, *m*, and *k* of the points L and M, by their mutual intersections. Then, by drawing lines from O to P and O to N (which are not shewn), across the lines T I and U V, are got the points *n* and *p*. Or, if one of these points, suppose *n*, be thus found, the other point *p* may be

obtained by drawing $a\,p$ parallel to N P; for the latter is parallel to the intersection line T U.

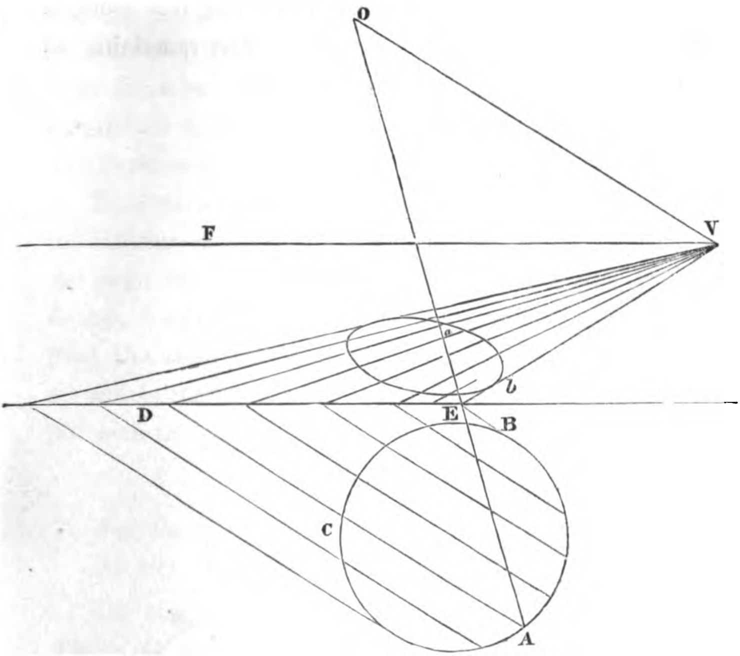
Fig. 41.



The projections of curve-lined figures are to be got by finding the projections of several of their points, and afterwards joining them neatly by hand. Thus in fig. 42, D E being the intersection, and V F the vanishing line, and

O the point of sight, and A B C an original circle, placed as in the foregoing problem, the projection *a* of any point

Fig. 42.

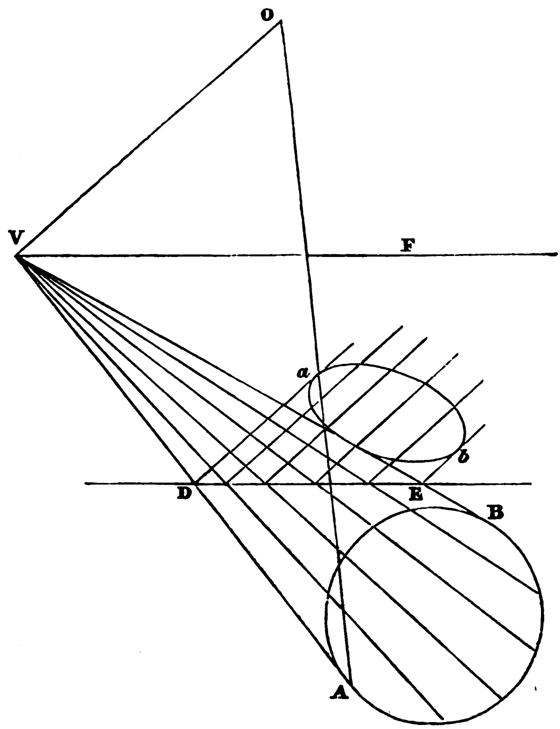


A may be found by drawing at pleasure A D, and O V parallel to it; then drawing D V and O A meeting in *a*, the point sought; D, according to the construction in fig. 33, being the intersection, and V the vanishing point of the line A D. And the several lines A D being drawn parallel to one another, the same vanishing point V may serve for them all. Another point *b*, for example, the projection of the point B, is found by drawing a visual ray from B to O, and where it crosses the line E V, is the position of the projection *b*.

This is a beautiful illustration of the fact, that all lines on any original plane that are parallel to each other have the same vanishing point.

Or, as in fig. 43, V.F being the directing line brought into the picture, as in fig. 40, and the rest remaining as

Fig. 43.



before; drawing at pleasure A D cutting D E and V F in D and V, then drawing O V and D *a* parallel to it, the projection *a* is got by drawing O A cutting D *a* in *a*. And the same directing point V being used for all the lines drawn from any other points in the original circle, as for example, the line B V from the point B, then E *b* is

parallel to V O, and also to D *a*; and so are all the lines between D *a* and E *b* drawn from the several points where the lines drawn from the circle to V cross the intersection line D E.

This clearly illustrates that the visual projection of all right lines on a plane which have the same director are parallel to each other, however they may vary in their directions in the original object.

To impress a most important fact on the memory of the student, it may here be stated, that if the distance of the point of sight O from the line V E, in the two last figures, be greater than the distance of the point of sight from the centre of the picture, then the original planes on which the circles are placed make a less angle than 90° with the planes on which they are projected.

PROBLEM VIII.

To find the projection of any figure in a plane parallel to the picture.

The projection being similar to its original (by *Cor.* 2. *Theor.* 4), this is done by making an exact copy of the original figure; making the homologous sides in the proportion explained in *Cor.* 3 of the same *Theorem.*

PROBLEM IX.

Having given the intersection of a plane, and its vanishing line, with its centre and distance of that vanishing line; to find the original of any projection given on the picture.

Every thing being disposed in fig. 41, as in problems 6 and 7; let it be proposed to find the original of the figure *k l m n p*. Having continued the projections *k l*,

l m, *m n*, and *p k*, till they cut the intersection and vanishing line in their intersections Q, R, T, and their vanishing points G, H, I, and V; then draw O G, O H, O V, and O I; next draw Q K, R M, T N, and U P, parallel to the former, respectively meeting in L, M, and K, which will be the originals of *l*, *m*, and *k*. Draw O *p* and O *n*, which will cut U K and T M in the originals P and N of *p* and *n*. Lastly, drawing N P, you have the original figure sought K L M N P.

DEMONSTRATION.

This construction is evident from *problem* 7, it being the reverse of it.

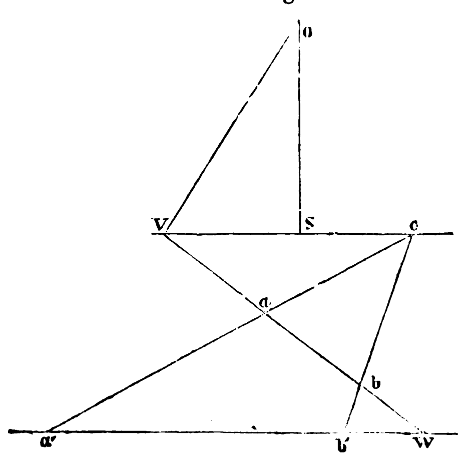
N. B.—In the same manner one may go back to the original figure by the directors, as in fig. 40.

PROBLEM X.

The same things being given; to find only the length of the original of a visual projection given.

Let *a b* (fig. 44) be the projection given; the rest

Fig. 44.



of the figure being to be understood as in the foregoing problems. Continue $a\ b$, till it cuts the vanishing line in its vanishing point V, and draw V O. In the vanishing line take V c equal to V O, and draw $c\ a$ and $c\ b$, cutting the intersection in a' and b'. Then will $a'\ b'$ be the length sought of the original of $a\ b$.

DEMONSTRATION.

Let W be the intersection of $a\ b$. V c being equal to V O the distance of the vanishing point V; and W a' being parallel to V S; the point c may be considered as the point of sight, and W $b'\ a'$ as the original line; and $c\ a'$ and $c\ b'$ as visual rays producing the projection $a\ b$.

Corol. The length $a'\ b'$ may be found by a scale and compasses, making

$$a'b' : \mathrm{V}c \text{ (or to VO)} :: ab \times \mathrm{WV} : a\mathrm{V} \times b\mathrm{V}.$$

PROBLEM XI.

Having given the vanishing line of a plane, with the centre and distance of that vanishing line, and the visual projection of a line in that plane; to find the visual projection of another line in that plane, making a given angle with the former.

Let O (fig. 39) be the point of sight placed as in the foregoing problems; G H being the vanishing line and $a\ b$ the given projection; it being required to draw a F, so that the original of the angle $b\ a$ F may be equal to a given angle. Continue $a\ b$ to its vanishing point G. Draw G O and O I, making G O I equal to the given angle, and cutting the vanishing line at I. Then draw I a F, which will be the line sought.

DEMONSTRATION.

The figure being understood as in the foregoing problems, let A B be the original of *a b*, and consequently be parallel to O G. For the same reason, A F parallel to O I is the original of *a* F, I being its vanishing point. But A B and A F being parallel to O G and O I, the angle B A F is equal to G O I, which is equal to the given angle by the construction. Wherefore *b a* F representing the angle B A F, represents that given angle. Which was to be done.

N. B.—If it had been required to make *a b* E to represent the angle A B E, the angle G O H must have been made equal to the complement of the angle A B E to two right angles.

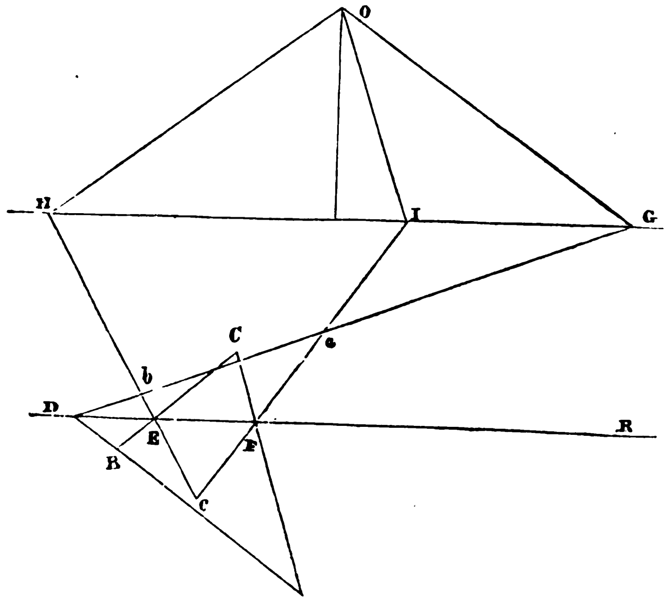
PROBLEM XII.

Having given the vanishing line of a plane, with the centre and distance of that vanishing line, and the visual projection of one side of a triangle of a given species in that plane; to find the visual projection of the whole triangle.

The projections of the sides wanting are to be found by the foregoing problem, the angles of the triangle being given. Thus having given the projection *a b* (fig. 45) of the side A B of the triangle A B C, the vanishing point I of the side *a c* is found by making the angle I O G equal to the angle C A B; and the vanishing point H of the side *b c* is found by making the angle H O G equal to the complement of the angle C B A to two right angles.

N. B.—If the vanishing point of the given line *a b* is out of reach, you may proceed thus: taking any line

Fig. 45.



D R (parallel to the vanishing line H G by *Theor*. 6.) for the intersection; by means of two lines H *b* E, I *a* F, drawn at pleasure through *b* and *a*, find the orignals A and B of the points *a* and *b* (by *Prob*. 9) and draw A B. Then on the side A B complete the original triangle, and find the projections of the sides wanting by *Prob*. 7.

PROBLEM XIII.

Having given the vanishing line of a plane, its centre and distance, and the visual projection of one side of any figure in that plane, to find the visual projection of the whole figure.

Resolve the whole figure given into triangles, by means of diagonals; and find the projections of those triangles

one after another (by *Prob.* 12), beginning with those that have the line given for one of their sides.

The same thing may be done several ways by the application of the foregoing problems, as is most convenient in every particular case. This will be best understood by a few examples.

Example 1.—Fig. 46. In this example I K is the vanishing line, S its centre, and S O its distance; and A B parallel to I K is the given projection of one side of a regular hexagon. Having drawn O G parallel to I K, the original of A B being parallel to the picture (by *Cor.* 2. *Theor.* 5.), the vanishing points H, I, K of the sides and Diagonals B C, F E and A D; A F, B E and C D; and A C, are found by *Prob.* 11, making the angles H O G 60°, I O G 120°, K O G 30°. Then, drawing A K and B H, is got the point C; drawing A H and C I is got D; drawing D E parallel to I K and A S is got E (for S is the vanishing point of A E, the original of the angle E A B being a right angle, as is G O S.) Lastly, drawing E H and A I, is got F, which completes the figure sought.

Example 2.—Fig. 47. In this figure the visual projection *m r p t q s* of the figure M R P T Q S (which is the direct projection of a regular icosaedron reposing on one of its faces) is found, having given the projection *a b* of the side A B, V X being the vanishing line, and O the point of sight reduced to the picture, as in the foregoing *Problems.* The original direct projection is described by making two concentric and parallel regular hexagons, A F B I C H and R M S Q T P, whose homologous sides are in the proportion of the parts of a line cut in extreme

and mean proportion, (see *Def.* 3. *Lib.* 6. *Elem.*) and drawing the lines, as is obvious enough in the figure.

Having continued *a b* to its vanishing point V, the vanishing points W and X of the other two sides of the triangle *a b c* are found by *Prob.* 11. Then drawing a pleasure s p parallel to V X, and drawing W *a* and W *b* cutting it in a and b, and dividing a b in k, d, e, l, in the same proportion as A B is divided in K, D, E, L, and draw k W, d W, e W, l W, are got the projections *k*, *d*, *e*, *l* of the points K, D, E, L, (by *Prob.* 3.) Then drawing *d* X, *e* W, and *e* X, are got the points *f*, and *g*; and drawing *g* V, are got the points *h* and *i*. Drawing *k* X and *l* W, is got the point *m*, and *n* (which is the projection of *N*) by the intersection of *k* X with *d* W already drawn. Then making d o and o p each equal to m d, and drawing o W, p W, are got the points *o* and *p* (by *Prob.* 3.) Then drawing *p* V is got the point *q*, by its intersection with *l* W already drawn. Drawing *o* W and *n* V is got *r*: and making m s equal to m d, and drawing s W is got *s*. Lastly, drawing *s* X cutting *r o* W already drawn, is got *t*. The rest is done by joining the points found, as is evident enough in the scheme.

Example 3.—Fig. 48. In this example is found the visual projection of the direct projection of a regular dodecaedron, having the projection of one side given, by returning to the original figure, by *Prob.* 9, and then proceeding by *Prob.* 7. I shall leave the reader to exercise himself in considering this scheme, and only observe, that the original direct projection is made by describing two concentric and parallel decagons, whose homologous

sides are the segments of a line cut in extreme and mean proportion.

Example 4.—Fig. 49. In this figure, C D being the vanishing line and O the point of sight reduced to the picture as in the foregoing problems, the visual projection N A B M L P of a regular octaedron, having given the visual projection of A B, is found as follows. Having continued A B to its vanishing point C, the vanishing point G of A K and N M, is found by *prob.* 11. making the angle C O G equal to 60 degrees. Then by *prob.* 10. taking any line *b l* parallel to C D for the intersection, and making C D equal to C O, and G H equal to G O, and drawing D A and D B cutting *b l* in *a* and *b*, *a b* is the length of the original of A B; and drawing H A cutting *b l* in a′, and making a′ *l* equal to *a b*, and drawing *l* H cutting A G in L, is got the visual projection A L, (the original of it being equal to a *l*, and consequently to the original of A B, a *l*′ and *a b* being equal.) Then dividing *a b* and a′ *l* each into three equal parts by the points *e*, *f*, *i*, *k*, and drawing *e* D, *f* D, *i* H, *k* H, are got the points E, F, I, K, (by Prob. 3.) Then drawing F G, K C, E I, are got the points M, N, P, which complete the figure.

PROBLEM XIV.

Having given the centre and distance of the picture and the vanishing line of a plane; to find the vanishing point of lines perpendicular to that plane.

Let A B, fig. 50, be the vanishing line given, and C the centre of the picture. Draw C A perpendicular to A B, and C O parallel to it and equal to the distance of the picture. Draw A O and O D perpendicular to it,

cutting C A in D, which will be the vanishing point sought.

DEMONSTRATION.

Imagine the triangle A O D to be turned up on the plane of projection, so that C O may be perpendicular to it, O being brought into the point of sight. This being done, the plane passing through the point O and the line A B will be the parallel of the original plane, and the line O D will be perpendicular to it, and consequently will be the parallel of lines perpendicular to the original plane. Wherefore D is the vanishing point of those perpendiculars.

N. B.—1. When the vanishing line A B passes through the centre of the picture, that is, when the original plane is perpendicular to the picture, the point D will be infinitely distant, the line O D being parallel to A D; and the projections of the lines perpendicular to the plane proposed will all of them be perpendicular to A B, they having to meet the line A C, which is perpendicular to it at an infinite distance, and consequently they will be parallel to one another, which they ought to be upon another account, their originals being all parallel to the picture.

N. B.—2. But when the original plane is parallel to the picture, the distance C A will be infinite, and consequently O A will be parallel to C A, and O D will coincide with O C, making the point D to fall into the centre of the picture C, agreeably to *Cor.* 2. *Theor.* 3.

N. B.—3. C D is the third proportional to A C and C O; as also is A D to A C and A O.

N. B.—4. O D is the distance of the vanishing point D.

PROBLEM XV.

Having given the centre and distance of the picture; to find the vanishing line, its centre, and distances of planes that are perpendicular to those lines that have a certain given vanishing point.

Let C, fig. 50, be the centre of the picture, and D the vanishing point given. Draw D C and C O perpendicular to it and equal to the distance of the picture. Then draw D O and O A perpendicular to it, cutting D C in A. Perpendicular to D C draw A B, which will be the vanishing line sought, A being its centre, by *Theor.* 1. and O A its distance.

The construction follows necessarily from the construction of the foregoing problem, and the observations at the end of that may be applied here.

PROBLEM XVI.

Having given the centre and distance of the picture, through a given point to draw the vanishing line of a plane that is perpendicular to another plane whose vanishing line is given, and to find the centre and distance of that vanishing line.

Let A B, fig. 50, be the vanishing line given, and C the centre of the picture, and let E be the point given. Find the vanishing point D of lines perpendicular to the original planes of A B, by *Prob.* 14. Draw D E, which will be the vanishing line sought. Draw C F cutting D E at right angles in F, and F will be the centre of the

vanishing line D E, by *Theor.* 1. Make a right angled triangle whose base is C F, and its perpendicular equal to the distance of the picture, and its hypothenuse will be the distance of the vanishing line D E, by *Cor. Th.* 1.

DEMONSTRATION.

Because the plane, whose vanishing line is sought, is perpendicular to the other plane, its vanishing line must pass through the vanishing point D of lines perpendicular to that other plane, because some of those lines are in the plane sought. Therefore D E is the vanishing line sought. The rest needs no demonstration.

Corol. 1.—If F C be continued till it cuts the vanishing line given in B, B will be the vanishing point of lines perpendicular to the original plane of the vanishing line D E. For that vanishing point is in the line F C by the construction of *Prob.* 14, and it is in the vanishing line given by the demonstration of the present problem.

Corol. 2.—And, therefore, if the vanishing lines A B and D E meet in G, the points B, D, and G, will be the vanishing points of the three legs of the solid angle of a cube, which are perpendicular to one another. And, drawing D B—B G, G D, and D B will be the vanishing lines of the three planes that contain that solid angle.

Corol. 3.—The distance of the vanishing line D G is equal to the line F P, the point P being the intersection of the line F C with a circle described on the diameter D G.

PROBLEM XVII.

Having given the centre and distance of the picture, and the vanishing point of the common intersection of two

planes that are inclined to one another in a given angle, and the vanishing line of one of them; to find the vanishing line of the other of them.

Let C, fig. 50, be the centre of the picture, B G the given vanishing line of one of the planes, and B the vanishing point of their common intersection, and H the angle of their inclination to one another. Find the vanishing line G D of planes perpendicular to the lines whose vanishing point is B (by *Prob.* 15.) Let that vanishing line cut the vanishing line given in G. In G D find the vanishing point E of lines making the given angle H with the lines whose vanishing point is G (by *Prob.* 11), that is, in B C F perpendicular G F D take F P equal to the distance of the vanishing line G D (found by *Prob.* 15), and draw P G and P E making the angle E P G equal to H. Draw B E which will be the vanishing line sought.

DEMONSTRATION.

Imagine the triangle G P E to be turned up on the line G E so that the point P may be in the point of sight perpendicular over the centre of the picture C. In that case the planes B P G, G P D, and E P B, will be the parallels of three original planes whose vanishing lines are B G, G D, and E B; that original plane, whose vanishing line is G D being perpendicular to the other two by the construction, because it is perpendicular to their common intersection, whose vanishing point is B. Therefore the original planes whose vanishing lines are B G and B E are inclined to one another in the angle E P G, that is, in the angle H, (for the inclination of two planes is always measured in a plane perpendicular to

their common intersection. Therefore B G being the vanishing line given, B E is the vanishing line sought.

N. B.—The centre of the vanishing line B E is found by drawing a line perpendicular to it from C (by *Theor.* 1), and then its distance is found as was found the distance P F in *Prob.* 16.

PROBLEM XVIII.

Having given the centre and distance of the picture and the vanishing line of one face of any solid figure proposed, and the visual projection of one line in that face; to find the visual projection of the whole figure.

By means of the visual projection given, find the direct projection of the figure proposed on the plane of that face whose vanishing line is given by *Prob.* 13. Then by *Prob.* 16 find the vanishing line of a plane perpendicular thereto, and describe the direct projection of the solid by the help of the lines already given in the first direct projection, by *Prob.* 13. Lastly, by the intersections of the projections of perpendiculars of the two direct projections which are on planes perpendicular to each other, will be found the several points of the projection required.

Otherwise—Having found the direct projection of the face whose vanishing line is given, by means of the visual projection of the line given, find the vanishing lines of the adjacent faces, by *Prob.* 17, and describe their projections by the help of the lines given in the projection of the first face, and so on till the whole visual projection sought is completed.

This is a general description of the method to be used in putting any figures proposed into *perspective;* but in the

practice of particular cases several expedients may be used in the various application of the foregoing problems. But all this will be best understood by examples.

Example 5.—Fig. 51. In this figure is found the visual projection of a regular dodecaedron, having given the visual projection A B of one side parallel to the picture by means of direct projections on two planes which are perpendicular to each other; F G, parallel to A B, being the given vanishing line of the face A B C D E, F its centre, H the centre of the picture, and H O its distance. To avoid confusion of lines, the two direct projections are removed from the space taken up by the visual projection sought in the following manner. For one direct projection, draw at pleasure *a b* parallel to A B, and at a sufficient distance from it, and then drawing I A and I B cutting *a b* in *a* and *b*, the line A B is transferred to *a b;* I being the vanishing point of lines perpendicular to the face A B C D E, whose vanishing line is F G, found by *Prob.* 14. This being done, the whole direct projection is described on the line *a b*, by *Prob.* 13, as it is described in *Ex.* 3.

For the other direct projection; I H F passing through the centre of the picture is taken for its vanishing line as being most convenient; the direct projection in this case being the most simple, and the projections of lines perpendicular to it being all of them perpendicular to F I, by Note 1, *Prob.* 14. Having drawn G A from the vanishing point G of the line *ae* in the direct projection, and *e* I cutting it in E, is got the visual projection A E. Then drawing A *a* and E *e* both parallel to G F, (and consequently representing perpendiculars to the other direct

projection, as is already said) and *ae* at pleasure passing through F, and cutting them in *a* and *e*, is got *ae*; by the means of which the whole of the second direct projection is described by *Prob.* 13.

Having thus got the two direct projections, any point K of the visual projection sought is got by drawing *k* K parallel to F G and *k* I, from the corresponding points *k* and *k*, meeting each other in K.

For understanding the projection of a cube, which appears in this figure, after what has been already said, it is sufficient to inform the reader, that two of its vanishing lines are F G, and F I, and the third is a line passing through I parallel to F G.

As to the shadows, which are supposed to be cast by the sun on the plane of the face A B C D E of the dodecaedron; the shadow *uv* of any line V *v* is found as follows. S is the given vanishing point of all the rays of light, which being supposed to come from the sun, are to be considered as parallel. Therefore I S passing through the vanishing point I of the line V *v* and the vanishing point S of the rays, is the vanishing line of the plane made by all the rays passing through the line V *v*, and projecting the shadow *v u*. And I S cutting the vanishing line F G of the plane the shadow is cast on in *s*, *s* is the vanishing point of the shadow *v u*, which is the intersection of the plane of the shade, whose vanishing line is I *s*, and the plane the shadow is cast on (*s* being the vanishing point of that intersection by *Cor.* 2. *Theor.* 7).

Having therefore drawn *v s*, and V S cutting it in *u*, *v u* is the shadow of the line *v* V.

N.B.—In the original of the direct projection in the

present figure, the points *e, m, s, p,* are the angles of a square. The lines *o n, a l, q r,* being parallel to *e m,* and *o q, n r* being parallel to *e p,* and equal to *a l, t k, o n, q r,* are equal, and *o n, e m, a l,* are in the continued geometrical proportion of the smaller segment to the greater, of a line cut in extreme and mean proportion.

Several lines mentioned in this scheme are not actually drawn, to avoid confusion.

Example 6.—Fig. 52. In this scheme the vanishing line of the ground, which the piece of building, &c. stands upon, is A I C K B, passing through the centre of the picture C, the distance of the picture being equal to C O. B G and A H are the vanishing lines of the upright planes, *a b d e,* D E N, &c. and *a b c,* D F *n m,* &c. G and H being the vanishing points of the lines *b e,* and *m n,* which touch the upper corners of the two flights of steps, and consequently A G and B H are the vanishing lines of the planes that touch the upper or under edges of the steps.

The given side *h r* of the base of the regular tetraedron is parallel to the vanishing line A B. The point *u,* which is the projection of the centre of the base *h r p,* and the seat of the vertex *o,* is found by drawing *p q* parallel to A B, and I *r* cutting it in *q,* and then drawing C *p* and *h q* meeting in *u.* C L perpendicular to A B is the vanishing line of a plane perpendicular to the base *h r p* standing on *u p,* and passing through the line *p o,* whose vanishing point is L, found (by *Prob.* 11.) by making C Q equal to the distance of the picture, and the angle L Q C equal to the original of the angle *u p o.* L K is the vanishing line of the face *u p o,* by the help of which

that face is described, having given *o p r* (by *Prob.* 12), as the face *h r p* was described on *h r*. The regular octaedron, and icosaedron, are described by their direct projections, which method is sufficiently explained in the foregoing example. I will only inform the reader, that in the direct projection A B C D E F G H of the icosaedron, the lines C D, A F, G H are equal, as also are A C, F D, B E; and A F is to B E, as the smaller segment is to the greater of a line cut in extreme and mean proportion.

The light is supposed to come from the sun, and the rays are parallel to the picture and to the line A M; so that the shadow P of any point D, is found by drawing N P through its seat parallel to A B, and D P parallel to A M cutting it in P. For the shadow of the tetraedron, which is turned up on the end of the steps; having in the same manner found *s*, (which would be the shadow of *o* on the ground, if the steps were away), drawing *s h* and *s p*, are got the shadows *h o* and *p o*. Let *s u* and *s p* cut the lower edge of the steps in *y* and *x*; then drawing *x t* perpendicular to A C, and cutting *o s* in *t*, is got the shadow *t* of the vertex *o* on the end of the steps, and drawing *x t* is got that part of the shadow of *o p*, which falls on that end. And in the same manner is got the shadow of *o h*. All the rays being parallel to A M, and A being the vanishing point of D F, A M is the vanishing line of the plane made by the rays which pass through the line D F, and B M being the vanishing line of the wall the shadow F *f* is cast on; M is the vanishing point of the common intersection of those two planes, that is, of the shadow F *f* of the line D F.

Example 7.—Fig. 53. In this scheme A C B pass-

ing through the centre of the picture C, is the vanishing line of the ground; A is the vanishing point of the edge E F, which the beam rests upon, and the other edges parallel to it; D is the vanishing point of the edge G H, &c. and D B perpendicular to A B, is the vanishing line of the plane E G H; P and Q are the vanishing points of the sides M K, and M N, of the regular tetraedron, and consequently P Q is the vanishing line of the triangle K N M; L is the light, and I its seat on the ground. Having B E *g*, that is the intersection of the upright plane E G H with the ground, and consequently *g*, where H G meets it, is the intersection of the edge H G with the ground. B E cuts the edge S R of the wall S R *h* in R, and consequently R *h* perpendicular to A B is the intersection of the wall with the plane E G H, and consequently *h*, where R *h* and G H meet, is the intersection of the line G H with the wall. D L *l* is the projection of a line parallel to the original of G H (because of the vanishing point D) and B *l* is its seat, wherefore *l* is its intersection with the ground.

Now the originals of L *l* and H G being parallel, they are in the same plane, viz. in the plane which makes the shadow of H G; but *l g* represents the intersection of that plane with the ground; wherefore *l g* continued is part of the shadow, and drawing L G cutting it in *g*, *g* is the shadow of G, and *g* S is that part of the shadow of H G which is on the ground. Then drawing S *h* and L H cutting it in *h*, *h* S is the other part of that shadow against the wall.

Having drawn *g p* and T D both parallel to A B, D *g*, and *g p*, are the projections of two lines in a plane, whose

vanishing line is D T; and T is the vanishing point of the common intersection of that plane with the plane of the triangle K M N (by *Corol.* 2. *Theor.* 7.) P Q T being its vanishing line. Therefore drawing T*p* cutting *g* D in V, V is the intersection of the line *g* G H D, with the plane of that triangle K M N. Then *l g* cutting M K in *r*, drawing *r t* V, *r t* is that part of the shadow of G H, which falls on the triangle K M N.

Having thus explained the manner of finding the shadow of G H, the rest needs no explanation.

Example 8.—Fig. 54. In this scheme C is the centre of the picture; and C A the vanishing line of the ground, and of the surface of the water which gives the reflections; and S is the vanishing point of the rays of light, which are supposed to come from the sun.

The shadow of the perpendicular line B D is found thus: S A drawn perpendicular to C A gives the vanishing point of A of the shadow on the ground B*f*. Then in the circumference of the base of the cylinder (which is parallel to the picture, its axis being perpendicular to it, and consequently having the vanishing point C), taking any point E, and finding its seat on the ground *e*, drawing C E, and C *e* cutting B A in *f*, and then drawing *f* P perpendicular to C A and cutting C E in P, is got one point P of the shadow on the surface of the cylinder. And in the same manner are got all the other points of that shadow. To prove this, the reader need only consider, that the original of *e* E P *f e* is an upright plane cutting the cylinder in E P, and *f* P is the shadow of B D on that plane. Wherefore P is the point where that shadow falls on the surface of the cylinder. Any point Q

of the shadow of the circumference of the inward cylinder on its surface, is found thus. Having drawn C S, parallel to it is drawn at pleasure G H cutting that circumference in G and H. Then drawing G S and C H meeting in Q, Q is the point sought. For C being the vanishing point of the axis of the cylinder, as well as of C H, H Q is in the surface of the cylinder; and G H being parallel to C S, is the projection of a line parallel to the picture, in a plane whose vanishing line is C S (and its original being parallel to the picture, is in the base of the cylinder, which is parallel to the picture). Therefore the originals of H G, H C, and G S being in the same plane, Q is the projection of the point where the ray of light, whose projection is G S, cuts the surface of the cylinder; that is, it is the projection of the shadow of the original of the point G in the circumference of the base, on the inward surface of the cylinder.

The point *b* being the seat of the point D on the surface of the water, the reflection *d* of the point D is found by continuing the perpendicular D *b* till *b d* is equal to *b* D. This is evident, because the known law of reflections is that the reflections of all objects appear to be as much on one side of the reflecting plane, as the real objects are on the other side of it. In A S, making A *s* equal to A S, any point *q* in the shadow on the surface of the inward cylinder in the reflection, is found in the same manner as Q in the real figure, using the point *s* instead of S.

The shadow of the cylinder on the surface of the cone, is found by such another expedient, as the shadow of the line B D on the surface of the cylinder.

Example 9.—Fig. 55. In this scheme every thing else being easily understood by what has been already explained, I shall only shew. the manner how the reflection is found in the looking-glass of the picture on the eazle.

A is the centre of the picture, and A B the vanishing line of the ground; the distance of the picture being equal to A B. A C is the vanishing line of the picture on the eazle, and C D the vanishing line of the looking-glass.

Through *a*, where the edge *b a* of the leg of the table cuts the surface of it, drawing *a e,* and through *b* drawing *b d,* both parallel to A B; *b d* cutting the intersection *c d* of the surface of the picture on the eazle with the ground in *d;* and then drawing *d e* parallel to A C, and cutting *a e* in *e*, and then drawing A*e*, is got the projection A*e* of the common intersection of the surface on the table, and of the picture on the eazle. For *a e* being parallel to A B, is the projection of a line on the surface of the table parallel to the picture; and for the same reason *b d* is the projection of a line on the ground, and *d e* is the projection of a line in the plane of the picture on the eazle, both of them parallel to the picture; *a b* is also the projection of a line parallel to the picture. Therefore *a b d e* is the projection of a trapezium parallel to the picture, whose angle *e* is in the common intersection of the surface on the table, and of the picture on the eazle. But A, being the common intersection of the vanishing lines of those two planes, is the vanishing point of their common intersection, and therefore *e*A is the projection of that intersection by *Cor.* 2. *Theor.* 7. For the same reason *o* being the projection of the point where the sur-

face of the glass touches the table, and E being the common intersection of the vanishing lines A B and C D, *o* E is the projection of the common intersection of the surface of the table and the surface of the glass. Therefore *f* where *o* E and *e* A meet, is the projection of the point where the three planes meet, of the surface of the table, the glass, and the picture on the eazle. Therefore drawing *f* C, it is the projection of the common intersection of the picture on the eazle and the looking-glass.

Having found the vanishing point P of lines perpendicular to the plane of the looking-glass, whose vanishing line is C D, by *Prob.* 14, drawing P A through the vanishing point A of the line G H, and cutting C D in D, D is the vanishing point of the seat of G H on the plane of the glass. Therefore G H cutting C *f* in *i*, D *i* is the projection of that seat. Then drawing G P, cutting D *i* in *k*, *k* is the seat of the point G on the glass. Wherefore in G P making *k g* to represent a line equal to that represented by G *k* (by *Prob.* 3.) *g* is the projection of the reflection of G, and *g i* is the reflection of G *i*, and drawing P H cutting *g i* in *h*, *g h* is the reflection of G H. And in the same manner may be found any other lines in the reflection.

The reflection of the picture on the eazle may also be described by its vanishing line, in the same manner as the projection of the picture itself was described; for in P A D making *a* D to represent a line equal to that represented by A D, *a* is the vanishing point of the reflected line *g h*, and C *a* is the vanishing line of the reflected picture on the eazle.

PROBLEM XIX.

Having given the visual projection of a line divided, and its vanishing point; to find the proportion of the parts of the original.

Let A B (fig. 36) be the given projection, divided in C, and V its vanishing point. Draw at pleasure V O, and $a\,b$ parallel to it, and from any point O in the line O V draw O A, O B, O C, cutting $a\,b$ in a, b, and c. Then will the original of A C be to the original of C B, as $a\,c$ is to $c\,b$.

Corol. $ac : cb :: AC \times BV : BC \times AV$.

PROBLEM XX.

Having given the visual projection of a line divided into two parts, and the proportion of the original; to find its vanishing point.

Let A B (fig. 36) be the projection given, divided in C. Through C draw at pleasure $a\,C\,b$, and in it make $a\,C$ to $C\,b$ as the original of A C is to the original of C B, and draw $a\,A$ and $b\,B$ meeting in O. Parallel to ab draw O V cutting A B in V, which will be the vanishing point sought.

Corol. $BV : BA :: Ca \times CB : Cb \times AC - Ca \times CB$.

These two last problems, with their corollaries, follow easily from *Prob.* 3 and its *Cor.*

PROBLEM XXI.

Having given the visual projection of a triangle, with its vanishing line, its centre and distance; to find the species of the original triangle.

Let $a\,b\,c$ (fig. 45) be the projection given, H G its

vanishing line, and S its centre, and S O perpendicular to H G, and equal to its distance. Having continued the sides of the projection given, till they cut the vanishing line in their vanishing points G, H, I, draw G O, O H, and O I, and the originals of the angles *b a c*, *a b* H, *a c b*, will be equal to G O I, G O H, I O H, respectively (by *Prob.* 11). Whence the species of the original triangle is given.

PROBLEM XXII.

Having given the visual projection of a triangle of a given species, and its vanishing line; to find the centre and distance of that vanishing line.

Let A B C (fig. 56) be the given projection, and F D its vanishing line. Continue the sides of the projection till they cut the vanishing line in their vanishing points D, E, F. Bisect D E and E F in G and H, and draw G I and H K perpendicular to F D, making G I to G E as radius is to the tangent of the angle represented by B A C, and K H to E H as radius is to the tangent of the angle represented by B C A; so that E I G and F K H may be equal to those angles. With the centres I and K and the radii I E and K E, describe two circles cutting each other in O, and draw O S cutting F D at right angles in S. Then will S be the centre, and S O the distance sought.

DEMONSTRATION.

Supposing S to be the centre, and S O the distance of the vanishing line F D, the originals of the angles B A C and B C A will be equal to D O E and E O F (by *Prob.* 11). But, by the nature of the circle, D O E and

F O E are equal to G I E and H K E, which, by the construction, are equal to the angles that ought to be represented by B A C and B C A. Therefore S is the centre, and S O the distance sought.

PROBLEM XXIII.

Having given the visual projection of a trapezium of a given species; to find its vanishing line, and the centre and distance of that vanishing line.

Let *a b c d* (fig. 56) be the projection given. Draw the diagonals *a c*, *b d*, meeting in *e*, and, by the proportions of the originals of *a e*, *e c*, and *b e*, *e d*, find the vanishing points E and F of the lines *a c* and *b d* (by *Prob.* 19). Draw F E, which will be the vanishing line sought. Then, by the given species of the original of the triangle *a b e*, find the eentre S and distance S O (by *Prob.* 21).

PROBLEM XXIV.

Having given the visual projection of a right angled parallelopiped; to find the centre and distance of the picture, and the species of the original figure.

Let A B C D E F G (fig. 57) be the projection given. Continue the projections of the parallel sides till they meet in their vanishing points H, I, K, and draw H I, H K, I K, which will be the vanishing lines of the several faces of the figure sought, containing a solid right angle. Draw K L perpendicular to H I, and H M perpendicular to K I, meeting in S; which will be the centre of the picture (by *Cor.* 2, *Prob.* 16). Then, on the diameter L K describe a circle, and draw S O perpendicular to L K, cutting it in O, and O S will be the distance of the picture

(by *Prob.* 14), L O K being a right angle upon account of the circle. Lastly, find the distance of the vanishing lines K I and I H (by *Cor.* 3, *Prob.* 16, M and L being their centres, by *Theor.* 1), and then find the species of the originals of the faces D A F E and D A B C (by *Prob.* 20).

Corol.—When the vanishing line of one of the faces (suppose I H) passes through the centre of the picture, the vanishing point K of the sides perpendicular to it, will be at an infinite distance; by which means the situation of L K will be indetermined. So that the species of the face A B C D may be taken at pleasure, and then the centre and distance of the picture may be found by *Prob.* 20. And in this case, if it were only required that the projection proposed should represent a right angled parallelopiped in general, the place of the point of sight might be any where in the circumference of a circle described on the diameter H I, and in a plane perpendicular to the picture. This I leave as a hint that may be useful to the painters of scenes in theatres.

Description of a method by which the representations of figures may be drawn on any surface, however irregular it may be.

From what has been said in this book to explain the principles of painting, especially in the definitions, it is evident that the sense of the second Theorem may be extended to any surface that figures are painted on, whether concave or convex, or however irregular. So that if the surface of the picture A B C (fig. 13) be of any

form whatsoever, the projection *fg* of the original line F G, will still be the intersection of the picture with the plane of the triangle F G O. But the parallel O V is in that plane (*Theor.* 3). Hence it follows, that if the flame of a lamp be so placed as to cast the shadow of O V on any point B of the line F G, it will cover the whole line F G, and at the same time cover the line V*fg* B on the picture, which is the projection of F G; all the rays coming from the lamp, and passing by the line O V, in that case making a plane, which coincides with the plane of the triangle O F G. The same thing will happen if a person should place his eye so as to make any point of the line F G to appear to be covered by the line O V; in that case O V will seem to cover all the projection V*fg*. So that if a lamp be so placed as to make the shadow of O V to pass through any one point of the projection V*g*, it will coincide with the whole; and if a person places himself so as to make O V to seem to cover any one point of the same projection, it will seem to cover the whole. Hence I imagine, that the following method may be of use for drawing the projections of any figures on any surface—suppose on the walls and cupolas of churches, the walls and ceilings of great rooms, the scenes of theatres, &c.

Choose some principal line in the design to be drawn, and having by some proper method found the projections of its extreme points, through the point of sight pass a thread parallel to the original line, and cast the shadow of it on those two points already found, and that shadow marked with a crayon will be the projection of that principal line; or, if in any particular case it happens to be

more convenient, place your eye so as to make that parallel thread to seem to cover those two points already marked, and instruct an assistant to mark out the projection wanted. Suppose, for example, *p h* (fig. 52) to be that principal projection. Then, to find the projection (for example *o*), of any other point in the figure to be described, imagine that point to be the vertex of a triangle, whose base is the original of the projection already found, and place the thread (passing still through the point of sight) in a parallel situation to the original of one of the legs of that triangle, and cast its shadow on the proper extremity of the projection given, and mark it as before, and you will have the indefinite projection of that leg (suppose it to be *h o*). Do the same by the other leg, and, by the intersection of those projections (suppose of *h o* and *p o*), you will have the projection of the point sought. And by this method may be found the projections of any figures whatsoever. I shall not enlarge upon this method, not having had an opportunity of putting it in practice; for which reason Ionly propose it as a hint, which I leave to be further considered of by the curious.

A NEW THEORY

FOR

MIXING COLOURS,

TAKEN FROM

SIR ISAAC NEWTON'S OPTICS.

Although my design was only to treat of *Linear Perspective*, and not to discourse of all the parts of *Painting*, yet as this book will be most useful to those who practise that art, I thought it would not be improper, nor unentertaining to the readers, if I took this occasion to publish some thoughts I have had concerning the mixture of colours, which occurred to me upon considering Sir Isaac Newton's Theory of Light and Colours, in his most excellent Treatise on *Optics*.

In colours these two things are to be considered: the hue (which is properly what may be called the colour), and the strength of light and shadow. For as different colours, suppose red and green, may have the same strength of light; so two things, that are one of them much darker than the other, may still have the same hue, as a light blue and a dark blue.

With respect to the hue, these two things are to be considered: 1st, the species of colour, and, 2d, the perfection and imperfection of colour under the same species. Colours differ in species, as blue and red; and colours of the same species differ in degree of perfection, as the red of a field-poppy is much more perfect than the red of a brick. This quality of perfection and imperfection in the colours, by the painters is expressed by the terms bright, or clean; or simple, and broken—which is taken from their method of making the imperfect colours, by the mixture of other colours, which is called breaking the colours. With respect to this quality of colours, Sir Isaac Newton, in the book already mentioned, shews, that every ray of light has its proper colour, which it always carries with it, and never loses, in whatever manner it happens to be reflected or refracted. These natural colours of the rays are the bright simple colours; and the natural order of them, as they appear when they are separated by the refraction of a prism, is—red, orange, yellow, green, blue, indigo, violet. All the less perfect or broken colours are made by the composition and mixture of these simple colours; as yellow rays mixed with blue rays make a green, but not so perfect as the simple natural rays that are green; and red and yellow rays make an orange colour, but not so perfect as the natural orange-coloured rays. And by a just proportion of all the natural rays together is produced whiteness, which is indifferent to all the simple colours, and cannot be said to incline more to one colour than to another. By white I mean any colour between the lightest white and the darkest black; for as we are not now considering the degrees of light

and shade, all the colours from black to white are to be considered as of the same hue.

According to the observation of the nature of whiteness, it appears that the broken colours are a medium between the simple colours and white; and the more broken a colour is, the nearer it is to white, and the further it is from white, the more simple it is.

Having thus explained the nature of the colours; and the effect of their mixture, in order to find exactly what colour will be produced by the mixture of any colours given, Sir Isaac disposes the colours in the following manner. Let there be a circle made A D F A (fig. 58), and let the circumference be divided into seven parts A B, B C, C D, D E, E F, F G, G A, in the same proportion to one another as the fractions, $\frac{1}{9}$, $\frac{1}{16}$, $\frac{1}{10}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{16}$, $\frac{1}{9}$, which are the proportions of the musical notes, *sol, a, fa, sol, la, mi, fa, sol.* Between A and B place all he kinds of red, from B to C place all the kinds of orange, from C to D place all the kinds of yellow, from D to E place all the kinds of green, from E to F place all the kinds of blue, from F to G place all the kinds of indigo, and from G to A place all the kinds of violet. Having thus disposed the simple colours, the centre of the circle O will be the place of white. And between the centre and the circumference are the places of all the broken compounded colours, those nearest the centre being the most compounded, and those furthest from it being the least compounded. As in the line O 1, all the colours at 1, 2, 3, 4, are of the same species—that is, green inclining towards blue; but the colour at 1 is the simple natural colour; that at 2 is something compounded or

broken; that at 3 is more broken; and that at 4 is still more broken.

The colours being thus disposed, to know what colour results from the mixture of any colours given, find the centre of gravity of the places of the colours given, and that will shew the character of the compound. For example, suppose I would know what colour would result from the mixture of two parts of the simple yellow at P, with three parts of the simple blue at Q: I find the centre of gravity 3 of the points P and Q; that is, I draw P Q, and having divided it into five parts (which is the sum of three and two), I take the point 3 three parts from P, because there are three parts of blue, and two parts from Q, because there are two parts of the colour at P. Then drawing O 3 cutting the circumference in 1, by the place of the point 1 (which is between D and E, but nearer to E), I find the mixture is a green, inclining towards blue; but because 3 is near the middle between the centre and the circumference, the colour is pretty much broken. To make the same thing more clear by another example; suppose I would know what would result from a mixture of two parts yellow at P, three parts blue at Q, and five parts red at R. First I find the place 3 of the mixture of the yellow and the blue, as before. Then drawing the line 3 R, because there are five parts of the colour at 3, and five parts of colour at R, I divide it into ten parts, and take the point *r* five parts distant from R. By this means *r* is the centre of gravity of the three colours at P, Q, and R, and is consequently the place of the mixture; which, by drawing O *r* cutting the circumference in *s*, I find to be an orange a little inclining

towards red, and because r is much nearer the centre than the circumference, the colour is very much broken. And thus one may proceed in other cases.

Again, having given the place of any compound colour, one may find what colours may be mixed to compound it. Thus, having given the colour at 3, drawing any line P 3 Q through 3, the colour proposed may be made by a mixture of the colours in P and Q, taking such a proportion of them as is expressed by the lines 3 P and 3 Q; that is, taking of the colour P as much as is in proportion to 3 Q, and as much of the colour Q as is in proportion to 3 P. Or, having drawn O 3 passing through the points 1, 2, 4, the same colour may be produced by mixing the colours in 2 and 4 in proportion to the lines 4.3 and 2.3; or it may be produced by breaking the simple colour at 1 with white (which is at O) in the proportion of the lines 3.1 and 3 O. And thus in other cases.

The proportions hitherto mentioned of the colours to be used in the mixtures, relate to the quantity of the rays of light, and not to the materials of which artificial colours are made. Wherefore, if several artificial colours were to be mixed according to these rules, and some of them are darker than others, there must be a greater proportion used of the darker materials, to produce the hue proposed, because they reflect fewer rays of light in proportion to their quantities; and a smaller proportion must be used of the lighter materials, because they reflect a greater quantity of light.

If the nature of the material colours which are used in painting was so perfectly known as that one could

tell exactly what species of colour, how perfect, and what degree of light and shade each material has with respect to its quantity, by these rules one might exactly produce any colour proposed, by mixing the several materials in their just proportions. But though these particulars cannot be known with sufficient exactness for this purpose, besides the tediousness that would be in practice, to measure the colours according to their exact proportions; yet the knowledge of this theory may be of great use in painting. Suppose, for example, I had a palette provided with the several colours at *a, b, c, d, e:* suppose, for instance, at *a* carmine; at *b* orpiment; at *c* pink; at *d* ultramarine; at *e* smalts; and I had occasion to make a broken green, such as I judge should be placed at *x*. Looking round the point *x*, I see that it does not lie a great deal out of a line drawn through *c* and *d;* therefore I conclude, that mixing the colours *c* and *d* will come very near what I want. But because *x* is nearer to the centre O than the line *c d*, having brought my tint as near as I can to what I want, suppose to *z*, I look from *z* cross *x* for some colour opposite to *z*, to break the tint with, and I find the nearest to be *a*; therefore by mixing the colour *a* I bring the composition to the tint I have occasion for. If the colour *a* carries the tint too much towards the line O D, I put a little more of the colour *d*, which brings it into the right place. Or, having got the tint *z*, I might have broken it with white, whose place is at the centre O. Or, putting a greater proportion of the colour *d* instead of *a*, I may afterwards break the tint by means of the colour *b*. And, in the same manner, by only inspecting this scheme, one may see in what man-

ner to make any tints whatsoever, that can be produced by the colours that one uses. Thus, it may be seen that red and yellow make a broken orange colour, which may still be more broken by adding blue, or indigo, or violet, which are to be taken one or other, as one would have the tint inclined more to the yellow or to the red; blue bringing it toward the yellow and breaking it much; and violet carrying it towards the red, and not breaking it so much.

From these principles one may see the reason why the materials of the brightest and simplest colours are the most valuable, and of them why the lightest are most to be esteemed. The simplest colours are the most valuable, because they cannot be produced by mixture; for mixture always breaks the colours. Suppose *a, b, c, d, e,* to be all the colours you have, then drawing lines to join the points *a, b, c, d, e,* all the tints that can be produced by those colours will have their places within the area of the polygon *a b c d e.* That the lighter colours are more valuable than the dark ones is, because black does not break the colours so much as white; so that it is easier to make the clean dark tints with light colours and black, than to make the bright ones light with dark colours and white. For by what has been shewn, white breaks the colours very much, but black being nothing but the absence of light, only darkens the colours. Though, upon account of the imperfections of the materials that are in use, black does also break the colours something, because there is no material so perfectly black as to have no colour at all, as one may see by the best blacks having lights and shades. There will be other exceptions also to be made in the application of these observations to

practice, upon account of the particular qualities of the materials some colours are made of. If all the colours were as dry powders, which have no effect upon one another, when mixed, these observations would exactly take place in the mixing of them. But some colours are of such a nature, that they produce a very different effect upon their mixture to what one would expect from these principles. So that it is possible there may be some dark materials, which, when diluted with white, may produce cleaner and less compounded colours than they gave when single; as some colours do very well to glaze with, which do not look well laid on in a body. But these properties of particular materials I leave to be considered by the practitioners in this art.

THE END.

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Plate I.

Fig. 13.

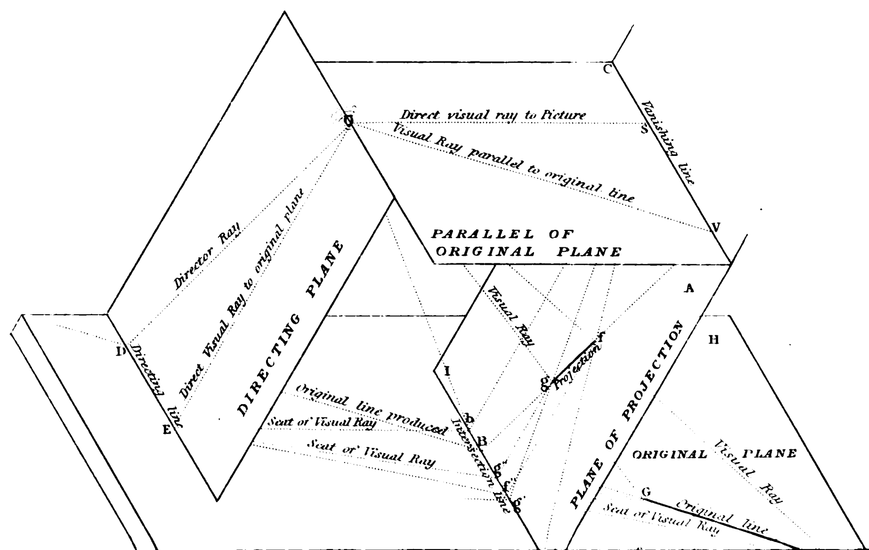

Plate II.

Fig. 14.

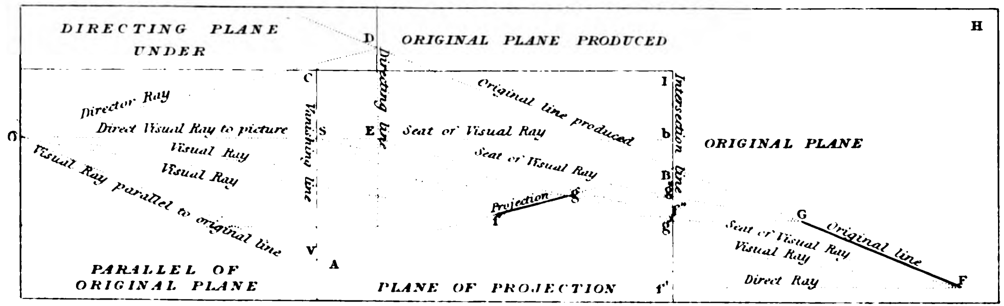

Fig. 15.

Plate III.

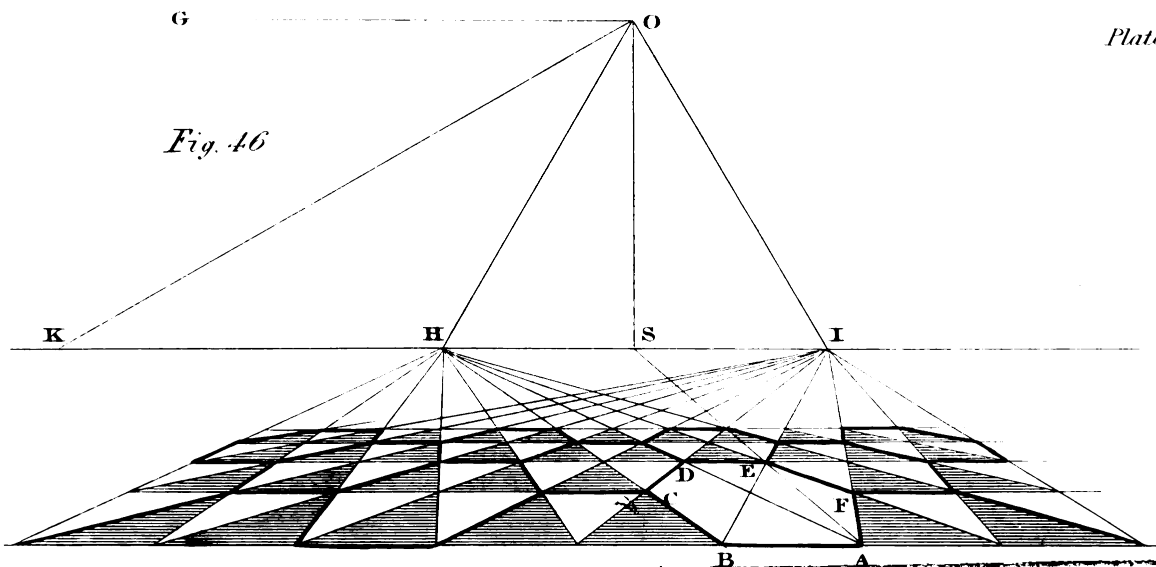

Fig. 46

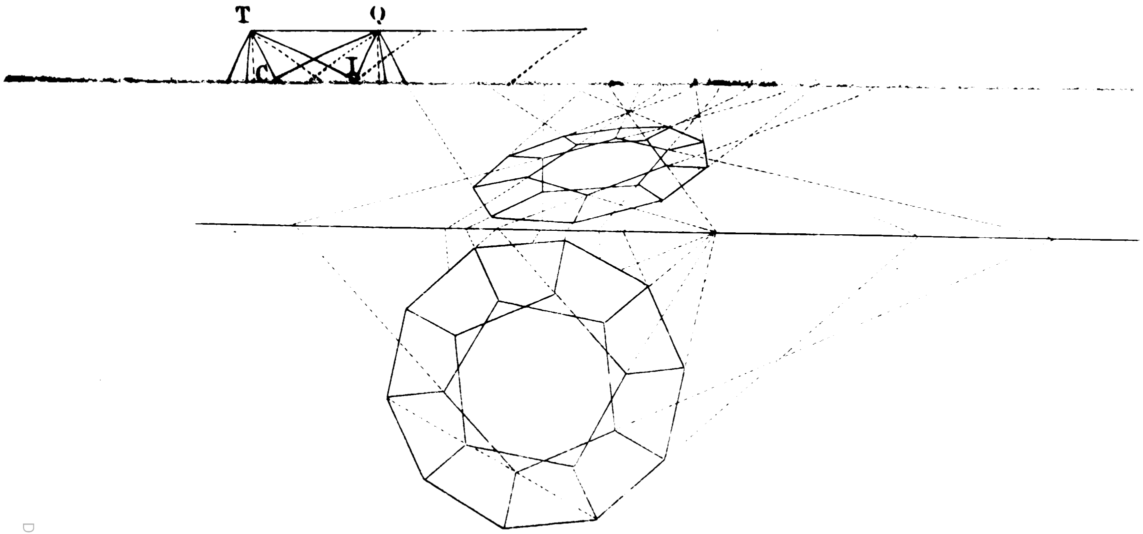
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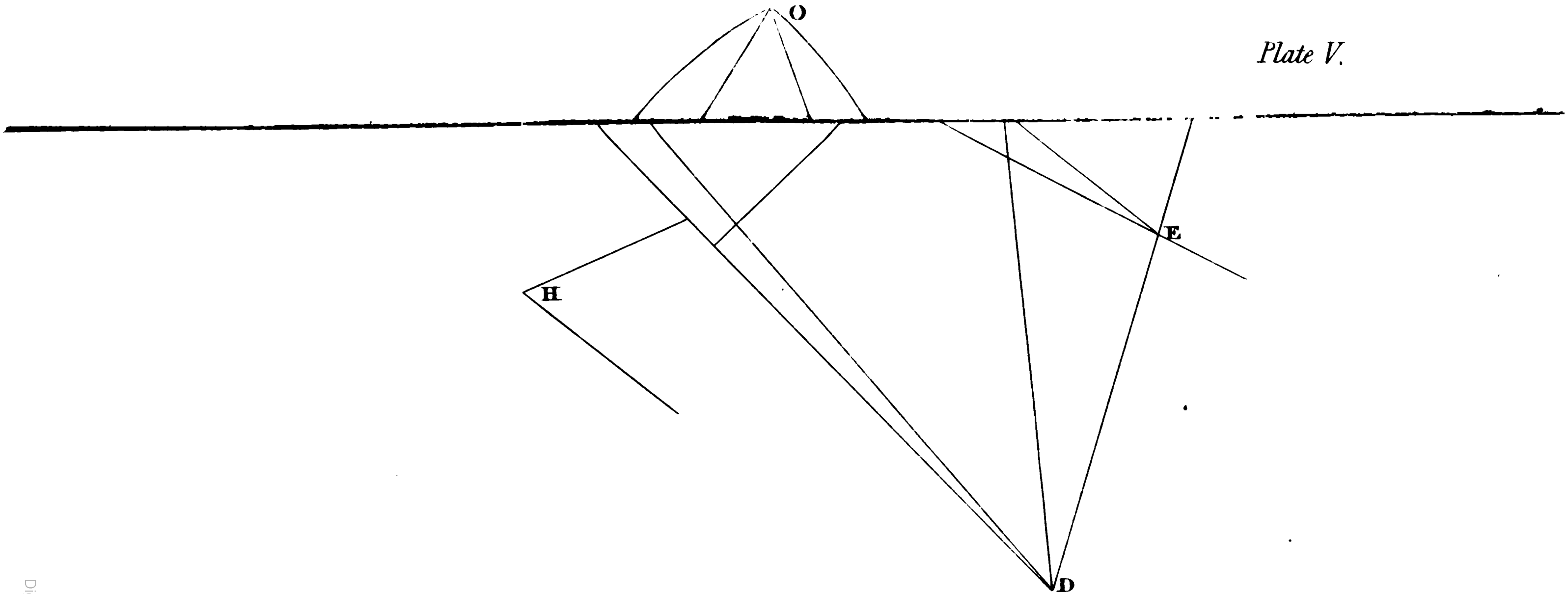
Plate V.



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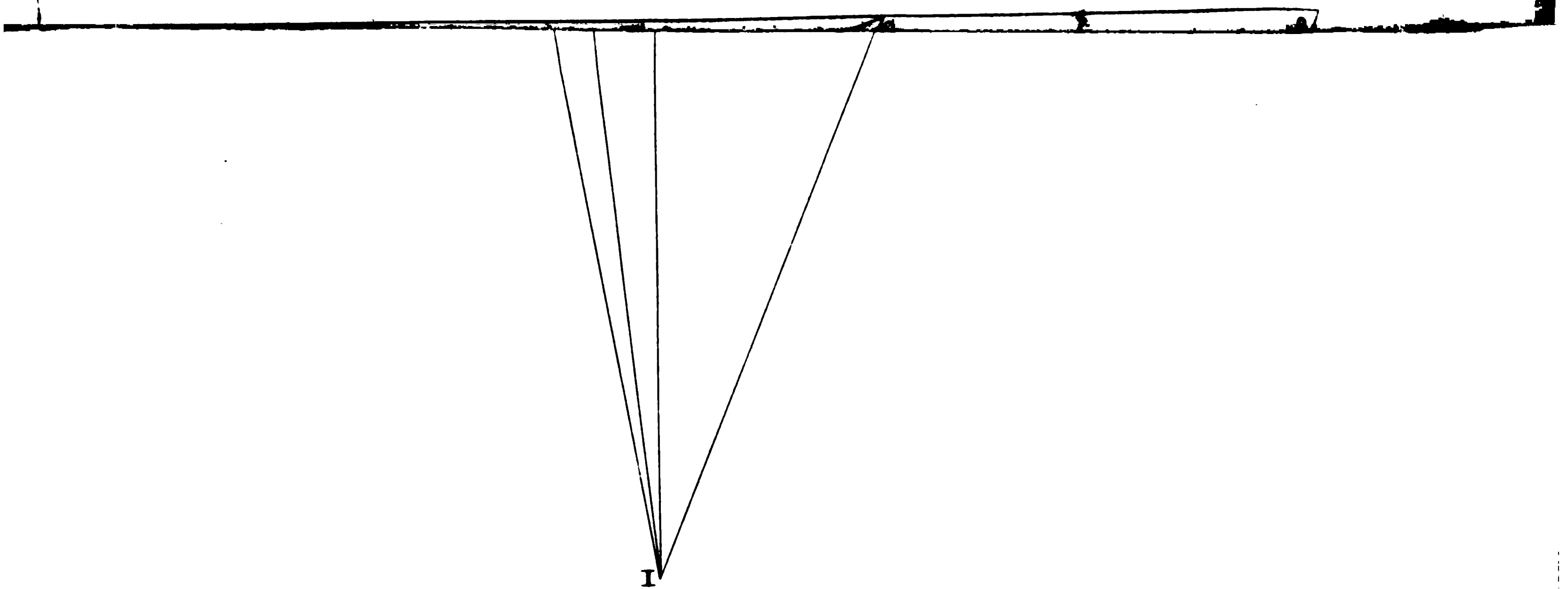

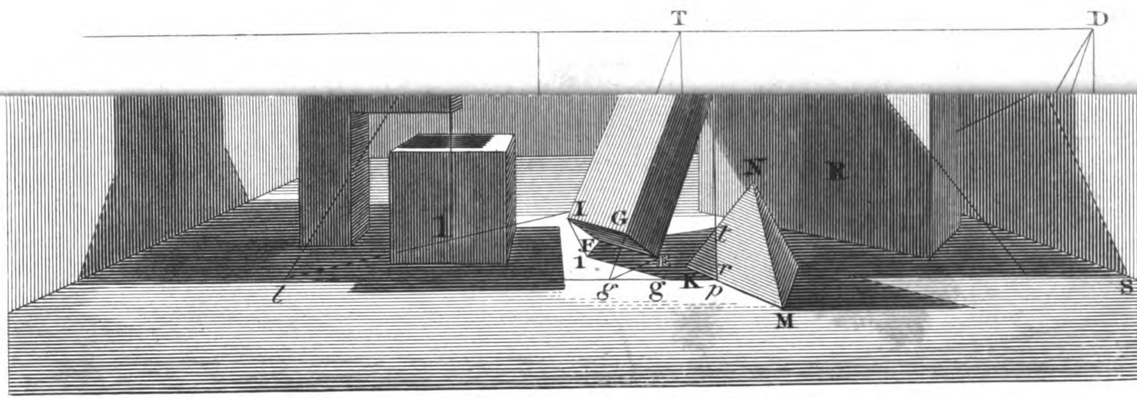
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Plate II.

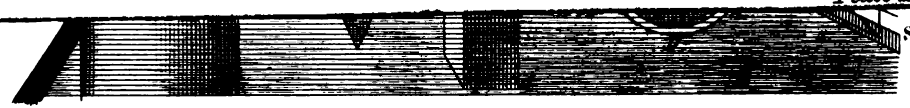


Plate X.

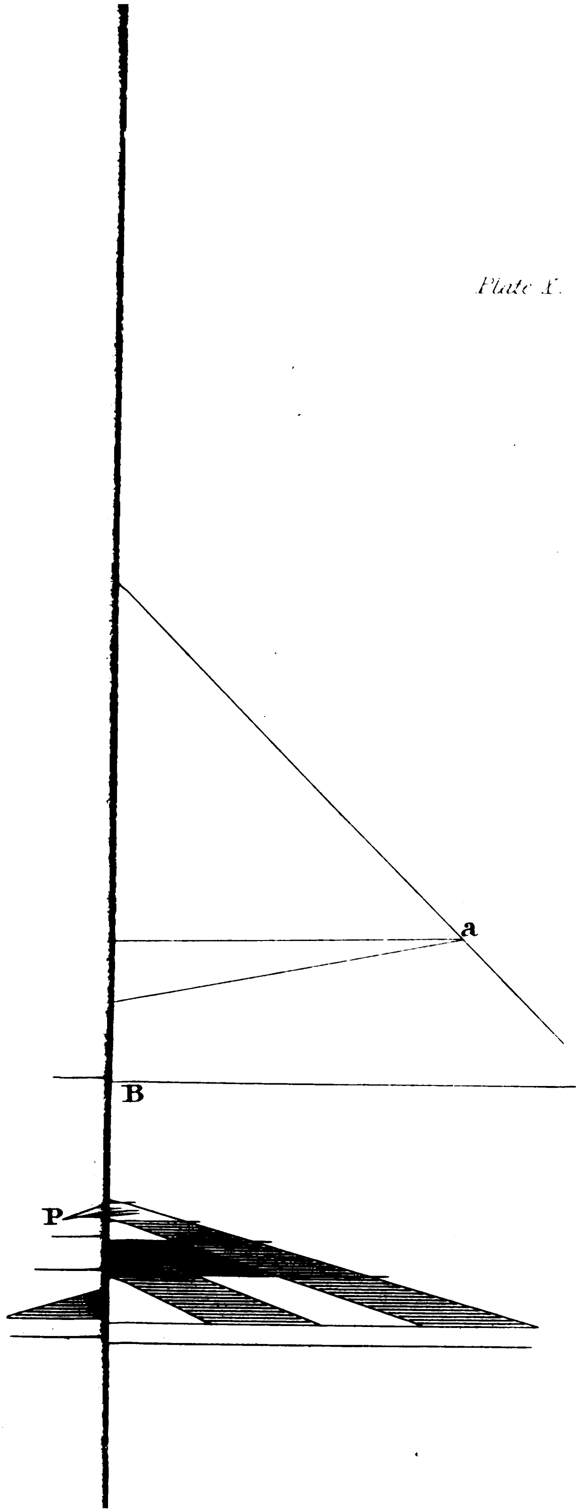


Plate XI.

Fig. 56

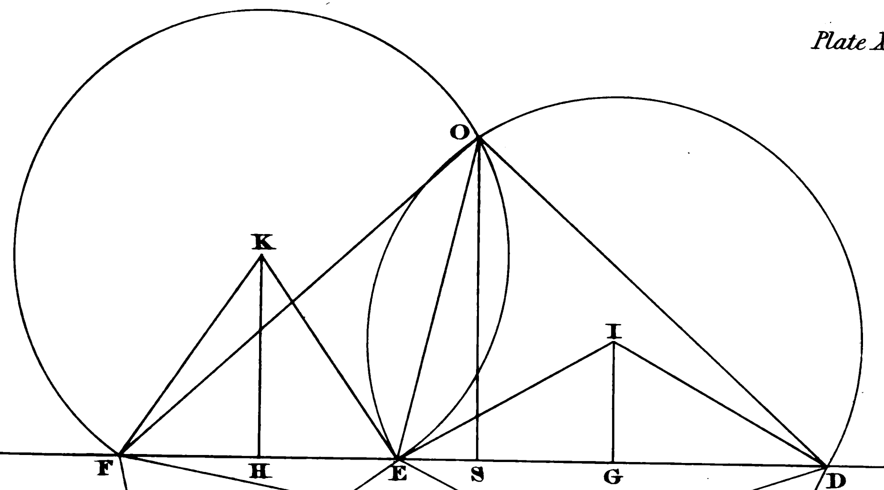
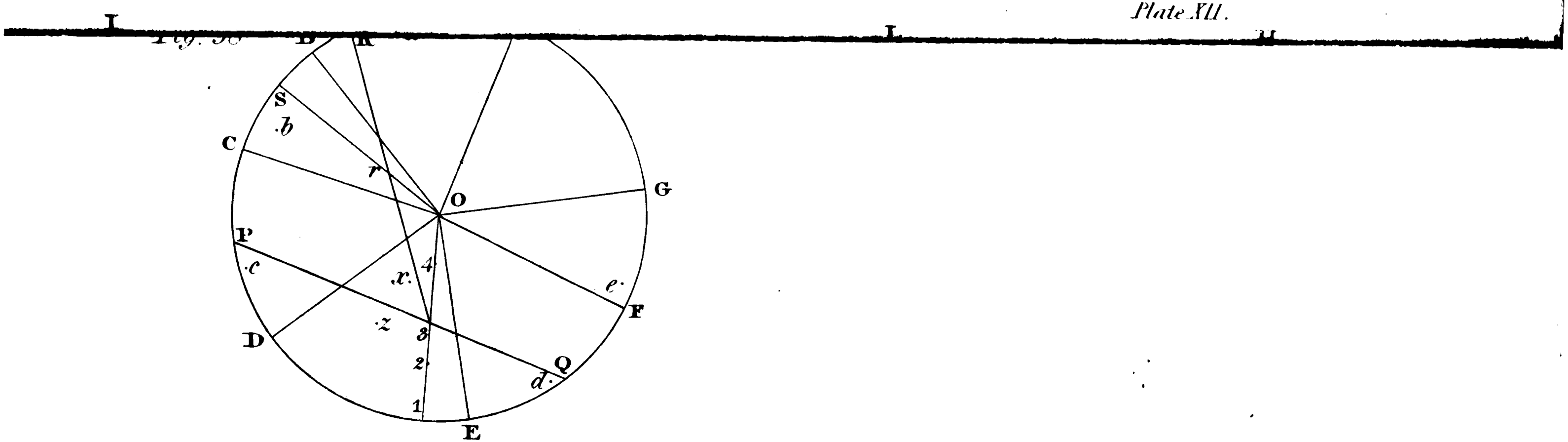


Plate XII.



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Taylor, Brook, 1685-1731

Dr. Brook Taylor's Principles...

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